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Abstract

I derive a theoretical model of choice bracketing from two behavioral axioms in an expected utility framework. The first behavioral axiom establishes a direct link between narrow bracketing and correlation neglect. The second behavioral axiom identifies the reference point as the place where broad and narrow preferences are connected. In my model, the narrow bracketer is characterized by an inability to process changes from the reference point in different dimensions simultaneously. As a result, her tradeoffs between dimensions are distorted. While she disregards interactions between actual outcomes, she appreciates these interactions mistakenly with respect to the reference point.

JEL codes: D03, D11, D91
Keywords: narrow bracketing, correlation neglect, reference dependence, axiomatic foundation

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1 Introduction

The amount of decisions that we face and the interdependencies between all of these decisions force us to apply a simplified view of the world. We isolate decisions from one another to be able to make them at all. Following Read et al. (1999b) this mental procedure is referred to as choice bracketing. A decision maker who assesses all of her decisions jointly to find the optimal combination is referred to as a broad bracketer. A narrow bracketer takes some or all of her decisions in isolation, disregarding their interdependencies. As a result, the combination of decisions that a narrow bracketer makes is rarely optimal.

Empirical and experimental evidence shows that narrow bracketing affects behavior in many important economic settings, potentially causing substantial inefficiencies. For example, in labor supply decisions, narrow bracketing is associated with negative wage elasticities (Camerer et al., 1997) and distortions of willingness to work (Fallucchi and Kaufmann, 2021). In decision-making under risk, it yields first-order stochastically dominated choices (Rabin and Weizsäcker, 2009; Tversky and Kahneman, 1981).[1] In consumption decisions, it is linked to observations of non-fungibility of money (Hastings and Shapiro, 2018; Abeler and Marklein, 2017) and diversification bias (Read and Loewenstein, 1995). And, the endowment effect in market trade is commonly explained by narrow bracketing in combination with loss-aversion (Kahneman et al., 1990).[2] Furthermore, on a methodological level, research in experimental economics oftentimes relies on the implicit assumption that subjects narrowly bracket their decisions in the laboratory, disregarding all decisions outside of the laboratory.[3]

Despite ample evidence of both prevalence and relevance of narrow bracketing, we still lack a generally applicable theoretical model of this important behavioral bias. Providing such a model is the main contribution of my paper. In particular, my model resolves two major shortcomings of the existing literature. First, virtually across the board, existing models of choice bracketing are restricted to unidimensional (monetary) outcomes. While covering a large set of relevant applications, including, for example, portfolio choice[4] and retirement savings decisions[5], these models cannot be applied to settings where outcomes are multidimensional. This excludes a variety of important applications ranging from basic consumption basket choice to complex multiattribute negotiations. Second, most existing theoretical accounts of narrow bracketing are incompatible with the basic economic principle of budget balance[6]. Inspired by the experimental literature on choice bracketing, they restrict attention to settings in which a subject's choice in one intermediate decision has no influence on the choices available to her in any other intermediate decision[7]. Thereby, they circumvent a major obstacle towards formalizing narrow bracketing: While nar-

[1] On a related theoretical note, Mu et al. (2020) show that the common observation of risk-aversion for small-stake gambles is, under the assumption of broad bracketing, incompatible with respecting stochastic dominance.

[2] Adding to these examples, in portfolio choice, narrow bracketing is connected to under-diversification (Kumar and Lim, 2008) and reduced willingness to take risk (Thaler et al., 1997; Gneezy and Potters, 1997). In retirement savings decisions, narrow bracketing is associated with under-annuitization (Brown et al., 2008) and misallocation of contributions in 401(k) plans (Choi et al., 2009). In intertemporal decisions, narrow bracketing fosters the choice of vices over virtues (Read et al., 1999a), while it can help overcome self-control problems (Koch and Nafziger, 2020).

[3] Andreoni et al. (2018) provide evidence for the validity of this assumption in the context of intertemporal choice.

[4] See Barberis and Huang (2009); Barberis et al. (2006); Benartzi and Thaler (1995)

[5] See Brown et al. (2008); Choi et al. (2009)

[6] Zhang (2021) and Barberis and Huang (2009) provide notable exceptions. I discuss these papers in detail in the following section.

[7] See, e.g., Ellis and Freeman (2020); Rabin and Weizsäcker (2009); Tversky and Kahneman (1981)

row bracketing is associated with isolating intermediate decisions from one another, reasonably satisfying budget balance requires making tradeoffs across these intermediate decisions.

Building on a choice-theoretic foundation in the context of expected utility, I derive a model of choice bracketing from basic behavioral regularities. The model is highly tractable and applicable to a large variety of economic settings. It is the first theoretical model of choice bracketing that allows for multidimensional outcomes. Capturing narrow bracketing within the framework of simultaneous decision-making, I am able to resolve the general incompatibility of narrow bracketing and budget balance. My model can, therefore, be used to predict the direction of and extent to which the behavior of a narrow bracketer departs from the behavior of a fully rational decision maker in environments with scarce resources and complex interactions between outcome dimensions. Finally, my model can be used to derive meaningful predictions for the behavior of a narrow bracketer who is not loss-averse at the same time, isolating the two behavioral biases from one another.

Consider, as a leading example, the labor supply decisions of soon-to-be parents. Before their baby arrives, they have to decide how many hours each of them will work in the first year of their baby's life in order to notify their employers about any parental leave requirements. Likely, at this point, they also have to make a rough plan for the hours each of them will work in the second year of their child's life to arrange childcare in due course. These are two in themselves complex decisions with non-trivial and potentially uncertain outcomes. But, when combined, they interact in ways that make treating them in isolation, as a narrow bracketer would, likely to cause substantial misallocation of resources. For example, in many professions, there are complementarities between the hours worked at any point in time with more hours today resulting in, e.g., higher wages, more fulfilling responsibilites, and more flexibility tomorrow. Furthermore, time is a scarce resource. Most likely, these soon-to-be parents face a binding constraint reflecting their overall time available over the course of two years as well as varying prices associated with hours worked at different points in time. While this paper will not, by any account, provide a comprehensive analysis of the described example, it is these kinds of problems with both non-trivial interactions between intermediate decisions and budgetary constraints that my model is particularly suited to be applied to.

A decision maker (DM) faces a series of intermediate decisions. Together, these intermediate decisions comprise the prospect she receives. A prospect is a probability distribution on a multidimensional outcome set. Each prospect is decomposed into several subprospects representing the intermediate decisions. There is one subprospect for each dimension of the outcome set. The subprospect corresponding to a given dimension of the outcome set is the marginal distribution on that dimension induced by the prospect it comprises. Using the multidimensionality of the outcome set to break down DM's overall decision problem into intermediate decisions in this way is novel in relation to the literature. It allows me to model narrow bracketing within the standard framework of simultaneous decision-making. While this means that my model cannot capture potential differences in behavior with respect to the sequence of intermediate decisions, it ensures that tractability remains uncompromised.

DM is characterized by two preference relations on prospects. Her *broad preference relation* captures her "true" preferences. If DM brackets broadly, she makes choices in line with her broad preference relation. If DM brackets narrowly, her choices are governed by her *narrow preference* *relation* instead. DM's narrow preference relation is characterized by a *system of brackets*. The system of brackets partitions the subprospects that make up an overall prospect into distinct groups (brackets). I take the system of brackets as given.[8] It determines the degree to which DM brackets narrowly. While a fully narrow DM puts each subprospect into a distinct bracket, a fully broad DM has only one bracket including all subprospects that comprise the overall prospect.

I derive a representation for DM's narrow preference relation from her broad preference relation and two behavioral axioms. I do so in the framework of expected utility. My first behavioral axiom specifies the mistake that a narrow bracketer makes. It identifies *correlation neglect* as the central flaw of narrow decision making. A narrow DM considers the subprospects inside a given bracket in isolation, disregarding all subprospects outside of that bracket. Of course, if these other subprospects are entirely independent of the considered subprospects, there is no harm done in disregarding them. If, however, these other subprospects are correlated with the considered subprospects or there are important interdependencies between the subprospect outcomes, disregarding them becomes a problem. Correlation neglect is not only a well-documented behavioral trait in general (see, e.g., Rees-Jones et al., 2020; Enke and Zimmermann, 2019; Eyster and Weizsäcker, 2016), it is also at the very heart of the behavior attributed to narrow bracketing in Tversky and Kahneman (1981)'s seminal study on the topic. Nevertheless, the two behavioral biases are mostly treated as separate phenomena in the literature.[9] My axiomatization clarifies their close connection and makes their overlap explicit.

My second behavioral axiom ties the narrow preference relation to its broad couterpart. The broad and narrow preference relations belong to one and the same DM. While the one captures DM's true preferences, the other captures the choices she makes. Therefore, the narrow preference relation may depart from the broad preference relation only if that departure can be rationalized by DM's bracketing behavior. In principle, a narrow bracketer disregards all interdependencies between subprospects across brackets. I assert, however, that the narrow bracketer is not entirely ignorant with respect to these across-bracket interdependencies. I assume that there exists a specific outcome, which I call the *reference point*,[10] at which she retains her ability to process all brackets simultaneously. Intuitively, the reference point captures an outcome that DM is used to and therefore comfortably able to keep the overview of. As long as changes from the reference point happen in only one bracket at a time, narrow and broad preference relation must agree with each other. On the flip side, disagreements of broad and narrow preferences may occur only between prospects that differ from each other and the reference point in more than one bracket.

The derived expected utility representation of the narrow preference relation is additively separable across brackets. The narrow bracketer's expected utility from a given prospect can be decomposed into a sum of expected utilities from its bracketwise subprospects. Additive separability is implied by my correlation neglect axiom. The axiom that ties the narrow preference relation to its broad counterpart via the reference point imposes further structure on the narrow bracketer's

[8]For models of endogeneous bracket formation in the context of intertemporal decision making see, e.g., Galperti (2019); Hsiaw (2018); Koch and Nafziger (2016). Relatedly,Kőszegi and Matějka (2020) present a model of how people form mental budgets.

[9]For an exception, consider Zhang (2021).

[10]The concept of a reference point was introduced by Kahneman and Tversky (1979) in the context of prospect theory. In that model, the reference point determines whether and to what extent an outcome is perceived as a gain or a loss. There is no guarantee that a reference point in that sense must overlap with what my axiom specifies. Nevertheless, I adopt this terminology because of the shared intuition of the reference point as a focal outcome which potentially influences a decision maker's choices.

bracketwise expected utilities. For a given bracket, the expected utility function of the narrow bracketer is equivalent to the broad bracketer's expected utility function with all outside-bracket outcomes fixed at the reference point.

My representation theorem (Theorem 1) reveals that, when evaluating a prospect, the narrow bracketer can be modeled as using the same expected utility function as the broad bracketer. However, she applies that expected utility function separately to each bracket in her system of brackets. For each bracket, she evaluates the broad expected utility function at the subprospects inside that bracket while keeping all other subprospects fixed at the reference point. Finally, she takes the sum of all of these bracketwise expected utilities. As a result, the narrow bracketer disregards any interactions between subprospects across brackets. However, she appreciates these interactions mistakenly with respect to her reference point.

My model of choice bracketing is simple in the sense that the derived representation of the narrow preference relation can be treated in exactly the same way as any broad expected utility representation. We can, thus, use the standard economics toolbox and the large body of existing results from microeconomic theory to study the choices of a narrow bracketer. In particular, the model can be applied to standard constrained (expected) utility maximization problems. As discussed above, narrow bracketing is not readily compatible with the principle of budget balance. While narrow bracketing is associated with a decision maker's inability to think multidimensionally, budget balance requires her to make tradeoffs between dimensions. My model resolves this incompatiblity of narrow bracketing and budget balance by introducing the reference point. At the reference point, the narrow bracketer retains her ability to think multidimensionally. However, since she is unable to process changes from the reference point in different dimensions simultaneously, her tradeoffs between dimensions are distorted.

To demonstrate the effects that narrow bracketing has on behavior in basic economic settings, I apply my model to the economics 101 constrained utility maximization problem with two goods. Although my model is readily applicable to decision-making under risk, analyses of Barberis and Huang (2009) and Barberis et al. (2001) have already demonstrated how additive separability of the narrow bracketer's utility function implies the choice patterns observed in these settings with unidimensional (monetary) outcomes. Instead, my application focuses on the simplest and most intuitive setting involving multidimensional outcomes, a setting that existing models do not accommodate. While, in the spirit of classic consumer choice, you may think of the two goods in my application as, say, apples and bananas, they can just as well represent a soon-to-be parent's decisions about the hours she/he wants to work in the first and, respectively, second year of her/his child's life.

Additive separability of the narrow preference representation means that any interactions between the two goods in her bundle are disregarded by the narrow bracketer. This disregard is nicely illustrated by the shape of the narrow indifference curves in comparison to their broad counterparts as summarized in Proposition 1. If the goods have negative interactions akin to substitutabilities, the narrow indifference curves are more convex than their broad counterparts. If the goods have positive interactions akin to complementarities, the narrow indifference curves are less convex than their broad counterparts. Intuitively, the more convex an indifference curve, the more complementary are the two goods. Thus, a narrow bracketer regards two substitutable goods as more complementary than they actually are and vice versa for two complementary goods. Returning to our soon-to-be parent, if she/he brackets narrowly, it will seem to her/him like the hours worked in the first year of her child's life are less complementary to the hours worked in the second year of her child's life than they will eventually turn out to be.

While disregarding interactions for the bundle she chooses, the narrow bracketer is, however, not fully ignorant of their existence. She mistakenly appreciates the interactions separately for each good dimension with respect to her reference point in the respective other dimension. In other words, the narrow bracketer does not consider changes from the reference point for the two goods simultaneously. When thinking about an alteration of her bundle away from the reference point in one good dimension, she keeps the respective other good dimension fixed at its reference point level. As a result, the tradeoffs she makes are distorted. Proposition 2 clarifies how the direction and severity of the resulting divergence between chosen and optimal bundle depends on the reference point. Returning to our soon-to-be parent once again, due to, for example, past work experience, she/he might have the reference point of an equal division of hours worked across the two years. However, with big changes ahead, this reference point is unlikely to remain optimal in the future. Suppose the parent's optimal division of hours across the coming two years is slightly imbalanced, say, 40/60. This division optimally trades off the parent's wish to spend more time away from work in the first year with the implications this has for her/his work-related outcomes due to positive interactions between the hours worked across years. According to Proposition 2, the narrow bracketing parent's decision, however, will be distorted towards an overly imbalanced division because she/he mistakenly judges the change away from the reference point for each of the two years in isolation.

Finally, bridging my results to applications involving negotiations, I study the implications of choice bracketing in an Edgeworth-box exchange economy assuming status-quo reference points (Proposition 3). I find that, starting from any initial endowment allocation, in the case of positive interactions the volume of trade is higher if the trading parties bracket narrowly. In contrast, in the case of negative interactions narrow bracketing results in a lower volume of trade. This result has important implications for how the procedures of negotiations affect their outcomes. Especially, it calls into question the general practice of splitting up multidimensional negotiations, negotiating every aspect of a deal separately, since this might induce the involved parties to bracket narrowly. Returning to our example one last time, now focusing on the intra-household negotiations associated with both parents' labor supply decisions, these results suggest that under narrow bracketing the resulting distribution of hours worked in the first two years will turn out to be overly unequal, with both parents mutually agreeing to a distribution that is not in their best interest.

The paper proceeds as follows. After a discussion of the related literature, I present the model and its foundations in Section 2. In Section 3, I derive predictions of my model for constrained utility maximization and an exchange economy. Section 4 concludes.

1.1 Related literature

Ellis and Piccione (2017) present a related axiomatic analysis of correlation misperception in portfolio choice. In an otherwise standard expected utility framework, the authors suggest a weakened monotonicity axiom, allowing the decision maker to hold arbitrary beliefs about the correlations between assets. Their representation boils down to subjective expected utility using a probability distribution of portfolio returns based on potentially misspecified beliefs about asset correlations. Akin to the system of brackets in my model, they characterize the decision maker by a system of *understanding classes* of assets such that correlations are correctly perceived within but misperceived across classes. In contrast to Ellis and Piccione, I capture correlation neglect within full-fledged expected utility. This is possible because of the explicit distinction between broad and narrow preferences. Intuitively, the narrow bracketer's correlation neglect is absorbed by her misconstrued notion of utilities associated with outcomes, leaving the perception of probabilities intact.[11] Further, in contrast to Ellis and Piccione's focus on monetary outcomes, I indroduce correlation neglect in a framework with multidimensional outcomes. This allows me to study its implications beyond the misperception of probability distributions, demonstrating additional misperception of interactions between outcome dimensions.

Zhang (2021) axiomatizes narrow bracketing and correlation neglect with respect to the evaluation of risk from two different sources. He characterizes full narrow bracketing as evaluating a utility function at two separate certainty equivalents, one for each of the two marginal distributions over outcomes induced by the two sources of risk. Similar to my model, Zhang's formulation of full narrow bracketing subsumes correlation neglect. He further presents a model of asymmetric narrow bracketing which does not imply correlation neglect. By requiring the existence of dimension-wise certainty equivalents, Zhang's model implicitly relies on the assumption that his two outcome dimensions can be meaningfully collapsed into one. As a result, it does not accommodate the rich preference interactions between dimensions studied here. Furthermore, by distinguishing between broad and narrow preferences, my model departs from Zhang's in providing a benchmark with respect to which the gravity of a narrow bracketer's mistakes beyond violations of stochastic dominance can be evaluated.

Camara (2021) relates narrow bracketing to computational tractability. In a closely comparable theoretical framework, he shows that for the maximization of expected utility to be computationally feasible in a reasonable amount of time, the utility function has to satisfy a weakened version of additive separability. Thus, Camara's results provide a behavioral rationale for exactly the type of narrow bracketing modeled in this paper: adhering to expected utility under the simplifying assumption that utility is (partly) additively separable. Interestingly, Camara goes on to show that such behavior is far from optimal under tractability constraints. That is, there exist tractable decision algorithms that can generate significantly higher payoffs. This result demonstrates that narrow bracketing is likely to be suboptimal not only compared to (potentially infeasible) broad bracketing but continues to be so once we impose reasonable constraints on cognitive ability.

Barberis and Huang (2009) present a theoretical model of choice bracketing for unidimensional outcomes.[12] Similar to my approach, they remedy the incompatibility of narrow bracketing and budget balance by assuming that the narrow bracketer evaluates her (broad) utility function separately for each intermediate decision and then maximizes the sum of all these individually evaluated utilities. My analysis contributes to theirs in three respects. First, it provides a choice theoretic foundation for the additive formulation of Barberis and Huang. Second, it extends the

[11] On a similar note, Yaari (1987) demonstrates a dual approach to modeling risk aversion, via probability weighting instead of utility curvature as in expected utility.

[12] The model extends and improves on an earlier version presented in Barberis et al. (2001) and has been used to study choice bracketing in applications including portfolio choice (Barberis and Huang, 2009; Barberis et al., 2006; Benartzi and Thaler, 1995), asset pricing (Barberis and Huang, 2001; Barberis et al., 2001), and self-control problems (Koch and Nafziger, 2016; Hsiaw, 2018).

set of possible applications considerably by allowing for multidimensional outcomes. Third, by explicitly modeling the system of brackets, it allows for more subtle forms of partial narrow bracketing. Barberis and Huang capture partial narrow bracketing through a global-plus-local utility function. That is, they model the partial narrow bracketer as evaluating a weighted average of broad and fully narrow utility. While this formulation is simpler than my approach of capturing partial narrow bracketing via the system of brackets, this simplicity comes at a cost. It blurs the very basic intuition of choice *bracketing* and does not allow for investigations of the effects that a change in the system of brackets has on the behavior of a narrow bracketer.

Experimental results of Ellis and Freeman (2020) provide empirical evidence that the costs of simplicity as imposed by the global-plus-local formulation of Barberis and Huang (2009) may outweigh its benefits. The authors propose revealed-preference conditions for testing the consistency of behavior with different forms of bracketing. They apply their tests using decision problems involving two intermediate decisions within three different contexts, spanning portfolio choice, social allocations, and consumption choice. Across all contexts, virtually none of their subjects can be classified as partial narrow bracketers as modeled by Barberis and Huang. At the same time, the authors find ample evidence of full narrow bracketing. In particular, $40-44\%$ of their subjects behave consistently with full narrow bracketing while only $0-15\%$ are consistent with broad bracketing. These results are in line with my model in which full narrow bracketing is the only type of narrow bracketing that can occur in the studied two-dimensional decision problems.

In the context of consumer choice, my analysis of choice bracketing reveals a tight connection to budgeting, which besides narrow bracketing is another aspect of mental accounting as outlined by Thaler (1999). Budgeting describes the intuitively appealing idea that, when choosing a large consumption basket, the consumer follows a two-stage procedure (Gilboa et al., 2010). In the first stage, the budgeting stage, she optimally distributes her budget across general categories of goods like clothing, food, and entertainment. Then, in the second stage she decides separately for each good category how to allocate her category budget from the first stage across the individual goods belonging to that category. Such a budgeting procedure is generally admissibile if and only if the utility function is additively separable across good categories (Gorman, 1959; Strotz, 1957, 1959). Thus, additive separability of my narrow preference representation implies that a narrow bracketer can be interpreted as using the described budgeting procedure although her broad preferences do not allow it. This interpretation is in line with what Blow and Crawford (2018) call "pure mental accounting", i.e. budgeting with good categories across which (broad) preferences are not separable. The nonparametric conditions that the authors provide to test consistency of such mental accounting with consumer choice data can, therefore, serve as necessary, albeit not sufficient, conditions for consistency with my model of narrow bracketing as well.

Staying within consumer choice, a recent related literature shows how a consumer's limited attention to price or preference shocks provokes behavior akin to the narrow consumer's behavior in my model. For different definitions of limited attention, papers by Kőszegi and Matějka (2020), Lian (2020), and Gabaix (2014) show that in reaction to such a shock in one good dimension, the inattentive consumer behaves as if she (partially) disregards interactions of that good with the other goods in her bundle. I model narrow bracketing more directly without taking a stance on the origins of the associated behavior. In contrast to the models based on limited attention, my model has bite also in settings with perfect information on prices and preferences. Indeed, experimental evidence suggests that narrow bracketing readily occurs even in such deterministic settings (see, e.g., Ellis and Freeman, 2020; Rabin and Weizsäcker, 2009).

2 The model

2.1 Theoretical framework

The outcome set X is a Cartesian product $\prod_{i \in I} X_i$. I is a finite set $\{1, 2, ..., n\}$ indexing the dimensions of an outcome $x \in X$. Let $\mathcal{P}$ denote the set of all finite discrete probability distributions on the set of all subsets of X. A *prospect* $P \in \mathcal{P}$ is a probability distribution over the multidimensional outcomes assigning to each outcome $x \in X$ its probability $P(x)$. If $P \in \mathcal{P}$, then $0 \leq P(x) \leq 1$ for all $x \in X$ and $\sum_{x \in X} P(x) = 1$.

The domain of preference is the set of all prospects. A decision maker (DM) is characterized by two preference relations on the set of prospects. Her *broad preference relation* $\succcurlyeq_b$ and her *narrow preference relation* $\succcurlyeq_n$. Consider prospects $P, Q \in \mathcal{P}$. [$P \succcurlyeq_b Q$] indicates that P is weakly preferred to Q according to $\succcurlyeq_b$. As usual, [$P \succ_b Q$] indicates [$P \succcurlyeq_b Q$ and not $P \succcurlyeq_b Q$] while [$P \sim_b Q$] indicates [$P \succcurlyeq_b Q$ and $P \succcurlyeq_b Q$]. The indications apply analoguously to $\succcurlyeq_n$.

I interpret DM's broad preference relation as capturing her true preferences in the sense that if she brackets broadly, her choices are in line with $\succcurlyeq_b$. If DM brackets narrowly, her choices may not be in line with her true preferences. I interpret $\succcurlyeq_n$ as the preference relation that governs the narrow DM's choices.

Assumption 1 (Richness)**.** Every probability distribution over outcomes that takes only finitely many values is available in the preference domains of $\succcurlyeq_b$ and $\succcurlyeq_n$.

So far, my theoretical framework closely follows the literature on multiattribute utility theory (see e.g. Keeney and Raiffa, 1993; Fishburn, 1965, 1967). To accomodate the idea of choice bracketing, I now carry the multiattribute nature of outcomes over to the prospect that generates them.

Let $\mathcal{P}_i$ be the set of all finite probability distributions on X_i. For every prospect $P \in \mathcal{P}$ there exists an element $P_i \in \mathcal{P}_i$ which is the marginal distribution on X_i induced by P. Refer to P_i as *subprospect i* of prospect P. Any prospect $P \in \mathcal{P}$ is thus associated with a collection of subprospects corresponding to its outcome dimensions, $(P_1, P_2, ..., P_n)$.

The decomposition of prospects into subprospects captures that a DM's overall decision for a specific prospect is the result of several intermediate decisions. In each intermediate decision, DM chooses a subprospect. Taken together, these subprospects then generate the overall prospect. In the multidimensional outcome arising from this prospect, each dimension represents the outcome of one subprospect.

As long as DM brackets broadly, i.e. makes choices in line with $\succcurlyeq_b$, the above decomposition of prospects is redundant. A broad bracketer chooses the same prospect independent of whether this choice is the result of just one or several intermediate decisions. A narrow bracketer, however, does not keep track of the interdependencies between all intermediate decisions. Therefore, a narrow bracketer's overall decision for a specific prospect may depend on whether it is decomposed into subprospects or not.

In its most extreme form, narrow bracketing means that DM decides about each subprospect in isolation, disregarding its interdependencies with any other subprospect she chooses. I allow for less extreme forms of narrow bracketing in which DM retains her ability to process subsets of her intermediate decisions jointly. Therefore, I define a *system of brackets* characterizing the narrow preference relation. The system of brackets partitions the collection of subprospects that generate the overall prospect into distinct groups (brackets).

The *system of brackets* B characterizing $\succcurlyeq_n$ is a set $\{B_1, B_2, ..., B_m\}$ of nonempty subsets of the outcome dimension index set I with $\bigcup_{j=1}^{m} B_j = I$. We refer to B_j as *bracket* j of the system of brackets B. Let $\mathcal{P}^j$ be the set of all finite discrete probability distributions on the set of all subsets of the outcome set in bracket B_j, $X^j := \prod_{i \in B_j} X_i$. For every prospect $P \in \mathcal{P}$ there exists an element $P^j \in \mathcal{P}^j$ which is the marginal distribution on X^j induced by P. We refer to P^j as the *jth bracket prospect* of P. Given a system of brackets B, each prospect P induces a collection of bracketwise prospects, $(P^1, P^2, ..., P^m)$, and each outcome x can be written as a collection of bracketwise outcomes, $x = (x^1, x^2, ..., x^m)$ where $x^j = (x_i)_{i \in B_j}$ for $j = 1, 2, ..., m$.

When a prospect $P \in \mathcal{P}$ is deterministic, i.e. $P(x) = 1$ for some $x \in X$, I refer to that prospect directly by its outcome x. Similarly, I refer to a deterministic subprospect $P_i \in \mathcal{P}_i$ by its outcome $x_i \in X_i$ and to a deterministic bracketwise prospect $P^j \in \mathcal{P}^j$ by its bracketwise outcome $x^j \in X^j$.

Given two prospects $P, Q \in \mathcal{P}$, denote by $(P^j, Q^{-j}) \in \mathcal{P}$ the prospect generated by combining the jth bracket prospect P^j of P with all but the jth bracket prospects of Q. Given two outcomes $x, y \in X$, denote by $(x^j, y^{-j}) \in X$ the outcome that combines the jth bracket outcome, x^j, in x with all but the jth bracket outcomes in y.

In gerneral, you can think of the multidimensional nature of outcomes in my framework in two ways. First, in line with what is normally thought of in the multiattribute utility literature, the outcomes of different subprospects may as such be qualitatively different from one another, naturally giving rise to a multiattribute formulation. For example, the overall outcome could be a consumption basket which is comprised of many individual goods, the different outcome dimensions, each of which was individually put into the basket by DM on her way through the supermarket. Second, capturing the possibility of narrow bracketing in cases where outcomes do not have a multiattribute nature as such, I allow for a distinction between outcome dimensions that are qualitatively the same but are the result of distinct intermediate decisions. For example, the overall outcome could be total money earnings from a portfolio comprised of the earnings from a collection of assets, the outcome dimensions, each of which was puchased individually by DM.

2.2 Axiomatic foundation

In the following, I derive a utility representation for the narrow preference relation $\succcurlyeq_n$ from the broad preference relation $\succcurlyeq_b$. I do so in the framework of expected utility (EU), implicitly assuming that the axioms underlying the EU representation are fulfilled for each of the two preference relations $\succcurlyeq_b$ and $\succcurlyeq_n$.[13]

[13] For axiomatizations of EU see, for example, Fishburn (1970) and Wakker (2010).

Assumption 2 (EU)**.**

(1) There exists a function $u : X \to \mathbb{R}$, the *broad utility function*, such that for all prospects $Q, R \in \mathcal{P}$, $Q \succcurlyeq_b R \Leftrightarrow EU(Q) \geq EU(R)$ with $EU(P) := \sum_{x \in X} P(x)u(x)$. u is unique up to positive affine transformation.

(2) There exists a function $\tilde{u} : X \to \mathbb{R}$, the *narrow utility function*, such that for all prospects $Q, R \in \mathcal{P}$, $Q \succcurlyeq_n R \Leftrightarrow \widetilde{EU}(Q) \geq \widetilde{EU}(R)$ with $\widetilde{EU}(P) := \sum_{x \in X} P(x)\tilde{u}(x)$. $\tilde{u}$ is unique up to positive affine transformation.

My approach for finding a utility representation of the narrow preference relation proceeds as follows. I ask myself two basic questions about the behavior of a narrow bracketer. The answers to these questions are captured in my two behavioral axioms.[14] Together with Assumption 2 (EU), these two behavioral axioms determine the shape of the narrow bracketer's preference representation.

What is the narrow bracketer's mistake? First, I restrict attention to the narrow preference relation. The following behavioral axiom clarifies what exactly it is that the narrow bracketer misses when choosing between two prospects.

Axiom 1 (correlation neglect)**.** For any two prospects $P, Q \in \mathcal{P}$, if all bracketwise prospects induced by P and Q on the system of brackets B are the same, i.e. $P^j = Q^j$ for all $j \in \{1, 2, ..., m\}$, then $P \sim_n Q$.

Axiom 1 states that the narrow bracketer is ignorant with respect to the correlation between the bracketwise prospects that comprise an overall prospect. When making a choice between two prospects, she only considers the individual bracketwise subprospects without keeping track of the overall prospects they comprise. Therefore, any two prospects that are comprised of the same subprospects, i.e. that induce the same marginal distributions on all bracketwise outcome sets, look exactly the same to her. This holds irrespective of whether the overall prospects, i.e. the joint distributions on the overall outcome set, are the same as well. Of course, Axiom 1 only has bite in the sense that it harms the narrow bracketer if there are meaningful interactions between subprospect outcomes across brackets. Only then does the correlation structure of a prospect matter for the broad preference relation and only then does the correlation neglect axiom imply that the narrow preference relation deviates from its broad counterpart.

Axiom 1 is supported by a host of experimental studies documenting correlation neglect in economically relevant decision problems including, for example, forecasting tasks (Enke and Zimmermann, 2019), portfolio choice (Eyster and Weizsäcker, 2016), and student-to-school matching (Rees-Jones et al., 2020). From a decision-theoretic perspective, Axiom 1 is a weaker form of *marginal independence* as used in axiomatizations of additively separable multiattribute expected utility (Wakker, 2010; Fishburn, 1965). Marginal independence requires the conditions in Axiom

[14] An earlier version of this paper, which constituted my job market paper, included an experiment providing preliminary evidence for the validity of my behavioral axioms. While the main goal of the experiment was to demonstrate the empirical testability of my model more generally, its results provide first evidence in support of my axioms. However, to conclusively assess the validity of my model will require more comprehensive tests which shall be the subject of future work. For brevity, I decided to exclude the experiment from the current paper. Information about the experiment and its results can be found in the job market paper version of this paper which remains available on my personal website (paulinevorjohann.com).

1 to hold on the level of subprospects instead of bracketwise prospects. Therefore, it overlaps with Axiom 1 in the case of full narrow bracketing. Fishburn (1967) introduces an assumption equivalent to Axiom 1 to establish additive separability of multiattribute expected utility with respect to subsets of attributes. I make heavy use of the results from that paper in the proof of my representation theorem. In particular, they allow me to establish additive separability of the narrow bracketer's preference representation with respect to bracketwise outcomes.

In conjunction with EU (Assumption 2), the narrow bracketer's disregard of correlations between bracketwise subprospects as captured by Axiom 1 carries over to a general disregard of preference interactions between the dimensions of outcomes across brackets even in decisions under certainty. More precisely, a bracketwise version of *preferential independence* as defined by Keeney and Raiffa (1993) is implied once we introduce Axiom 1 to the EU framework.[15] That is, given EU and Axiom 1, the narrow bracketer's preferences over deterministic prospects (outcomes) that vary in only one bracketwise outcome are independent of the specific levels at which we fix all other outcome dimensions.

Where are broad and narrow the same? Axiom 1 pins down the narrow bracketer's mistake. I now identify the instances in which the narrow bracketer's choice should not deviate from her true preferences. The following axiom considers the connection between the narrow preference relation and its broad counterpart.

Axiom 2 (Reference Point)**.** There exists an outcome $r \in X$, the *reference point*, such that for any two prospects $P, Q \in \mathcal{P}$, if the bracketwise prospects induced by P and Q differ from each other and r in at most one bracket, i.e. $P^j = Q^j = r^j$ for all but at most one $B_j \in \{B_1, B_2, ..., B_m\}$, then $P \succsim_b Q \Leftrightarrow P \succsim_n Q$.

Axiom 2 states that there exists an outcome, the reference point, which ties together broad and narrow preference relation. At the reference point, the narrow bracketer is perfectly able to consider all brackets jointly. Furthermore, she can properly process changes from the reference point as long as they occur inside only one bracket at a time. In that sense, Axiom 2 bounds the irrationality of the narrow preference relation. It allows for departures from the broad preference relation only for prospects that differ from each other and the reference point in more than one bracket. As a result, the narrow bracketer appreciates all preference interactions between subprospects within brackets. Furthermore, she is never fully ignorant of the interactions between subprospects across brackets. In particular, at and around the reference point (as defined by Axiom 2), she fully appreciates interactions across brackets by making choices that are in line with her true (broad) preferences.

There is ample evidence of context-dependence of preferences in general and the influence of reference points on choice in particular (see, e.g., Camerer et al., 1997; Kahneman et al., 1990; Tversky and Kahneman, 1981). Most of this evidence is focused on the specific notion of reference points defined in Kahneman and Tversky (1979)'s prospect theory. In that model, the reference

[15] Preferential independence can, for example, be derived as an implication of the interplay between Axiom 1 and the classic independence axiom as well as transitivity, both of which are implied by EU (see, e.g., Fishburn, 1970). To see this, consider any two outcomes x and y such that $x \succsim_n (y^j, x^{-j})$. By independence, we have for prospects P and Q with $P(x) = P((x^j, y^{-j})) = 0.5$ and $Q((y^j, x^{-j})) = Q((x^j, y^{-j})) = 0.5$ that $P \succsim_n Q$. Now, by Axiom 1, it follows that we have for prospect R with $R(x) = R(y) = 0.5$ that $R \sim_n Q$. Thus, by transitivity, $P \succsim_n R$. Finally, by independence, $(x^j, y^{-j}) \succsim y$, establishing the bracketwise version of preferential independence.

point constitutes an outcome against which all other outcomes are evaluated to determine whether they are perceived as gains or losses. In contrast, a reference point in the sense of Axiom 2 constitutes the benchmark against which the narrow bracketer evaluates multidimensional prospects bracket-by-bracket. Arguably, these two conceptualisations of a reference point have no explicit overlap. They do, however, share the main intuition of the reference point as a focal outcome against which prospects are evaluated. Furthermore, in both capacities, the reference point can have a profound influence on a decision-maker's choices without consequentially impacting her decision problem as such.

It is worth noting here that none of my theoretical results or model predictions will rely on movements of the reference point. This is in contrast to large parts of the literature on reference-dependence in the sense of prospect theory where changes in the reference point are integral to explaining observations of preference reversals across outcome equivalent decision tasks. In that sense, my model's agnosticism about how the reference point is determined may be regarded as a more secondary concern. Essentially, the reference point will be treated as a fixed parameter in the narrow bracketer's preference representation and can, as such, be identified from suitable choice data. Rather, any choice-inconsistencies of interest will be captured by differences in how subprospects are bracketed, i.e. via changes in the system of brackets.

2.3 Representation theorem

I am now ready to state my representation theorem for the narrow preference relation.

Theorem 1 (Narrow Preference Representation)**.** *Under Assumptions 1 (Richness) and 2 (EU), Axioms 1 (Correlation neglect) and 2 (Reference point) hold if and only if for all prospects $P \in \mathcal{P}$ and corresponding bracketwise prospects $P^j \in \mathcal{P}^j$*

$$\widetilde{EU}(P) = \sum_{j=1}^{m} \widetilde{EU}_j(P^j) \quad \text{with} \quad \widetilde{EU}_j(P^j) := \sum_{x^j \in X^j} P^j(x^j)\tilde{u}_j(x^j)$$

where $\tilde{u}_j : X^j \to \mathbb{R}$ for brackets $B_j \in \{B_1, B_2, ..., B_m\}$ are bracketwise utility functions with

$$\tilde{u}_j(x^j) := u(x^j, r^{-j}) \quad \forall x^j \in X^j \tag{1}$$

where $u(\cdot, r^{-j})$ denotes the broad utility function evaluated at the reference point for all brackets except bracket j, r^{-j}, which is treated as a fixed parameter of $\tilde{u}_j$.

Proof.

Step 1: The narrow utility function is additively separable across brackets. This result follows from Axiom 1 (Correlation neglect) using the results of Fishburn (1967). I restate his Theorem 1 translated to my framework:

Theorem (Fishburn, 1967)**.** *Under Assumptions 1 (Richness) and 2 (EU), Axiom 1 (Correlation neglect) holds if and only if there exist bracketwise utility functions $\tilde{u}_j : X^j \to \mathbb{R}$ for all brackets*

$B_j \in \{B_1, B_2, ..., B_m\}$ *such that*

$$\widetilde{EU}(P) = \sum_{j=1}^{m} \widetilde{EU}_j(P^j) \quad \textit{with} \quad \widetilde{EU}_j(P^j) := \sum_{x^j \in X^j} P^j(x^j)\tilde{u}_j(x^j)$$

for all prospects $P \in \mathcal{P}$ *and corresponding bracketwise prospects* P^j. $\widetilde{EU}$ *is unique up to positive affine transformation.*

Step 2: The jth bracket utility function corresponds to the broad utility function evaluated at the reference point outside of bracket j. This result follows from Axiom 2 (Reference point). Consider any two prospects $P, Q \in \mathcal{P}$ with correspoinding bracketwise prospects P^j, Q^j such that $P^j = Q^j = r^j$ for all but at most one $B_j \in \{B_1, B_2, ..., B_m\}$. Without loss of generality, take $B_j = B_1$ as the bracket for which P^j, Q^j and r^j may differ. By Assumption 2 (EU) for the broad preference relation, $P \succcurlyeq_b Q$ if and only if $EU(P) \geq EU(Q)$. We can rewrite P and Q as (P^1, r^{-1}) and (Q^1, r^{-1}), obtaining $EU(P^1, r^{-1}) \geq EU(Q^1, r^{-1})$. We thus have

$$P \succcurlyeq_b Q \quad \Leftrightarrow \quad \sum_{x^1 \in X^1} P^1(x^1)u(x^1, r^{-1}) \geq \sum_{x^1 \in X^1} Q^1(x^1)u(x^1, r^{-1}). \tag{2}$$

Similarly, by Assumption 2 (EU) for the narrow preference relation, $P \succcurlyeq_n Q$ if and only if $\widetilde{EU}(P) \geq \widetilde{EU}(Q)$. Rewriting P and Q as above, we obtain

$$P \succcurlyeq_n Q \quad \Leftrightarrow \quad \sum_{x^1 \in X^1} P^1(x^1)\tilde{u}(x^1, r^{-1}) \geq \sum_{x^1 \in X^1} Q^1(x^1)\tilde{u}(x^1, r^{-1}).$$

Now, by Step 1 we can rewrite the above expression as

$$P \succcurlyeq_n Q \quad \Leftrightarrow \sum_{x^1 \in X^1} P^1(x^1)\tilde{u}_1(x^1) + \sum_{j=2}^{m} \tilde{u}_j(r^j) \geq \sum_{x^1 \in X^1} Q^1(x^1)\tilde{u}_1(x^1) + \sum_{j=2}^{m} \tilde{u}_j(r^j)$$

and simplify it to

$$P \succcurlyeq_n Q \quad \Leftrightarrow \sum_{x^1 \in X^1} P^1(x^1)\tilde{u}_1(x^1) \geq \sum_{x^1 \in X^1} Q^1(x^1)\tilde{u}_1(x^1). \tag{3}$$

Now, by Axiom 2 (Reference point) $P \succcurlyeq_b Q \Leftrightarrow P \succcurlyeq_n Q$. Combining expressions 2 and 3 we therefore have

$$\sum_{x^1 \in X^1} P^1(x^1)u(x^1, r^{-1}) \geq \sum_{x^1 \in X^1} Q^1(x^1)u(x^1, r^{-1}) \quad \Leftrightarrow \quad \sum_{x^1 \in X^1} P^1(x^1)\tilde{u}_1(x^1) \geq \sum_{x^1 \in X^1} Q^1(x^1)\tilde{u}_1(x^1).$$

The above statement requires $\tilde{u}_1$ to be a positive affine transformation of u evaluated at r^{-1}. Now, by Axiom 2 (Reference point) this requirement holds for all bracketwise utility functions in the sequence $\tilde{u}_1, \tilde{u}_2, ..., \tilde{u}_m$. Furthermore, by Theorem 2 of Fishburn (1967) a transformation of a bracketwise utility function $\tilde{u}_j$ cannot be performed individually, i.e. without appropriately transforming all other bracketwise utility functions in accordance with the admissible transformations of $\widetilde{EU}$.[16] □

[16]For a detailed discussion of the admissible transformations on the sequence of functions $\widetilde{EU}_1, \widetilde{EU}_2, ..., \widetilde{EU}_m$ see Fishburn (1967).

The first part of Theorem 1 is essentially a restatement of Fishburn (1967)'s Theorem 1. Applied to my setting, his finding implies that under Assumptions 1 (Richness) and 2 (EU) Axiom 1 (Correlation neglect) holds if and only if the narrow utility function $\tilde{u}$ is additively separable across brackets. For each bracket B_j in the system of brackets characterizing the narrow preference relation, there exists a bracketwise utility function $\tilde{u}_j$, mapping the jth bracket outcome to the real numbers. The narrow utility function can be written as the sum of all bracketwise utility functions. This means that we can write the narrow expected utility of a prospect $P \in \mathcal{P}$ as a sum of bracketwise expect utilities from all bracketwise prospects P^j induced by P.

The important new insight of my Theorem 1 is that the jth bracket utility function, $\tilde{u}_j$ is equivalent to the broad utility function keeping all outcomes except the jth bracket outcome fixed at the reference point. This means that we can interpret the narrow bracketer as actually using the same utility function she would use if she bracketed broadly. However, she applies that utility function separately to each bracket in her system of brackets. For a given bracket, she evaluates her broad utility function at the outcomes inside that bracket while keeping all outside-bracket outcomes fixed at their reference point levels. Finally, her overall utility from a specific outcome is determined by the sum of all of these bracketwise evaluated utilities.

To illustrate the content of Theorem 1, consider the special case of $n = 2$ such that every prospect consists of two subprospects and suppose the system of brackets characterizing $\succsim_n$ separates these two subprospects into distinct brackets. Consider any prospect $P \in \mathcal{P}$. The expected utility of the broad bracketer is given by

$$EU(P) = \sum_{x \in X} P(x)u(x). \tag{4}$$

Theorem 1 implies that the expected utility of the narrow bracketer can be expressed as

$$\widetilde{EU}(P) = \sum_{x_1 \in X_1} P_1(x_1)u(x_1, r_2) + \sum_{x_2 \in X_2} P_2(x_2)u(r_1, x_2) = \sum_{x \in X} P(x)[u(x_1, r_2) + u(r_1, x_2)] \tag{5}$$

with u equivalent across the two expected utility formulas.

The narrow bracketer's expected utility representation is an additively separable version of its broad counterpart. Consider the first formulation of $\widetilde{EU}(P)$ in (5) and compare it to the broad expected utility formula in (4). $\widetilde{EU}(P)$ is additively separable across brackets. It consists of the sum of two separate expected utility formulas, one evaluating the first subprospect P_1 and one evaluating the second subprospect P_2. This additive separability reflects the fact that any correlation between the two subprospects is disregarded by the narrow bracketer. By evaluating their expected utilities separately, she treats them as if they were entirely independent.

Furthermore, the narrow bracketer disregards any interactions between the outcomes of the two subprospects. This is nicely illustrated by the second formulation of $\widetilde{EU}(P)$ in (5). The utility that a narrow bracketer derives from an outcome x of the overall prospect P is, again, additively separable across brackets. Instead of evaluating the broad utility function at the overall outcome x as in (4), she evaluates the broad utility function separately for each bracket, once at the outcome of the first subprospect x_1 and once at the outcome of the second subprospect x_2. Since she never evaluates the broad utility at x_1 and x_2 jointly, she does not keep track of possible complementarities or substitutabilities between the two subprospect outcomes.

However, since the narrow bracketer uses the same utility function in her evaluation as the broad bracketer, she is never fully ignorant of the existence of interactions between the two outcome dimensions. She simply appreciates these interactions mistakenly with respect to the reference point. When the narrow bracketer evaluates the outcome of the first subprospect x_1, she keeps the outcome of the second subprospect fixed at r_2 and vice versa. Thus, while she considers the interdependencies between x_1 and r_2 as well as the interdependencies between r_1 and x_2, she fails to keep track of the interdependencies between x_1 and x_2. As a result, her tradeoffs between the outcome dimensions are distorted.

2.4 Discussion

Budget balance A major obstacle towards modeling narrow bracketing is that there exists a tension between the behavioral bias and the economic principle of budget balance. Intuitively, narrow bracketing is associated with "...making each choice in isolation" (Read et al., 1999b). Adhering to this basic intution, one might be drawn to model the narrow bracketer as sequentially making each decision in a set of concurrent decisions as if it were the only decision she faces overall. Such a modeling approach works nicely when applied to the specific environments studied in large parts of the experimental literature on choice bracketing. These experiments are designed such that the specific option a decision maker chooses in one decision does not influence the set of options that are available to her in any other decision (see, e.g., Ellis and Freeman, 2020; Rabin and Weizsäcker, 2009; Tversky and Kahneman, 1981). However, the approach of modeling narrow bracketing as fully isolated decision making runs into serious problems when applied to economically more relevant settings in which decision makers face resource constraints which tie together the option sets of concurrent decisions.

For illustration consider the constrained utility maximization problem of a consumer who has a fixed budget to spend on food and clothing. Suppose the consumer narrowly brackets these two good categories. As long as her budget is tight enough, full isolation of her decisions in these two categories implies that the consumer spends her whole budget on either one of the two categories leaving nothing for the respective other category. Once she enters a, say, clothing store she fully ignores that she might also want to get dinner later on and therefore spends her whole budget on a new outfit. Only later, when she passes by her favourite restaurant she realizes how hugry she is. Of course, the irrationality displayed by the consumer's behavior in this example is not what we observe in reality and goes far beyond what we actually think of when we talk about narrow bracketing.

The example demonstrates that a reasonable model of narrow bracketing needs to balance the isolated nature of narrow decision making with the integrated thinking required for making meaningful tradeoffs across brackets to satisfy budget balance. By defining the narrow preference relation on the same multidimensional prospects as the broad preference relation, I implicitly model the narrow bracketer's decision making as simultaneous. Therefore, my framework allows me to cover the whole spectrum of isolation and integration in the narrow bracketer's decision making. Axiom 1 (Correlation neglect) imposes a limit on the ability of the narrow bracketer to integrate subprospects across brackets. This limit is balanced by Axiom 2 (Reference point) which retains the narrow bracketer's ability to integrate subprospects across brackets at and around the reference point. It is the combination of these two axioms that enables me to derive a representation of the narrow preference relation which captures the narrow bracketer's tendency to isolate intermediate decisions from one another and at the same time resolves the general incompatibility of this behavior with the principle of budget balance.

Mental accounting and budgeting Thaler (1999) defines mental accounting as "...the set of cognitive operations used by individuals and households to organize, evaluate, and keep track of financial activities". Choice bracketing is one component of such mental accounting. Another important component of mental accounting is *budgeting*. In the context of consumption choice, budgeting describes the assignment of goods into categories with a fixed budget for each category. An important implication of budgeting is the violation of monetary fungiblity across categories.

Already long before behavioral economics was introduced into the scientific debate, economists contemplated how a general but sufficiently tractalbe utility function capturing consumer behavior should look like. Strotz (1957) argues that it is intuitively appealing to think of the consumer as follwing a two-stage maximization procedure akin to budgeting. In the first stage, the consumer allocates her overall budget across general good categories like, for example, food, clothing, and travel. Then, in the second stage she considers each category in isolation and allocates the previously determined category budget across the individual goods inside that category. Gorman (1959) investigates the characteristics a utility function needs to have in order for the solution to a full constrained utility maximization problem to be equivalent to the solution obtained in the described two-stage-procedure. A necessary and sufficient condition for budgeting to be rational is that the consumer's utility is either additively separable across budget categories or separable with budgetwise utilities entering through an intermediate function that is homogeneous of degree one.

This reveals how in my model narrow bracketing implies a boundedly rational form of budgeting as discussed by Blow and Crawford (2018). The narrow bracketer's expected utility representation is additively separable across brackets. Thus, she behaves as if she employed the described two-stage budgeting procedure with budgeting categories equivalent to the brackets in her system of brackets. However, her broad expected utility representation is not generally additively separable across brackets. Therefore, such budgeting behavior is not generally admissible according to the narrow bracketer's true preferences.

3 Model predictions

3.1 Constrained utility maximization

Consider economics 101 consumption bundle choice. DM faces the problem of allocating a given budget or wealth w across two goods. She chooses a bundle $x \in \mathbb{R}^2_+$. We can write $x = (x_1, x_2)$ where x_1 denotes the amount of good 1 and x_2 denotes the amount of good 2. The per-unit prices of the two goods are p_1 and p_2 respectively.

As benchmark consider the maximization problem solved by a broad bracketer:

$$\max_{x_1,x_2} u(x_1,x_2) \quad \text{subject to} \quad p_1x_1+p_2x_2 \leq w. \tag{6}$$

Denote by $x^* = (x_1^*, x_2^*)$ the broad optimum, i.e. the argument that maximizes (6). I am interested in how a narrow DM's choice deviates from her broad optimum. Suppose DM brackets each good in her consumption bundle separately, i.e. $B = \{\{x_1\},\{x_2\}\}$. She solves

$$\max_{x_1,x_2} u(x_1,r_2)+u(r_1,x_2) \quad \text{subject to} \quad p_1x_1+p_2x_2 \leq w. \tag{7}$$

Denote by $\tilde{x} = (\tilde{x}_1, \tilde{x}_2)$ the narrow optimum, i.e. the argument that maximizes (7).

The following assumption assures that the broad consumer's optimization problem is well-behaved. By the subsequent lemma, this assumption also implies well-behavedness of the narrow consumer's optimization problem.

Assumption 3 (Quasi-concavity of u)**.** For all $x,y \in \mathbb{R}^2_+$ and all $\lambda \in [0,1]$, the broad utility function $u: \mathbb{R}^2_+ \to \mathbb{R}$ satisfies $u(\lambda x+(1-\lambda)y) \geq \min\{u(x),u(y)\}$.

Lemma 1 (Quasi-concavity of $\tilde{u}$)**.** *Assumption 3 implies that the narrow utility function* $\tilde{u}: \mathbb{R}^2_+ \to \mathbb{R}$ *is quasi-concave.*

Proof. For all $x,y,r \in \mathbb{R}^2_+$ and all $\lambda \in [0,1]$, quasi-concavity of u implies,

$$u(\lambda x_1+(1-\lambda)y_1,r_2) \geq \min\{u(x_1,r_2),u(y_1,r_2)\} \text{ and}$$

$$u(r_1,\lambda x_2+(1-\lambda)y_2) \geq \min\{u(r_1,x_2),u(r_1,y_2)\}.$$

Since $\tilde{u}(\lambda x+(1-\lambda)y) = u(\lambda x_1+(1-\lambda)y_1,r_2)+u(r_1,\lambda x_2+(1-\lambda)y_2)$, $\tilde{u}(x) = u(x_1,r_2)+u(r_1,x_2)$, and $\tilde{u}(y) = u(y_1,r_2)+u(r_1,y_2)$ it follows, that $\tilde{u}(\lambda x+(1-\lambda)y) \geq \min\{\tilde{u}(x),\tilde{u}(y)\}$. □

The direction in which the narrow optimum departs from its broad counterpart depends crucially on the type of interdependencies between the two goods captured by the sign of the broad utility function's cross-derivative.

Definition 1. Goods 1 and 2 have negative interactions if $\frac{\partial^2 u}{\partial x_1 \partial x_2} < 0$ for all $x \in \mathbb{R}^2_+$. They have positive interactions if $\frac{\partial^2 u}{\partial x_1 \partial x_2} > 0$ for all $x \in \mathbb{R}^2_+$. The two goods have no interactions if $\frac{\partial^2 u}{\partial x_1 \partial x_2} = 0$ for all $x \in \mathbb{R}^2_+$.

Roughly, negative interactions are associated with substitutabilities between the two goods while postive interactions are associated with complementarities between the two goods.[17]

In Section 2.3 (Representation theorem) I alluded to the fact that the additive separability of the narrow utility function implies that the narrow bracketer disregards interactions. In the context of consumption bundle choice, the following proposition illustrates this fact by comparing the indifference curves of the narrow bracketer to their broad counterparts. Like all further proofs, the proof of the proposition is relegated to the appendix.

[17]See Chambers and Echenique (2009) and Topkis (1998) for a detailed discussion on when a positive cross-derivative of the utility function implies complementarity.

Denote by $MRS(x)$ and $\widetilde{MRS}(x)$ the marginal rates of substitution between good 1 and good 2 at bundle $x = (x_1, x_2)$ for the broad and the narrow bracketer respectively, i.e. $MRS(x) = \frac{\partial u}{\partial x_1} / \frac{\partial u}{\partial x_2}$ and $\widetilde{MRS}(x) = \frac{\partial \tilde{u}}{\partial x_1} / \frac{\partial \tilde{u}}{\partial x_2}$.

Proposition 1 (Indifference curves)**.** *Assume goods 1 and 2 have either positive, negative, or no interactions. For any amount of good 1, x_1, there exists a corresponding amount of good 2, $f(x_1)$, such that $MRS(x_1, f(x_1)) = \widetilde{MRS}(x_1, f(x_1))$ where $f(r_1) = r_2$, $f(x_1) < r_2$ for $x_1 < r_1$ and $f(x_1) > r_2$ for $x_1 > r_1$. Furthermore,*

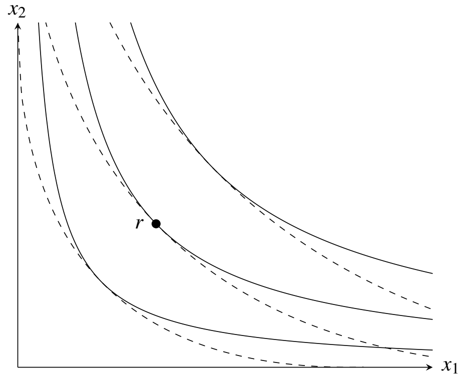
- *Positive interactions $\Rightarrow MRS(x) > \widetilde{MRS}(x)$ for all $x \in \mathbb{R}^2_+$ with $x_2 > f(x_1)$ and $MRS(x) < \widetilde{MRS}(x)$ for all $x \in \mathbb{R}^2_+$ with $x_2 < f(x_1)$*
- *Negative interactions $\Rightarrow MRS(x) < \widetilde{MRS}(x)$ for all $x \in \mathbb{R}^2_+$ with $x_2 > f(x_1)$ and $MRS(x) > \widetilde{MRS}(x)$ for all $x \in \mathbb{R}^2_+$ with $x_2 < f(x_1)$*
- *No interactions $\Rightarrow MRS(x) = \widetilde{MRS}(x)$ for all $x \in \mathbb{R}^2_+$.*

Proposition 1 states that at the reference point the slopes of broad and narrow indifference curves are the same. Furthermore, for every amount of good 1, there exists a corresponding amount of good 2 such that the slopes of broad and narrow indifference curves are the same at that bundle. If there are positive interactions between the two goods, the narrow indifference curve is flatter than the broad indifference curve to the left of that bundle and steeper than the broad indifference curve to the right of that bundle. Therefore, narrow indifference curves are less convex than their broad counterparts if the two goods have positive interactions. Conversely, narrow indifference curves are more convex than their broad counterparts if the two goods have negative interactions. Intuitively, the more convex the indifference curves, the more complementary are the two goods. Therefore, in the case of positive interactions, the narrow bracketer can be interpreted as behaving as if the two goods were less complementary than they actually are and vice versa for the case of negative interactions.

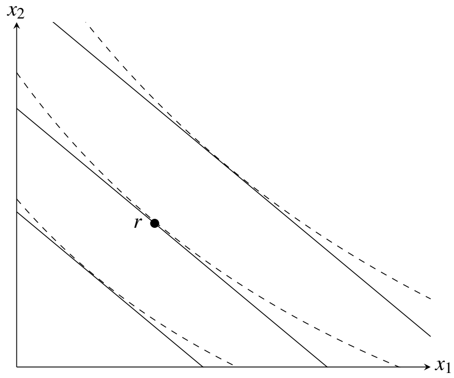
Figure 1 illustrates the content of Proposition 1 for two specific broad utility functions given the reference point r. Consider first Figure 1a. The figure shows the indifference curve maps of broad (solid) and narrow (dashed) bracketer for a broad utility function belonging to the Cobb-Douglas family. The utility function is characterized by complementarities which is reflected by the convex shape of the broad indifference curves. The corresponding narrow indifference curves are less convex than their broad counterparts, reflecting the fact that the narrow bracketer disregards the positive interactions between the two goods. However, at the reference point and at any bundle with a distribution of amounts between the two goods proportional to the reference point distribution, broad and narrow indifference curves have the same slope. This illustrates how the narrow bracketer's tradeoffs between the two goods remain undistorted at the reference point and proportional bundles.

In contrast, Figure 1b depicts the indifference curve maps of broad and narrow bracketer for a perfect substitutes broad utility function with negative interactions between the two goods[18]. Perfect substitutability between the two goods implies that the broad indifference curves are straight lines. The narrow bracketer, however, disregards the negative utility interactions between the two

[18] The utility function is widely used in the context of decision making under risk since it has the CRRA property.



(a) Positive interactions: $u(x_1,x_2) = \sqrt{x_1 x_2}$



(b) Negative interactions: $u(x_1,x_2) = \sqrt{x_1 + x_2}$

Figure 1: Comparison of broad (solid) and narrow (dashed) indifference curves with reference point r.

goods. As a result, her indifference curves are convex. She treats the two goods as more complementary than they are. Again, her tradeoffs at the reference point and at bundles proportial to the reference point remain undistorted.

The next proposition investigates how the narrow bracketer's chosen consumption bundle departs from her optimal consumption bundle.

Denote by $d(x,y)$ the Euclidean distance between two consumption bundles $x,y \in \mathbb{R}^2_+$, i.e. $d(x,y) := \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$.

Proposition 2 (Narrow optimum)**.** *Assume $w = p_1 r_1 + p_2 r_2$ and $r \neq x^*$. The follwing two statements hold at any interior solutions x^* and $\tilde{x}$ to the maximization problems (6) and (7) respectively.*

- *Positive interactions $\Rightarrow d(r,x^*) < d(r,\tilde{x})$*
- *Negative interactions $\Rightarrow d(r,x^*) > d(r,\tilde{x})$*

Proposition 2 states that for budget balanced reference points, unless $r = x^*$, the narrow optimum $\tilde{x}$ is further away (in terms of Euclidean distance) from the reference point than the broad optimum x^* if the two goods have positive interactions. Conversely, the narrow optimum $\tilde{x}$ is closer to the reference point if the the two goods have negative interactions.

Considering Proposition 1 (Indifference curves) in isolation, one might expect that the narrow bracketer's disregard of interactions between the two goods and the resulting shape of her indifference curves imply that the narrow bracketer underdiversifies in the case of positive interactions and overdiversifies in the case of a negative interactions. However, while this intuition is not generally flawed, it does not take into account the role that the reference point plays for the narrow bracketer's decisions. The important role of the reference point is clarified by Proposition 2.

While the narrow bracketer disregards the interdependencies between the goods in her bundle, she is not fully ignorant of their existence. However, she does not consider changes from the respective reference quantities for the two goods simultaneously. Thus, when thinking about an alteration in the amount she might purchase of good 1, from r_1 to $x_1 \neq r_1$, she keeps the amount of good 2 fixed at the reference quantity of good 2, r_2. The reverse holds for alterations in the amount she purchases of good 2. Therefore, the narrow bracketer's appreciation of the interactions between the two goods only occurs separately for the two quantities she purchases and mistakenly with respect to the reference quantity of the respective other good. This implies that the reference point has a profound influence on the narrow bracketer's choice.

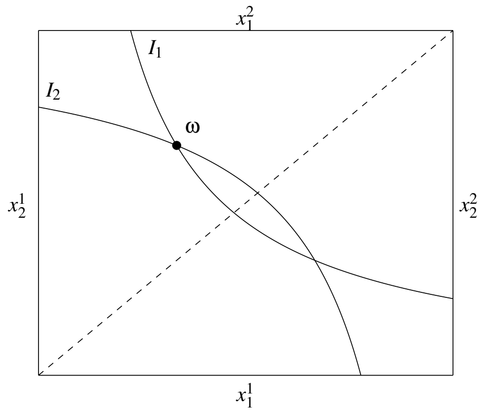
For example, if the goods have positive interactions, an unbalanced reference point with $r_1 > r_2$ pushes the narrow bracketer towards increasing her consumption of good 2 and decreasing her consumption of good 1. This happens because the high reference quantity of good 1, r_1, makes investments in good 2 seem more attractive than investments in good 1, which are in the narrow bracketers mind combined with the relatively low reference quantity of good 2, r_2. Now, if the optimal consumption basket of the broad bracketer x^* prescribes $x_1^* \leq x_2^*$, the fact that the narrow optimum $\tilde{x}$ is pushed further from the reference point r compared to the broad optimum x^* in this constellation always implies that the bundle chosen by the narrow bracketer is less diversified than the bundle chosen by the broad bracketer. If, however, the broad optimum x^* prescribes $x_1^* > x_2^*$, the extra push away from r might induce the narrow bracketer to choose a more diversified consumption bundle than the broad bracketer even though she disregards the positive utility interactions between the chosen quantities x_1 and x_2. Depending on the constellation of reference point and broad optimum, we might therefore observe a narrow bracketer overdiversifying her consumption bundle compared to the broad optimum although the goods have positive interactions. Similarly, we might observe a narrow bracketer underdiversifying her conusmption bundle compared to the broad optimum although the goods have negative interactions.

Interestingly, if the goods have positive interactions the effect of the reference point on the narrow bracketer's chosen bundle goes into the opposite direction of the effect that loss-aversion implies in this setting. The chosen bundle of a loss-averse narrow bracketer is always closer to the reference point than the chosen bundle of a narrow bracketer without loss-aversion. Thus, while narrow bracketing in the case of negative interactions exacerbates the effects of loss-aversion, in the case of positive interactions it actually dampens the effects of loss-aversion. My results reveal that the reference point plays an important role in the decision making of a narrow bracketer independent of whether she is loss-averse or not.

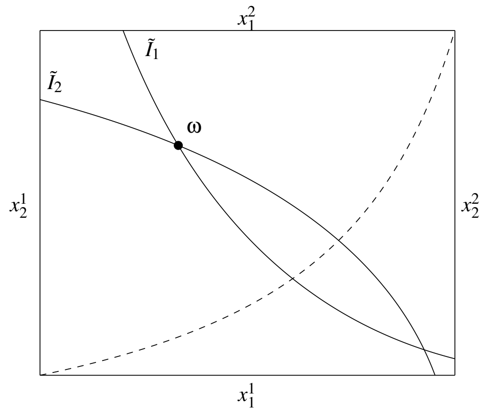
3.2 Exchange economy

Consider an exchange economy with two consumers $i = 1,2$ and two goods. Consumer i's consumption bundle is denoted by $x^i = (x_1^i, x_2^i)$. An allocation $x \in \mathbb{R}_+^4$ is an assignment of a consumption bundle to each consumer, i.e. $x = (x^1, x^2) = ((x_1^1, x_2^1), (x_1^2, x_2^2))$. The total endowments of goods 1 and 2 in the economy are given by $\omega_1 > 0$ and $\omega_2 > 0$ respectively. The initial endowment allocation is denoted $\omega = (\omega^1, \omega^2)$ with $\omega^1 = (\omega_1^1, \omega_2^1)$ denoting consumer 1's endowment such that consumer 2's endowment is given by $\omega^2 = (\omega_1 - \omega_1^1, \omega_2 - \omega_2^1)$. I assume $\omega_1^i, \omega_2^i \geq 0$ for $i = 1,2$. The systems of brackets for the two consumers are given by $B^i = \{\{x_1^i\}, \{x_2^i\}\}$ for $i = 1,2$.

I refer to the *broad economy* as the exchange economy in which both consumers bracket broadly and to the *narrow economy* as the exchange economy in which both consumers bracket narrowly. Furthermore, I refer to the *broad contract curve* as the set of Pareto optimal allocations of the broad economy and to the *broad core* as the set of Pareto optimal allocations that constitute Pareto improvements with respect to the initial endowment allocation in the broad economy.



(a) Broad economy



(b) Narrow economy

Figure 2: Edgeworth-box comparison of broad and narrow exchange economy with broad utilities $u^i(x_1^i, x_2^i) = \sqrt{x_1^i x_2^i}$ for $i = 1,2$ (positive interactions) and reference points $r^i = \omega^i$ for $i = 1,2$. In each Edgeworth-box, the lower left corner corresponds to consumer 1's origin and the upper right corner corresponds to consumer 2's origin. I_i and $\tilde{I}_i$ for $i = 1,2$ respectively denote consumer i's broad and narrow indifference curve reached at the initial endowment allocation ω. The dashed graph displays the contract curve of the respective economy. The part of the contract curve that is enclosed by the lense that opens up between the two indifference curves corresponds to the core of the economy.

Narrow contract curve and *narrow core* are defined analogously. It is a well known fact that any Walrasian equilibrium of an exchange economy is an element of its core (Mas-Colell et al., 1995).

The following proposition shows how choice bracketing systematically affects the volume of trade in the exchange economy.

Proposition 3 (Exchange economy)**.** *Assume that consumer i's reference point is equal to her initial endowment, i.e.* $r^i = \omega^i$ *for* $i = 1,2$*. For any inditial endowment allocation* ω *such that* $MRS^1(\omega^1) \neq MRS^2(\omega^2)$*, if two allocations* x *and* $\tilde{x}$ *are elements of the broad and narrow core respectively and they are not at the corner, then*

- *Positive interactions for both consumers* $\Rightarrow d(\omega, x) < d(\omega, \tilde{x})$.
- *Negative interactions for both consumers* $\Rightarrow d(\omega, x) > d(\omega, \tilde{x})$.

Proposition 3 states that starting from any inital endowment allocation there is more trade in the narrow exchange economy compared to its broad counterpart if the two goods have positive interactions. Conversely, there is less trade in the narrow exchange economy compared to its broad counterpart if the two goods have negative interactions.

Figure 2 illustrates the difference between a broad exchange economy and its narrow counterpart when there are positive interactions between the two goods. Consider first Figure 2a which shows the broad economy in an Edgeworth-box. At the initial endowment allocation ω consumer 1 holds a bundle that is unbalanced towards good 2 while consumer 2 holds a bundle that is unbalanced towards good 1. The indifference curves that the two consumers reach at this initial endowment allocation intersect. Any allocation inside the lense enclosed by the two indifference curves constitutes a Pareto improvement with respect to ω. In particular, redistributing a small amount of good 1 in exhange for a small amount of good 2 from consumer 2 to consumer 1 resulting in more balanced bundles makes both consumers better off. Performing a series of such small trades allows the consumers to arrive at the broad core which is located on the part of the contract curve that intersects with the lense. At the broad core, the consumers have reached a Pareto optimal allocation. Since in this example the broad contract curve is on the 45° line, any such allocation has the property that it equalizes the amounts of good 1 and good 2 allocated to a given consumer. Thus, in the given broad economy, we should expect the consumers to perform trades that move them from the initial endowment allocation towards an allocation that fully balances their consumption bundles.

Consider now the corresponding narrow exchange economy displayed in Figure 2b. As in the broad economy, the consumer's narrow indifference curves intersect at the initial endowment allocation. Furthermore, moving to an allocation which induces bundles that are more balanced between the two goods for both consumers constitutes a Pareto improvement. However, in the narrow economy, the overall set of allocations constituting a Pareto improvement with respect to ω extends much further to the lower right corner of the Edgeworth-box than in the broad economy. This is a direct consequence of the narrow consumers' disregard of the positive interactions between the good dimensions in their bundles. As stated in Proposition 1 (Indifference curves) positive interactions between the two goods imply that the narrow indifference curves are less convex compared to their broad counterparts. The narrow consumers perceive the two good dimensions of their bundles as less complementary than they actually are.

Relatedly, the narrow contract curve is not on the 45° line but bent towards the lower right corner of the Edgeworth-box. As a result, the bundles in the narrow core allocations are not balanced between the two goods. Instead, any allocation in the narrow core has the property that consumer 1's bundle is unbalanced towards good 1 and consumer 2's bundle is unbalanced towards good 2. Interestingly, the imbalance in the consumers' bundles at the narrow core is exactly opposite to the imbalance in the consumers' bundles at the initial endowment allocation. This property of the narrow core mirrors the logic of Proposition 2 (Narrow optimum). The consumers appreciate the positive interactions between the two good dimensions mistakenly with respect to their reference points. Akin to status-quo based reference points, consumers' reference points are assumed to be equal to their respective bundles in the initial endowment allocation. Consider consumer 1. Her bundle in the initial endowment allocation is unbalanced towards good 2. Due to the complementarity between the two good dimensions, the resulting high reference point in the second good dimension makes increases in the amount of good 1 seem relatively more attractive than they actually are. Similarly, the low reference point in the first good dimension makes increases in the amount of good 2 seem relatively less attractive than they actually are. This constellation implies a push of narrow consumer 1's preference towards bundles that are characterized by an imbalance opposite to the imbalance in her initial endowment, i.e. towards good 1. Similarly, consumer 2's preferences is pushed towards bundles that are imbalanced towards good 2. As a result, the volume of trade predicted for the narrow economy is larger than the volume of trade predicted for the broad economy.

4 Conclusion

Narrow bracketing affects individual decision making which is the very basis of almost all economic activity. Therefore, the potential implications of this behavioral bias go through the whole economy. Indeed, empirical evidence suggests that narrow bracketing adversely affects behavior in a vast variety of important economic settings. In this paper, I present a tractable and generally applicable theoretical model of choice bracketing. While existing models of choice bracketing are essentially restricted to one-dimensional outcome spaces, my model can capture multidimensional outcomes with non-trivial interactions between outcome dimensions. Furthermore, my model resolves the general incompatibility between narrow bracketing and budget balance. It, therefore, opens up the possibility to systematically study the effects of narrow bracketing in new economic applications ranging from complex contract negotiations to basic consumption bundle choice. I derive my model from basic behavioral assumptions. In contrast to a model that is designed to generate specific predictions in a given setting, it may therefore be more likely to make accurate predictions when applied across a variety of different settings. Finally, my model provides a theoretical framework that can inspire and organize future empirical research on choice bracketing.

An essential component of my model of choice bracketing is the reference point. It ties the narrow preference relation to its broad counterpart. However, my model takes the reference point as given and stays agnostic about where it comes from. In my applications, I show that the direction and extent of the deviation of a narrow bracketer's choices from her broad optimum crucially depends on the specific form of the reference point. Future research investigating the nature of reference points in narrow bracketing is therefore essential to further our understanding of this behavioral bias. Another important component of my model is the system of brackets. It characterizes the degree to which a decision maker brackets narrowly. For a given system of brackets, my model fully characterizes the representation of the narrow preference relation. A promising direction for future research is to identify a decision maker's system of brackets from choice data.

Experimental results suggest that the system of brackets characterizing the narrow preference relation is not set in stone. Instead, the extent to which a decision maker brackets narrowly depends on how easy it is for her to access information on the different dimensions of her decision problem simultaneously. Existing experimental apporaches, for example, vary whether subjects make decisions simultaneously or sequentially (see, e.g., Rabin and Weizsäcker, 2009; Read et al., 2001, 1999a) or whether rewards are aggregated or separated (see, e.g., Koch and Nafziger, 2020; Stracke et al., 2017; Gneezy and Potters, 1997). Exploiting treatments that induce changes in the bracketing behavior of subjects along these lines, future experimental research can reveal the empirical validity of my model. Importantly, since the predictions of my model do not rely on changes in the reference point, my model is testable without requiring knowledge of the reference points used by subjects.

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A Proofs of Section 3

A.1 Proof of Proposition 1 (Indifference curves)

Proof. The marginal rates of substitution for the broad and the narrow bracketer are

$$MRS(x_1,x_2) = \frac{\partial u}{\partial x_1}\bigg|_{(x_1,x_2)} \left(\frac{\partial u}{\partial x_2}\bigg|_{(x_1,x_2)} \right)^{-1} \quad \text{and} \quad \widetilde{MRS}(x_1,x_2) = \frac{\partial u}{\partial x_1}\bigg|_{(x_1,r_2)} \left(\frac{\partial u}{\partial x_2}\bigg|_{(r_1,x_2)} \right)^{-1}.$$

Thus, we obviously have $MRS(r_1,r_2) = \widetilde{MRS}(r_1,r_2)$. In this proof I focus on the case $\frac{\partial^2 u}{\partial x_1 \partial x_2} > 0$. The other two cases can are proven analogously. Consider pairs (r_1,x_2) with $x_2 > r_2$. The above expressions for the broad and narrow marginal rates of substitution reveal that the numerator of $\widetilde{MRS}(r_1,x_2)$ is equal to the numerator of $\widetilde{MRS}(r_1,r_2)$ and the denominator of $MRS(r_1,x_2)$ is equal to the denominator of $\widetilde{MRS}(r_1,x_2)$. Furthermore, by $\frac{\partial^2 u}{\partial x_1 \partial x_2} > 0$ we have that the numerator of $MRS(r_1,x_2)$ is larger than the numerator of $MRS(r_1,r_2)$. Together with $MRS(r_1,r_2) = \widetilde{MRS}(r_1,r_2)$ this implies that $MRS(r_1,x_2) > \widetilde{MRS}(r_1,x_2)$ for all $x_2 > r_2$. Similar reasoning reveals that $MRS(r_1,x_2) < \widetilde{MRS}(r_1,x_2)$ for all $x_2 < r_2$. Now, consider pairs (x_1,r_2) with $x_1 > r_1$. The above expressions for the broad and narrow marginal rates of substitution reveal that the denominator of $\widetilde{MRS}(x_1,r_2)$ is equal to the denominator of $\widetilde{MRS}(r_1,r_2)$ and the numerator of $MRS(x_1,r_2)$ is equal to the numerator of $\widetilde{MRS}(x_1,r_2)$. Furthermore, by $\frac{\partial^2 u}{\partial x_1 \partial x_2} > 0$ we have that the denominator of $MRS(x_1,r_2)$ is larger than the denominator of $MRS(r_1,r_2)$. Together with $MRS(r_1,r_2) = \widetilde{MRS}(r_1,r_2)$ this implies that $MRS(x_1,r_2) < \widetilde{MRS}(x_1,r_2)$ for all $x_1 > r_1$. Similar reasoning reveals that $MRS(x_1,r_2) > \widetilde{MRS}(x_1,r_2)$ for all $x_1 < r_1$. Finally, the full claim presented in the proposition follows by convexity of preferences as implied by positive interactions. □

A.2 Proof of Proposition 2 (Narrow optimum)

Proof. Focus on the case $\frac{\partial^2 u}{\partial x_1 \partial x_2} > 0$. The proof for $\frac{\partial^2 u}{\partial x_1 \partial x_2} < 0$ proceeds analogously. Since x^* and $\tilde{x}$ are interior solutions and $r \neq x^*$, it must hold that $MRS(x_1^*,x_2^*) = \frac{p_1}{p_2}$, $\widetilde{MRS}(\tilde{x}_1,\tilde{x}_2) = \frac{p_1}{p_2}$, and $MRS(r_1,r_2) \neq \frac{p_1}{p_2}$.

Now, suppose $MRS(r_1,r_2) < \frac{p_1}{p_2}$. Since $MRS(r_1,r_2) = \widetilde{MRS}(r_1,r_2)$, this holds iff $\widetilde{MRS}(r_1,r_2) < \frac{p_1}{p_2}$. Since x^* and $\tilde{x}$ are interior solutions and $w = p_1 r_1 + p_2 r_2$, $MRS(r_1,r_2) < \frac{p_1}{p_2}$ and $\widetilde{MRS}(r_1,r_2) < \frac{p_1}{p_2}$ imply that $x_1^*,\tilde{x}_1 < r_1$ and $x_2^*,\tilde{x}_2 > r_2$. Thus, by Proposition 1 $\frac{\partial^2 u}{\partial x_1 \partial x_2} > 0 \Rightarrow MRS(x_1^*,x_2^*) > \widetilde{MRS}(x_1^*,x_2^*)$ and $MRS(\tilde{x}_1,\tilde{x}_2) > \widetilde{MRS}(\tilde{x}_1,\tilde{x}_2)$. As $\frac{p_1}{p_2} = MRS(x_1^*,x_2^*) > \widetilde{MRS}(x_1^*,x_2^*)$ and $MRS(\tilde{x}_1,\tilde{x}_2) > \widetilde{MRS}(\tilde{x}_1,\tilde{x}_2) = \frac{p_1}{p_2}$ it must therefore hold that $\frac{\partial^2 u}{\partial x_1 \partial x_2} > 0 \Rightarrow d(r,x^*) < d(r,\tilde{x})$.

Suppose instead $MRS(r_1,r_2) > \frac{p_1}{p_2}$. Since $MRS(r_1,r_2) = \widetilde{MRS}(r_1,r_2)$ this holds iff $\widetilde{MRS}(r_1,r_2) > \frac{p_1}{p_2}$. Since x^* and $\tilde{x}$ are interior solutions and $w = p_1 r_1 + p_2 r_2$, $MRS(r_1,r_2) > \frac{p_1}{p_2}$ and $\widetilde{MRS}(r_1,r_2) > \frac{p_1}{p_2}$ imply that $x_1^*,\tilde{x}_1 > r_1$ and $x_2^*,\tilde{x}_2 < r_2$. Thus, by Proposition 1 $\frac{\partial^2 u}{\partial x_1 \partial x_2} > 0 \Rightarrow MRS(x_1^*,x_2^*) < \widetilde{MRS}(x_1^*,x_2^*)$ and $MRS(\tilde{x}_1,\tilde{x}_2) < \widetilde{MRS}(\tilde{x}_1,\tilde{x}_2)$. As $\frac{p_1}{p_2} = MRS(x_1^*,x_2^*) < \widetilde{MRS}(x_1^*,x_2^*)$ and $MRS(\tilde{x}_1,\tilde{x}_2) < \widetilde{MRS}(\tilde{x}_1,\tilde{x}_2) = \frac{p_1}{p_2}$ it must therefore hold that $\frac{\partial^2 u}{\partial x_1 \partial x_2} > 0 \Rightarrow d(r,x^*) < d(r,\tilde{x})$. □

A.3 Proof of Proposition 3 (Exchange economy)

Proof. For any elements x and $\tilde{x}$ of the respective broad and narrow cores, we have $MRS^1(x^1) = MRS^2(x^2)$ and $\widetilde{MRS}^1(\tilde{x}^1) = \widetilde{MRS}^2(\tilde{x}^2)$.

Focus first on $\frac{\partial^2 u^i}{\partial x_1^i \partial x_2^i} > 0$ for $i = 1,2$ and ω such that $MRS^1(\omega^1) > MRS^2(\omega^2)$. From Proposition 1 we know that since $r^i = \omega^i$, $MRS^i(\omega^i) = \widetilde{MRS}^i(\omega^i)$ for $i = 1,2$. Therefore, $MRS^1(\omega^1) > MRS^2(\omega^2)$ implies $\widetilde{MRS}^1(\omega^1) > \widetilde{MRS}^2(\omega^2)$.

By $MRS^1(\omega^1) > MRS^2(\omega^2)$ and $\widetilde{MRS}^1(\omega^1) > \widetilde{MRS}^2(\omega^2)$ it must hold for any interior broad and narrow core allocations x and $\tilde{x}$, that $x_1^1 \geq \omega_1^1$, $x_2^1 \leq \omega_2^1$, $\tilde{x}_1^1 \geq \omega_1^1$, and $\tilde{x}_2^1 \leq \omega_2^1$, with one of the two inequalities concerning x and $\tilde{x}$ holding strictly.

Now, consider any allocation y with $y_1^1 \geq \omega_1^1$ and $y_2^1 \leq \omega_2^1$, implying $y_1^2 \leq \omega_1^2$ and $y_2^2 \geq \omega_2^2$, where one of the two inequalities holds strictly. By Proposition 1 we have $MRS^1(y^1) < \widetilde{MRS}^1(y^1)$ and $MRS^2(y^2) > \widetilde{MRS}^2(y^2)$.

Thus, starting from the initial endowment allocation ω, increasing the amount of good 1 allocated to person 1 while decreasing the amount of good 2 allocated to person 1 reduces the difference between the broad marginal rates of substitution of persons 1 and 2 faster than the difference between the narrow marginal rates of substitution of persons 1 and 2. Therefore, it must hold that at any allocation in the broad core $x = (x^1, x^2)$, $\widetilde{MRS}^1(x^1) > \widetilde{MRS}^2(x^2)$ while at any allocation in the narrow core $\tilde{x} = (\tilde{x}^1, \tilde{x}^2)$, $MRS^1(\tilde{x}^1) < MRS^2(\tilde{x}^2)$, such that the Euclidean distance between the initial endowment allocation ω and any allocation in the broad core x, $d(x,\omega) = \sqrt{(\omega_1^1 - x_1^1)^2 + (\omega_2^1 - x_2^1)^2}$, is smaller than the Eucleadian distance between the intital endowment allocation ω and any allocation in the narrow core $\tilde{x}$, $d(\omega,\tilde{x}) = \sqrt{(\omega_1^1 - \tilde{x}_1^1)^2 + (\omega_2^1 - \tilde{x}_2^1)^2}$.

Focus now on $\frac{\partial^2 u^i}{\partial x_1^i \partial x_2^i} > 0$ for $i = 1,2$ and ω such that $MRS^1(\omega^1) < MRS^2(\omega^2)$. From Proposition 1 we know that since $r^i = \omega^i$, $MRS^i(\omega^i) = \widetilde{MRS}^i(\omega^i)$ for $i = 1,2$. Therefore $MRS^1(\omega^1) < MRS^2(\omega^2)$ implies $\widetilde{MRS}^1(\omega^1) < \widetilde{MRS}^2(\omega^2)$.

By $MRS^1(\omega^1) < MRS^2(\omega^2)$ and $\widetilde{MRS}^1(\omega^1) < \widetilde{MRS}^2(\omega^2)$ it must hold for any interior broad and narrow core allocations x and $\tilde{x}$, that $x_1^1 \leq \omega_1^1$ and $x_2^1 \geq \omega_2^1$, respectively $\tilde{x}_1^1 \leq \omega_1^1$ and $\tilde{x}_2^1 \geq \omega_2^1$, with one of each of the two inequalities holding strictly.

Now, consider any allocation y with $y_1^1 \leq \omega_1^1$ and $y_2^1 \geq \omega_2^1$, implying $y_1^2 \geq \omega_1^2$ and $y_2^2 \leq \omega_2^2$, where one of the two inequalities holds strictly. By Proposition 1 we have $MRS^1(y^1) > \widetilde{MRS}^1(y^1)$ and $MRS^2(y^2) < \widetilde{MRS}^2(y^2)$.

Thus, starting from the initial endowment allocation ω, decreasing the amount of good 1 allocated to person 1 while increasing the amount of good 2 allocated to person 1 reduces the difference between the broad marginal rates of substitution of consumers 1 and 2 faster than the difference between the narrow marginal rates of substitution of consumers 1 and 2. Therefore, it must hold that at any allocation in the broad core x, $\widetilde{MRS}^1(x) < \widetilde{MRS}^2(x)$ while at any allocation in the narrow core $\tilde{x}$, $MRS^1(\tilde{x}) > MRS^2(\tilde{x})$, such that the Euclidean distance between the initial endowment allocation ω and any allocation in the broad core x, $d(x,\omega) = \sqrt{(\omega_1^1 - x_1^1)^2 + (\omega_2^1 - x_2^1)^2}$, is larger than the Eucleadian distance between the intital endowment allocation ω and any allocation in the narrow core $\tilde{x}$, $d(\omega,\tilde{x}) = \sqrt{(\omega_1^1 - \tilde{x}_1^1)^2 + (\omega_2^1 - \tilde{x}_2^1)^2}$.

The proof for $\frac{\partial^2 u^i}{\partial x_1^i \partial x_2^i} < 0$ for $i = 1,2$ proceeds analogously. $\square$