Economics Department Discussion Papers Series

ISSN 1473 – 3307

Response surface regressions for critical value bounds and approximate p-values in equilibrium correction models

Sebastian Kripfganz and Daniel C. Schneider

Paper number 19/01

URL Repec page: http://ideas.repec.org/s/exe/wpaper.html

Response surface regressions for critical value bounds and approximate p-values in equilibrium correction models*

Sebastian Kripfganz† Daniel C. Schneider‡

February 27, 2019

Abstract

Single-equation conditional equilibrium correction models can be used to test for the existence of a level relationship among the variables of interest. The distributions of the respective test statistics are nonstandard under the null hypothesis of no such relationship and critical values need to be obtained with stochastic simulations. We compute more than 95 billion F-statistics and 57 billion t-statistics for a large number of specifications of the Pesaran, Shin, and Smith (2001, *Journal of Applied Econometrics* 16: 289–326) bounds test. Our large-scale simulations enable us to draw smooth density functions and to estimate response surface models that improve upon and substantially extend the set of available critical values for the bounds test. Besides covering the full range of possible sample sizes and lag orders, our approach notably allows for any number of variables in the long-run level relationship by exploiting the diminishing effect on the distributions of adding another variable to the model. The computation of approximate p-values enables a fine-grained statistical inference and allows us to quantify the finite-sample distortions from using asymptotic critical values. We find that the bounds test can be easily oversized by more than 5 percentage points in small samples.

Keywords: Bounds test; Cointegration; Error correction model; Generalized Dickey-Fuller regression; Level relationship; Unit roots

JEL Classification: C12; C15; C32; C46; C63

*We are grateful for comments by Mehdi Hosseinkouchack.

†Corresponding author: University of Exeter Business School, Department of Economics, Streatham Court, Rennes Drive, Exeter, EX4 4PU, UK. Tel.: +44-1392-722110; E-mail: S.Kripfganz@exeter.ac.uk

‡Max Planck Institute for Demographic Research, Konrad-Zuse-Straße 1, 18057 Rostock, Germany. Tel.: +49-381-2081245; E-mail: schneider@demogr.mpg.de

1 Introduction

The empirical analysis of time series data is often confronted with test statistics that have nonstandard distributions in the presence of a unit root. While the asymptotic distributions can be characterized as functions of stochastic processes such as Brownian motions, the corresponding quantiles that are needed to compute critical values for hypothesis testing are usually obtained with stochastic simulations. As an additional complication, the distributions of the test statistics generally depend on the specific assumptions about the data-generating process and the specification of the estimated model, in particular whether an intercept or time trend are allowed. In a multivariable model, the dimension of the variable space and the cointegration rank matter. Importantly, the finite-sample distributions of the test statistics depend on further characteristics of the estimation. While augmenting the regression model with additional stationary variables does not affect the asymptotic distributions of unit-root and cointegration tests, their influence on the finite-sample distributions can be nonnegligible. Given the vast number of empirically relevant regression specifications that lead to possibly different distributions, the tabulation of critical values quickly approaches space limits and is usually only done for a selected number of situations. This leaves blank areas that can be interpolated only to a limited extent.

All of these remarks apply to the Pesaran et al. (2001) bounds test for the existence of a level relationship in an unrestricted conditional equilibrium correction model. This test is highly prominent among empirical researchers, not least because it evades the necessity of pretesting for the existence of unit roots, assuming that all variables are integrated at most of order one. The test yields conclusive evidence if the value of the test statistic falls outside of the critical-value bounds established for the situations where all long-run forcing variables are purely integrated of either order zero, $I(0)$, or order one, $I(1)$.[1] Because the bounds procedure does not require that all variables are individually $I(1)$, the considered concept of a level relationship is broader than that of cointegration.

Pesaran et al. (2001) derive the asymptotic distributions of their test statistics under the null hypothesis of no level relationship and then use stochastic simulations to compute near-asymptotic critical values. However, the asymptotic distributions might be poor

[1] McNown et al. (2018) propose a bootstrap procedure for the Pesaran et al. (2001) test that allows for conclusive inference when the test statistic falls within the two bounds.

approximations of the actual distributions in small samples. Finite-sample critical values are tabulated by Mills and Pentecost (2001), Narayan and Smyth (2004), Kanioura and Turner (2005), and Narayan (2005), but they cover only a limited portion of the set of possible model specifications and sample sizes. Moreover, the precision of these critical values suffers from a relatively small number of replications in the respective simulations.

In this paper, we set out to systematically approximate the finite-sample and asymptotic distribution functions for the Pesaran et al. (2001) bounds test statistics. We fill the gaps regarding the critical values by estimating response surface (RS) models that predict the quantiles of the distributions as a function of the sample size, lag order, and number of long-run forcing variables. The RS technique was introduced into the field of unit-root testing and cointegration analysis by MacKinnon (1991) for a range of Dickey and Fuller (1979) and Engle and Granger (1987) tests, and has since been applied numerous times.

Ericsson and MacKinnon (2002) provide RS estimates for the cointegration t-statistic in single-equation conditional error correction models that comprise the Dickey-Fuller statistic as a special case. Both asymptotic and finite-sample critical values can be obtained from these estimates.[2] As an important extension, Cheung and Lai (1995a) estimate RS models for the augmented Dickey-Fuller unit-root test, acknowledging the influence of the lag order on the finite-sample distributions.[3] As a complement to the generalized Dickey-Fuller t-statistic, Pesaran et al. (2001) propose a related F-statistic to test for the existence of a level relationship in a conditional equilibrium correction model.[4] So far, the only RS estimates available for this F-statistic stem from Turner (2006) but they again cover only a narrow subset of the empirically relevant situations.

Our work improves and expands on the previous literature in several ways. With the stochastic simulation of more than 95 billion F-statistics and 57 billion t-statistics under several scenarios regarding the deterministic model components, number of variables, sam-

[2]Previously tabulated critical values for a small set of sample sizes can be found in Fuller (1976) and Dickey (1976) for the univariable and Banerjee et al. (1998) for the multivariable setting.

[3]Cook (2001) compares the response surfaces from Cheung and Lai (1995a) with those from MacKinnon (1991) and concludes that adjusting for the lag order leads to a gain in power. RS estimates for finite-sample critical values of other unit-root tests are provided by Cheung and Lai (1995b), Harvey and van Dijk (2006), Otero and Smith (2012, 2017), and Otero and Baum (2017). All of them take the lag order into account. Further related applications of the RS methodology include Sephton (1995, 2008, 2017), Carrion-i-Silvestre et al. (1999), and Presno and López (2003).

[4]In the univariable model with restricted intercept or time trend, this statistic reduces to the Dickey and Fuller (1981) unit-root F-statistic.

ple size, and lag order, we can draw smooth density functions to illustrate how the distributions of the Pesaran et al. (2001) bounds test statistics change along various dimensions. Being based on these large-scale simulations, our RS estimates are both comprehensive and precise. Tabulations for selected combinations of the critical-value determinants and interpolations between them become redundant. While previously reported critical values could not easily be extrapolated beyond the largest number of variables considered in the respective simulations, our modified RS approach does not impose a limit on the number of variables in the level relationship. We achieve this aim by exploiting the monotonically decreasing impact of adding another variable to the model.

Lastly, to facilitate a more informative statistical inference, we adopt the approach of MacKinnon (1994, 1996) to numerically approximate p-values and distribution functions.[5] Together with the critical values from our RS regressions, the approximate p-values can be computed with a program in the statistical software *Stata* (Kripfganz and Schneider, 2018). By comparing p-values, we can meaningfully quantify the finite-sample distortions of the bounds test. While these distortions are relatively small for the t-statistic, we find that the test based on the F-statistic at the 5% and 10% nominal levels can be easily oversized by more than 5 percentage points when using the asymptotic rather than the small-sample critical values. The distortions from ignoring the lag order of the variables in the regression model are less severe, but still relevant, and they can go in either direction.

2 Bounds testing for a level relationship

In this section, we provide a compact summary of the model and assumptions used by Pesaran et al. (2001) to derive the asymptotic distributions of their bounds testing procedure for the existence of a level relationship.

2.1 Equilibrium correction model

Let $\mathbf{z}_t$ be a column vector of $k+1$ random variables, generated by a vector-autoregressive (VAR) model of order q:

$$\mathbf{\Phi}(L)(\mathbf{z}_t - \mathbf{b}_0 - \mathbf{b}_1 t) = \boldsymbol{\epsilon}_t, \quad t = q+1, q+2, \ldots, T, \tag{1}$$

[5] MacKinnon et al. (1999) proceed similarly for cointegration tests in a vector error correction model.

where $\mathbf{\Phi}(L) = \mathbf{I}_{k+1} - \sum_{i=1}^{q} \mathbf{\Phi}_i L^i$ is a q-th order polynomial in the lag operator L with unknown $(k+1) \times (k+1)$ coefficient matrices $\mathbf{\Phi}_i$, and $\mathbf{b}_0$ and $\mathbf{b}_1$ are $(k+1)$-dimensional vectors of unknown intercept and trend parameters. The initial observations $\mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_q$ are assumed to be observed. By defining the long-run multiplier matrix $\mathbf{\Pi} = \sum_{i=1}^{q} \mathbf{\Phi}_i - \mathbf{I}_{k+1}$ and the short-run coefficient matrices $\mathbf{\Gamma}_i = -\sum_{j=i+1}^{q} \mathbf{\Phi}_j$, $i = 1, 2, \ldots, q-1$, we can rewrite the above VAR(q) model in vector equilibrium correction (VEC) form:

$$\Delta \mathbf{z}_t = \mathbf{a}_0 + \mathbf{a}_1 t + \mathbf{\Pi} \mathbf{z}_{t-1} + \sum_{i=1}^{q-1} \mathbf{\Gamma}_i \Delta \mathbf{z}_{t-i} + \boldsymbol{\epsilon}_t, \tag{2}$$

where $\Delta = (1 - L)$ is the first-difference operator, $\mathbf{a}_0 = -\mathbf{\Pi}\mathbf{b}_0 + (\mathbf{\Pi} + \mathbf{\Gamma})\mathbf{b}_1$, $\mathbf{a}_1 = -\mathbf{\Pi}\mathbf{b}_1$, and $\mathbf{\Gamma} = \mathbf{I}_{k+1} - \sum_{i=1}^{q-1} \mathbf{\Gamma}_i$. Let us partition $\mathbf{z}_t = (y_t, \mathbf{x}_t')'$ and the long-run multiplier matrix conformably as

$$\mathbf{\Pi} = \begin{pmatrix} \pi_{yy} & \boldsymbol{\pi}_{yx}' \\ \boldsymbol{\pi}_{xy} & \mathbf{\Pi}_{xx} \end{pmatrix}.$$

Furthermore, partition $\mathbf{\Gamma}_i = (\boldsymbol{\gamma}_{yi}, \mathbf{\Gamma}_{xi}')'$ and $\mathbf{\Gamma} = (\boldsymbol{\gamma}_y, \mathbf{\Gamma}_x')'$.

Pesaran et al. (2001) impose the following assumptions:

Assumption 1: The roots of $|\mathbf{I}_{k+1} - \sum_{i=1}^{q} \mathbf{\Phi}_i z^i| = 0$ satisfy $-1 < 1/z \leq 1$. The data-generating process of $\mathbf{z}_t$ is integrated at most of order unity.[6]

Assumption 2: The vector of errors $\boldsymbol{\epsilon}_t$ is independent multivariate normally distributed, $\boldsymbol{\epsilon}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{\Omega})$, with mean vector zero and positive-definite variance matrix $\mathbf{\Omega}$.

Assumption 3: The data-generating process of $\mathbf{x}_t$ is long-run forcing for the process of y_t, that is $\boldsymbol{\pi}_{xy} = \mathbf{0}$.

Assumption 4: The matrix $\mathbf{\Pi}_{xx}$ has rank r with $0 \leq r \leq k$.

Assumption 1 allows the individual elements of the vector $\mathbf{z}_t$ to be $I(0)$ or $I(1)$, or to be cointegrated. The cointegration order for the data-generating process of $\mathbf{x}_t$ is defined by Assumption 4. Consequently, the rank of the long-run multiplier matrix $\mathbf{\Pi}$ is either r or $r+1$. Assumption 3 implies that $\mathbf{\Pi}$ being of rank r corresponds to the parameter restriction $\pi_{yy} = 0$, while the rank $r+1$ necessitates $\pi_{yy} \neq 0$. Under Assumptions 3 and 4, we can

[6] See Pesaran et al. (2001) for a more formal statement of the last part of this assumption.

express the long-run multiplier matrix as $\mathbf{\Pi} = \boldsymbol{\alpha}_y \boldsymbol{\beta}_y' + \mathbf{AB}'$, where $\boldsymbol{\alpha}_y = (\alpha_{yy}, \mathbf{0}')'$ and $\boldsymbol{\beta}_y = (\beta_{yy}, \boldsymbol{\beta}_{yx}')'$ are $(k+1)$-dimensional vectors, and $\mathbf{A} = (\boldsymbol{\alpha}_{yx}, \mathbf{A}_{xx}')'$ and $\mathbf{B} = (\mathbf{0}, \mathbf{B}_{xx}')'$ are $(k+1) \times r$ matrices of full column rank, respectively.[7] With the normalization $\beta_{yy} = 1$, it follows $\pi_{yy} = \alpha_{yy}$. Clearly, $\mathbf{AB}' = \mathbf{0}$ if $r = 0$.

Under Assumptions 2 and 3, we can now obtain the following equilibrium correction (EC) model for y_t conditional on $\mathbf{x}_t$ and their past values $\mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_{t-1}$:

$$\Delta y_t = c_0 + c_1 t + \boldsymbol{\pi}' \mathbf{z}_{t-1} + \sum_{i=1}^{q-1} \boldsymbol{\psi}_i' \Delta \mathbf{z}_{t-i} + \boldsymbol{\omega}' \Delta \mathbf{x}_t + u_t, \tag{3}$$

with intercept $c_0 = -\boldsymbol{\pi}' \mathbf{b}_0 + [(\boldsymbol{\gamma}_y - \mathbf{\Gamma}_x' \boldsymbol{\omega})' + \boldsymbol{\pi}'] \mathbf{b}_1$ and trend coefficient $c_1 = -\boldsymbol{\pi}' \mathbf{b}_1$, and where $\boldsymbol{\pi} = (\pi_{yy}, \boldsymbol{\varphi}')'$, with $\boldsymbol{\varphi} = \boldsymbol{\pi}_{yx} - \mathbf{\Pi}_{xx}' \boldsymbol{\omega}$. Furthermore, $\boldsymbol{\psi}_i = \boldsymbol{\gamma}_{yi} - \mathbf{\Gamma}_{xi}' \boldsymbol{\omega}$ for all i. With the partition of the error term $\boldsymbol{\epsilon}_t = (\epsilon_{yt}, \boldsymbol{\epsilon}_{xt}')'$ and the conformably partitioned variance matrix

$$\mathbf{\Omega} = \begin{pmatrix} \omega_{yy} & \boldsymbol{\omega}_{xy}' \\ \boldsymbol{\omega}_{xy} & \mathbf{\Omega}_{xx} \end{pmatrix},$$

$\boldsymbol{\omega} = \mathbf{\Omega}_{xx}^{-1} \boldsymbol{\omega}_{xy}$ is obtained as the coefficient vector in the linear projection of ϵ_{yt} on $\boldsymbol{\epsilon}_{xt}$. The corresponding projection error u_t is independent normally distributed under Assumption 2, $u_t \sim \mathcal{N}(\mathbf{0}, \omega_{yy} - \boldsymbol{\omega}_{xy}' \mathbf{\Omega}_{xx}^{-1} \boldsymbol{\omega}_{xy})$.

A conditional level relationship between y_t and $\mathbf{x}_t$ exists if both $\pi_{yy} \neq 0$ and $\boldsymbol{\varphi} \neq \mathbf{0}$, and the data-generating processes of y_t and $\mathbf{x}_t$ are cointegrated if y_t is $I(1)$. In the opposite situation, $\boldsymbol{\pi} = \mathbf{0}$, the conditional EC model (3) only contains first-differenced terms such that no level relationship between y_t and $\mathbf{x}_t$ can exist and y_t must be $I(1)$. There are two degenerate cases. If just $\pi_{yy} = 0$, y_t is still $I(1)$ and there exists only a level relationship among the elements of $\mathbf{x}_t$ not involving y_t. If π_{yy} is the only nonzero element of $\boldsymbol{\pi}$, y_t is generated by a trend-stationary or $I(0)$ process not involving the levels of $\mathbf{x}_t$.

2.2 Bounds test

In the light of the two degenerate situations, the following testing procedure can be applied:

(1) Test the joint null hypothesis $H_0^{\boldsymbol{\pi}} : \boldsymbol{\pi} = \mathbf{0}$ versus $H_1^{\boldsymbol{\pi}} : \boldsymbol{\pi} \neq \mathbf{0}$.

[7] This decomposition is useful for the derivation of the asymptotic distribution of the t-statistic used by Banerjee et al. (1998) to test whether $\pi_{yy} = 0$. See Pesaran et al. (2001) for details.

(2) If $H_0^{\boldsymbol{\pi}}$ is rejected, test the single hypothesis $H_0^{\pi_{yy}}: \pi_{yy} = 0$ versus $H_1^{\pi_{yy}}: \pi_{yy} < 0$, under the additional assumption that either $r = 0$ or $\boldsymbol{\alpha}_{yx} - \mathbf{A}'_{xx}\boldsymbol{\omega} = \mathbf{0}$ if $0 < r \leq k$.

(3) If $H_0^{\pi_{yy}}$ is rejected, test the joint hypothesis $H_0^{\boldsymbol{\theta}}: \boldsymbol{\theta} = \mathbf{0}$ versus $H_1^{\boldsymbol{\theta}}: \boldsymbol{\theta} \neq \mathbf{0}$, where $\boldsymbol{\theta} = -\boldsymbol{\varphi}/\pi_{yy}$ are the long-run multipliers in the conditional level relationship between y_t and $\mathbf{x}_t$.

The reason for proceeding with steps (2) and (3) is that the alternative hypothesis $H_1^{\boldsymbol{\pi}}$ in step (1) does not rule out any of the two degenerate cases mentioned above. The latter are the subject of the hypothesis tests in steps (2) and (3). Only if all three null hypotheses are rejected, we can conclude that there is statistical evidence for the existence of a nondegenerate level relationship between y_t and $\mathbf{x}_t$.

As demonstrated by Pesaran et al. (2001), y_t is $I(1)$ under the null hypothesis in steps (1) and (2) and the respective test statistics have nonstandard asymptotic distributions. The additional assumption required for step (2) implies $\boldsymbol{\varphi} = \pi_{yy}\boldsymbol{\beta}_{yx}$. Consequently, under $H_0^{\pi_{yy}}$ we have again $\boldsymbol{\pi} = \mathbf{0}$ as in step (1), but $H_1^{\pi_{yy}}$ is more informative at the cost of imposing additional structure on the data-generating process. Without this assumption, the asymptotic distribution of the t-statistic would depend on nuisance parameters and tabulations of critical values for general purposes would become practically infeasible.[8]

For the long-run multipliers $\boldsymbol{\theta}$ that are the subject of step (3), Pesaran and Shin (1998) and Hassler and Wolters (2006) show that the ordinary least squares (OLS) estimator is super-consistent if $\mathbf{x}_t$ contains $I(1)$ regressors, and it is asymptotically normally distributed irrespective of the order of integration. This constitutes a practical advantage over tests directly based on $\boldsymbol{\varphi}$ because the latter have nonstandard distributions.[9] The remainder of this text is primarily concerned with the test statistics in steps (1) and (2).

The restricted VAR formulation (1) imposes constraints on the coefficients c_0 and c_1 in the conditional EC model (3) that ensure that the cointegration rank r does not affect the deterministic trending behavior.[10] Pesaran et al. (2001) distinguish five cases, depending

[8] See Pesaran et al. (2001) for a discussion. Banerjee et al. (1998) assume $r = 0$ and briefly argue that the critical values obtained under this assumption will lead to a conservative test if it is violated.

[9] McNown et al. (2018) propose a bootstrap procedure for the inference on the coefficients $\boldsymbol{\varphi}$ of the level regressors. Following the procedure of Pesaran et al. (2001) and Narayan (2005), Sam et al. (2018) tabulate critical values for a Wald test of joint insignificance of up to 7 long-run forcing variables in the level relationship.

[10] See Pesaran et al. (2000) for details.

on which deterministic components are included in the model specification and whether we disregard the implied restrictions on their coefficients or not:

(i) No intercept and no trend are included, $c_0 = c_1 = 0$,

(ii) A restricted intercept is included but no trend, $c_0 = -\boldsymbol{\pi}'\mathbf{b}_0$ and $c_1 = 0$,

(iii) An unrestricted intercept is included but no trend, $c_0 \neq 0$ and $c_1 = 0$,

(iv) An unrestricted intercept and a restricted trend are included, $c_0 \neq 0$ and $c_1 = -\boldsymbol{\pi}'\mathbf{b}_1$,

(v) An unrestricted intercept and an unrestricted trend are included, $c_0 \neq 0$ and $c_1 \neq 0$.

As emphasized by Pesaran et al. (2001), the data-generating processes under case (ii) and (iii) are identical, and similarly for cases (iv) and (v), but the Wald test statistics in step (1) and their asymptotic distributions differ under the null hypothesis $H_0^{\boldsymbol{\pi}}$. For the single-hypothesis test in step (2), the restrictions can be ignored.

Pesaran et al. (2001) argue that the critical values for the two polar cases of $\mathbf{x}_t$ being purely $I(0)$ or purely $I(1)$ provide lower and upper bounds, respectively, when the orders of integration and the cointegration rank r are unknown. They derive the asymptotic distributions of the Wald test statistic in step (1) and the t-statistic in step (2), respectively. Both statistics are functions of standard Brownian motions, de-meaned and de-trended where necessary, and depend on the cointegration rank r.[11]

3 Critical values and approximate p-values

Pesaran et al. (2001) use stochastic simulations to obtain near-asymptotic critical value bounds based on a sample size of 1000 time periods for the F-statistic under $H_0^{\boldsymbol{\pi}}$ in step (1) and the t-statistic under $H_0^{\pi_{yy}}$ in step (2).[12] They tabulate the critical values for the range of $k \in [0, 10]$ long-run forcing variables. Several other authors provide finite-sample critical values for a subset of the relevant situations. We summarize the existing literature in Table 1.[13] A number of authors tabulated critical values that require interpolations

[11] See Theorems 3.1 and 3.2 in Pesaran et al. (2001).

[12] The F-statistic is obtained by dividing the Wald statistic by $k + 1$ in cases (i), (iii), and (v), and by $k + 2$ in cases (ii) and (iv).

[13] The distributions of the cointegration test statistics resulting from the Engle and Granger (1987) two-stage procedure differ from those considered in the Pesaran et al. (2001) framework. Corresponding RS estimates can be found in MacKinnon (1991, 1996, 2010).

Table 1: Critical value tabulations in the previous literature

	$T-q$	q	k	$I(d)$	deterministics cases[+] F	t
Fuller (1976)	$25, 50, 100, 250, 500, \infty$	1	0	–	–	(i), (iii), (v)
Dickey (1976)	$25, 50, 100, 250, 500, 750, \infty$	1	0	–	–	(i), (iii), (v)
Dickey and Fuller (1981)	$25, 50, 100, 250, 500, \infty$	1	0	–	(ii), (iv)	–
MacKinnon (1991, 2010)	RS	1	0	–	–	(i), (iii), (v)
Cheung and Lai (1995a)	RS	≥ 1	0	–	–	(i), (iii), (v)
MacKinnon (1996)*	RS	1	0	–	–	(i), (iii), (v)
Banerjee et al. (1998)	$25, 50, 100, 500, \infty$	1	$[1, 5]$	1	–	(iii), (v)
Pesaran et al. (2001)	1000	0	$[0, 10]$	$0, 1$	(i)–(v)	(i), (iii), (v)
Mills and Pentecost (2001)	$22, 26$	1	3	$0, 1$	(i)–(v)	(i), (iii), (v)
Ericsson and MacKinnon (2002)*	RS	1	$[0, 11]$	1	–	(i), (iii), (v)
Narayan and Smyth (2004)	$22, 25, 30, 37$	0	2	$0, 1$	(ii)	–
Kanioura and Turner (2005)**	$50, 100, 200, 500$	$0/1$	$[1, 3]$	1	(iii)	(i)
Narayan (2005)	30–80 in steps of 5	0	$[0, 7]$	$0, 1$	(ii)–(v)	–
Turner (2006)	RS	1	$[1, 3]$	$0, 1$	(iii), (v)	–

Note: The regression model used to compute the F-statistics and t-statistics can be written as in equation (6) with q lags and k long-run forcing variables that are integrated of order d. For the unit-root tests, i.e. $k = 0$, the specifications are equivalent for $q = 0$ and $q = 1$.
*MacKinnon (1996) and Ericsson and MacKinnon (2002) provide computer programs that compute the critical values and approximate p-values.
**Kanioura and Turner (2005) compute their test statistics from different regression specifications. Their F-statistic is based on $q = 1$ and their t-statistic on $q = 0$. The latter is only tabulated for $k = 1$.
+MacKinnon (1996, 2010) and Ericsson and MacKinnon (2002) furthermore consider the t-statistic in the presence of a quadratic trend.

between the reported sample sizes. Accordingly, they are unanimously superseded by the estimates from RS regressions, whenever the latter are available and sufficiently precise.

Although unit-root tests are not the primary focus of our work, the Dickey-Fuller test statistics result as a special case in the univariable setting, $k = 0$. When there is no need for a lag augmentation, the RS estimates of MacKinnon (1996, 2010) and Ericsson and MacKinnon (2002) are the primary source for accurate finite-sample critical values, as far as the t-statistic is concerned. In many situations, however, serial error correlation threatens to undermine the validity of the test. A remedy is the augmented Dickey-Fuller test based on a higher-order autoregressive model. The test statistic remains the same, and Said and Dickey (1984) prove that its asymptotic distribution is unaffected as well. However, the degrees-of-freedom reduction affects the finite-sample distributions. The RS from Cheung and Lai (1995a) provides more accurate critical values in that situation. For the unit-root F-statistic, we are the first to provide comprehensive RS estimates.[14]

[14]Dickey and Fuller (1981) tabulate a few critical values for the restricted intercept or trend cases (ii) and (iv). While the F-statistic in the unrestricted cases (i), (iii), and (v) equals the square of the t-statistic, this is not true for the quantiles of the corresponding distributions. Consequently, separate critical values

In the multivariable setting, the lag order dependence of finite-sample critical values has been neglected completely so far. A stronger emphasis has been put on the number of variables in the level relationship. The RS estimates from Ericsson and MacKinnon (2002) cover the cointegration t-statistic for up to 11 long-run forcing variables that are purely $I(1)$. For the F-statistic, the coverage is much thinner. To date, only Turner (2006) provides such RS estimates, but merely for cases (iii) and (v) and a small number of up to 3 long-run forcing variables.

3.1 Monte Carlo simulations

To improve upon and substantially expand existing critical-value tabulations via RS regressions, we start by computing empirical distribution functions (EDFs) for the F- and t-statistic under a variety of scenarios. The respective quantiles from these EDFs will be used in the subsequent RS analysis. For each replication in our Monte Carlo simulations, we generate the data according to the following processes that satisfy $H_0^{\boldsymbol{\pi}}$ and $H_0^{\pi_{yy}}$:

$$y_t = y_{t-1} + \epsilon_{yt}, \tag{4}$$

$$\mathbf{x}_t = \mathbf{P}\mathbf{x}_{t-1} + \boldsymbol{\epsilon}_{xt}, \tag{5}$$

for $t = 1, 2, \ldots, T + 50$ and with the initializations $y_0 = 0$ and $\mathbf{x}_0 = \mathbf{0}$. The first 50 observations are discarded. The elements of the vector of shocks $\boldsymbol{\epsilon}_t$ are independently drawn from the standard normal distribution. The matrix $\mathbf{P}$ equals either the zero or the identity matrix, depending on whether $\mathbf{x}_t$ is supposed to be purely $I(0)$ or $I(1)$.[15]

The test statistics are constructed from the unrestricted regression coefficients in a reparameterization of equation (3):

$$\Delta y_t = c_0 + c_1 t + \pi_{yy} y_{t-1} + \boldsymbol{\varphi}' \mathbf{x}_t + \sum_{i=1}^{q-1} \psi_{yi} \Delta y_{t-i} + \sum_{i=0}^{q-1} \boldsymbol{\psi}'_{xi} \Delta \mathbf{x}_{t-i} + u_t, \tag{6}$$

where $(\psi_{yi}, \boldsymbol{\psi}'_{xi})' = \boldsymbol{\psi}_i$ for all $i = 1, 2, \ldots, q-1$. The use of the contemporaneous $\mathbf{x}_t$ instead of the lagged $\mathbf{x}_{t-1}$ is advocated by Pesaran and Shin (1998). It has the advantage that the short-run coefficients $\boldsymbol{\psi}_{xi}$ can be treated as unrestricted for all lag orders q, while in the

need to be obtained.

[15] The data-generating process is identical to the one used by Pesaran et al. (2001), besides the discarded observations.

representation (3) the presence of the term $\boldsymbol{\omega}'\Delta\mathbf{x}_t$ induces an overparameterization when $q = 0$.[16] In cases (i), (iii), and (v), under the null hypothesis $H_0^{\boldsymbol{\pi}}$, the F-statistic is used to test for joint insignificance of the level regressors y_{t-1} and $\mathbf{x}_t$ in equation (6). In cases (ii) and (iv), the respective exclusion restriction on the intercept c_0 or trend coefficient c_1 is added. Under $H_0^{\pi_{yy}}$, the t-statistic is computed for π_{yy}.

For each of the 2 integration orders and 5 deterministic model component cases, we run separate simulations for all combinations of $k \in [0, 10]$,

$$T \in \{18, 20, 22, 25, 28, 30, 32, 36, 40, 50, 60, 80, 100, 150, 200, 300, 400, 500, 1000\},$$

and $q \in \{0, 1, 2, 3, 4, 6, 8, 12\}$, subject to the restriction that there are at least twice as many observations as coefficients in equation (6) to ensure a sufficient number of degrees of freedom.[17] This yields a total of 9,528 simulation designs.[18] For each design, we run 100,000 replications and then repeat the entire procedure 100 times, which we refer to as 'meta replications'. We thus compute a total number of 9.528×10^{10} F-statistics and 5.744×10^{10} t-statistics.[19] To reduce the storage memory requirements for such a large number of test statistics, we first round the statistics to three digits after the decimal point and then apply a reversible transformation in terms of first differences of sorted statistics and occurrence counts.[20] The effect of rounding on the RS regressions is absolutely negligible.

The 10 million statistics for each configuration are sufficiently many to draw smooth probability density functions without the need for sophisticated kernel density estimators. With a bin width of 0.1, Figure 1 is obtained by connecting the points that result from counting the number of simulated test statistics for each bin (divided by the total number

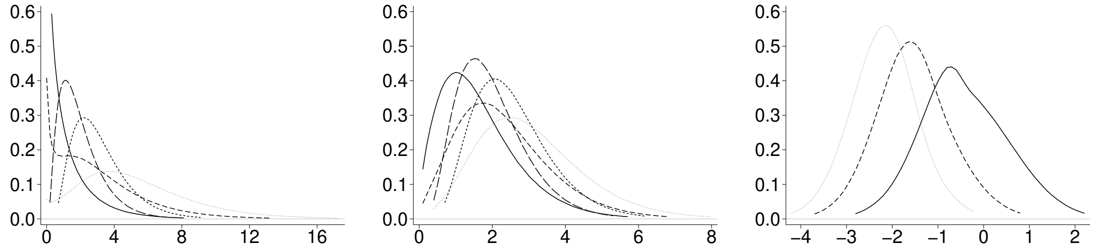
[16]The lag specification $q = 0$ can be obtained from the VAR(1) model in equation (1) by imposing the restriction $\boldsymbol{\omega} = \boldsymbol{\varphi}$.

[17]That is $\max(1, q) + k(q + 1) + \mathcal{I}(c_0 \neq 0) + \mathcal{I}(c_1 \neq 0) \leq (T - \max(q, 1))/2$, where $\mathcal{I}(\cdot)$ is an indicator function that equals unity if the respective deterministic component is included and zero otherwise. The effective sample size is $T - \max(q, 1)$. The distinction between $q = 0$ and $q = 1$ is irrelevant when $k = 0$.

[18]There are 1,960 simulation designs for case (i), 1,910 designs for cases (ii) and (iii) each, and 1,874 designs for cases (iv) and (v), respectively.

[19]There is no longer a computational reason as in MacKinnon (1996) for the use of meta replications instead of a single experiment with 10 million replications. His second argument, that meta replications provide an easy way to evaluate the experimental randomness, survives.

[20]Details on the compression procedure as well as other computational aspects are relegated to the Supplementary Appendix.



(a) F-statistic, $k = 0$ (b) F-statistic, $k = 2$ (c) t-statistic, $k = 0$

Figure 1: Probability density functions obtained from the 10^7 simulated test statistics for cases (i)-(v) with sample size $T = 1000$ and lag order $q = 1$. With increasing case number, the curves have shorter dashes. For $k = 2$, the upper-bound densities are shown.

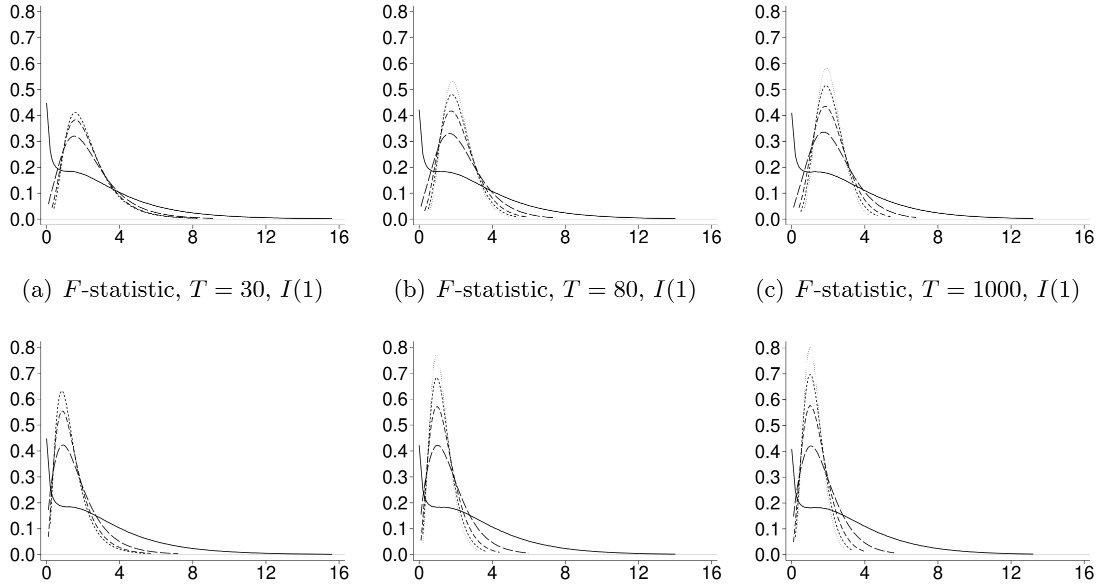
of test statistics and the bin width).[21] In particular for the F-statistic, the shape of the distributions varies quite a bit depending on the deterministic model components. This is illustrated in Figure 1 for a sample size of $T = 1000$ that was considered by Pesaran et al. (2001) in their simulation of near-asymptotic critical values.

In the univariable situation, $k = 0$, we observe unimodal densities in cases (ii) and (iv) with a restricted intercept or trend. In case (i) without any deterministic component, the density is zeromodal. The density in case (iii) with an unrestricted intercept looks similar in that it is downward sloping almost everywhere, but with a saddle point or tiny mode after the initial steep descent. In the unrestricted trend case (v), we observe a local minimum close to the origin. In the multivariable designs, all densities have the expected unimodal shape with positive skewness. For the t-statistic, the densities have the familiar bell shape but are not centered around zero. With increasing case number, the mode moves further away from zero and the dispersion becomes smaller. In the following, we restrict the discussion primarily to the empirically most often applied case (iii).

Figures 2 and 3 highlight the variation of the densities across the number of variables k, separately for different sample sizes. For the F-statistic, the probability mass around the mode is increasing in both k and T but the mode itself remains fairly stable. The shape of the distributions is quite similar when all long-run forcing variables are $I(1)$ compared to when they are $I(0)$. For obvious reasons, the corresponding quantiles are found closer to zero for the lower-bound distributions.[22] For the t-statistic, some differences arise.

[21] We restrict the plots of density and distribution functions to the quantile interval $p \in [0.005, 0.995]$.

[22] For $k = 0$, the upper-bound and lower-bound densities coincide.



(a) F-statistic, $T = 30$, $I(1)$ (b) F-statistic, $T = 80$, $I(1)$ (c) F-statistic, $T = 1000$, $I(1)$

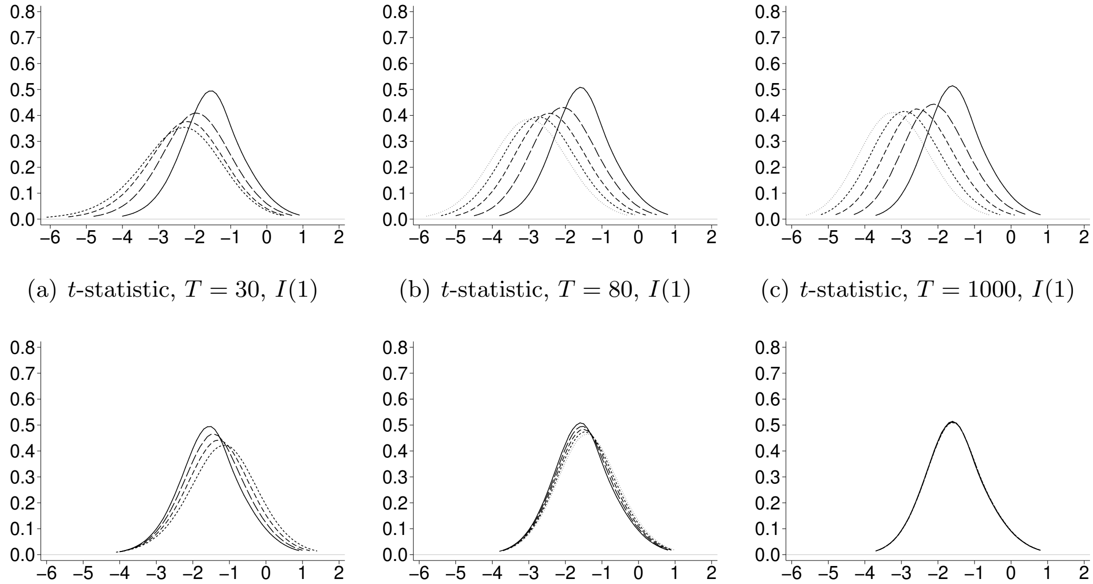
(d) F-statistic, $T = 30$, $I(0)$ (e) F-statistic, $T = 80$, $I(0)$ (f) F-statistic, $T = 1000$, $I(0)$

Figure 2: Upper-bound and lower-bound probability density functions obtained from the 10^7 simulated F-statistics in case (iii) with $k \in \{0, 2, 4, 6, 8\}$ variables and lag order $q = 1$. The solid curve refers to $k = 0$. With increasing k, the curves have shorter dashes.

The densities are as well less dispersed with larger sample size but more dispersed with increasing number of variables. While the upper-bound densities become more distinct with increasing sample size and their quantiles grow with k, the opposite is true for the lower bound. As formally shown by Pesaran et al. (2001), the distributions of the t-statistic asymptotically no longer depend on the number of $\mathbf{x}_t$ variables when all of them are $I(0)$.[23]

We can construct such probability density functions for any of our simulation designs. By sorting the 10^7 simulated test statistics in ascending order, it is straightforward to obtain the corresponding quantiles of interest. For example, in case (iii), the 95-th percentile of the F-statistic with $k = 2$ long-run forcing variables that are $I(1)$, $T = 1000$ observations, and a lag order of $q = 2$ is found to be 4.81. Pesaran et al. (2001) report a critical value of 4.85 for the same setup. The difference between these two numbers is within the range of the simulation uncertainty that can be measured by the variation across the 100 meta-replication EDFs, each of them based on 10^5 replications instead of the 10^7 replica-

[23] When $T = 1000$, the upper-bound densities for the t-statistic look very similar to the asymptotic density functions plotted by Ericsson and MacKinnon (2002).



(a) t-statistic, $T = 30$, $I(1)$ (b) t-statistic, $T = 80$, $I(1)$ (c) t-statistic, $T = 1000$, $I(1)$

(d) t-statistic, $T = 30$, $I(0)$ (e) t-statistic, $T = 80$, $I(0)$ (f) t-statistic, $T = 1000$, $I(0)$

Figure 3: Upper-bound and lower-bound probability density functions obtained from the 10^7 simulated t-statistics in case (iii) with $k \in \{0, 2, 4, 6, 8\}$ variables and lag order $q = 1$. The solid curve refers to $k = 0$. With increasing k, the curves have shorter dashes.

tions used to construct the aggregate EDFs. For our example, the observed quantiles fall into the interval $[4.77, 4.86]$ with a coefficient of variation of 0.29%. This number is close to the average of 0.30% over all simulation designs for the F-statistic. The further we go into the tail of the distribution, the more noisy the quantile estimates are. For the 99-th percentile, the average coefficient of variation is 0.51%. In the Supplementary Appendix, we show that the variation tends to shrink with larger T and larger k, and that it is larger for the lower than for the upper bound. For the t-statistic, the coefficient of variation is a bit smaller in absolute terms, on average 0.21% for the 95-th percentile and 0.33% for the 99-th percentile.

Due to the independence of the replications, we can infer statements about the precision of the aggregate EDFs. Since their number of replications exceeds that of the meta replications by factor 100, the respective coefficient of variation is an order of magnitude smaller than for a single meta replication. In the above example, this implies a coefficient of variation of 0.03% for the 95-th percentile of the F-statistic. By contrast, for 40,000 replications, as performed by Pesaran et al. (2001), it would be about 0.46% which is still a nonnegligible amount of variation. This is best seen by noting that their tabulated critical value of 4.85 corresponds to a p-value of 0.048 rather than 0.05 when we use our aggregate EDF as the reference distribution. Similar arguments apply to the finite-sample critical values tabulated by Narayan (2005) that do not comply with the monotonic decline of the finite-sample toward the asymptotic quantiles due to the experimental randomness.

3.2 Response surface regressions

The tabulation of all empirically relevant critical values would be cumbersome since it would stretch dozens of pages. Moreover, even though we have obtained EDFs from 9,528 simulation designs, they still do not cover the whole spectrum of sample sizes, lag orders, and variable counts. In the following, we thus estimate RS models that allow us to predict critical values for any point in this three-dimensional space.

For each meta replication and simulation design, we compute the quantiles of interest from the EDFs of both test statistics. In the previous literature, the most relevant quantiles have either been tabulated or used in RS regressions for a given number of k long-run forcing variables. The RS models are usually estimated by regressing the simulated quantiles on a polynomial in the reciprocal of the sample size. To account for the increasing relevance of the lag order in smaller samples, Cheung and Lai (1995a) have added a polynomial in the lag order divided by the sample size. The intercept in such a regression can be interpreted as the quantile of the asymptotic distribution.

In the Supplementary Appendix to this paper, we proceed similarly by estimating RS regressions for each quadruplet $\{c, k, d, p\}$, where c is the case regarding the deterministic model components, k is the number of long-run forcing variables with integration order d, and p is the level of the quantile. For the limited number of congruent scenarios, the estimated RS hardly differs from those of Turner (2006) for the F-statistic and MacKinnon (2010) and Ericsson and MacKinnon (2002) for the t-statistic. Yet, their critical values are no longer ideal for higher lag orders in equation (6). For most sample sizes, they are too conservative, to such an extent that even the asymptotic critical values would provide a better approximation. The Cheung and Lai (1995a) RS addresses this problem but is slightly skewed towards zero compared to ours.[24]

[24] See our Supplementary Appendix for a graphical comparison. Merely adjusting the sample size for the number of estimated coefficients, as done by Ericsson and MacKinnon (2002), does not prove to be a successful strategy.

Table 2: Response surface estimates, unrestricted deterministic terms

	F-statistic, case (i)						t-statistic, case (i)					
	α = 1%		α = 5%		α = 10%		α = 1%		α = 5%		α = 10%	
	$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$
$\theta_{0,0,0}$	1.370	2.428	1.294	2.362	1.237	2.296	-2.564	-7.317	-1.940	-6.695	-1.617	-6.353
$\theta_{1,0,0}$	10.654	14.007	6.241	7.954	4.337	5.360	–	28.072	–	28.027	–	27.949
$\theta_{2,0,0}$	-13.656	-26.006	-8.832	-15.927	-6.701	-11.865	–	-83.108	–	-82.935	–	-82.740
$\theta_{3,0,0}$	15.416	31.917	9.459	18.112	7.078	13.064	–	113.408	–	113.362	–	113.190
$\theta_{4,0,0}$	-6.763	-15.213	-4.000	-8.296	-2.952	-5.833	–	-53.657	–	-53.709	–	-53.665
$\theta_{0,1,0}$	43.83	89.09	22.34	46.70	15.23	31.65	-8.30	-7.72	-1.81	15.47	0.84	24.49
$\theta_{1,1,0}$	-300.21	-733.56	-141.43	-368.29	-91.73	-247.91	58.11	-27.39	10.67	-216.22	-7.30	-288.60
$\theta_{2,1,0}$	974.32	2450.58	426.74	1192.90	263.51	792.20	-190.47	251.06	-34.11	862.47	24.67	1095.71
$\theta_{3,1,0}$	-1361.79	-3547.99	-577.89	-1701.90	-347.94	-1121.25	268.59	-471.43	47.19	-1335.58	-35.49	-1663.50
$\theta_{4,1,0}$	652.69	1734.59	272.63	826.60	161.97	542.51	-128.90	256.89	-22.30	673.34	17.32	830.90
$\theta_{0,2,0}$	452.2	878.7	186.3	360.1	98.8	205.2	-77.8	-104.7	-17.9	19.3	-1.3	60.1
$\theta_{0,3,0}$	-2057	-4987	-1061	-2280	-573	-1327	409	369	71	-463	-31	-769
$\theta_{0,1,1}$	-0.75	-0.41	-0.57	-0.22	-0.49	-0.15	0.14	1.53	0.09	1.65	0.10	1.76
$\theta_{1,1,1}$	1.20	3.90	0.49	2.66	0.35	1.94	-0.55	-9.56	0.14	-9.37	0.26	-9.52
$\theta_{2,1,1}$	-9.03	-7.87	-6.08	-10.42	-4.56	-8.82	1.25	31.43	-0.69	29.58	-1.17	29.52
$\theta_{3,1,1}$	19.61	-3.61	14.78	10.74	11.60	11.15	0.21	-40.41	1.90	-38.66	2.31	-39.01
$\theta_{4,1,1}$	-12.45	5.66	-9.34	-3.76	-7.32	-4.66	-0.57	18.11	-1.14	17.58	-1.28	17.91
$\theta_{0,2,1}$	39.3	74.3	27.9	40.0	20.9	25.7	-12.9	-38.6	-4.9	-26.4	-1.8	-22.9
$\theta_{0,3,1}$	332	56	-76	-179	-108	-152	38	270	10	226	-9	213
$\bar{R}^2$	0.9980	0.9934	0.9982	0.9927	0.9977	0.9898	0.9716	0.9987	0.9249	0.9993	0.7784	0.9993
RMSE	0.0769	0.1146	0.0344	0.0453	0.0230	0.0271	0.0164	0.0313	0.0077	0.0218	0.0060	0.0210

Note: The RS regression model is equation (8). The dependent variable is the simulated α-quantile of the test statistic. $\bar{R}^2$ denotes the adjusted coefficient of determination, and RMSE the root mean square error.

Carrying out RS estimations separately for each k has two shortcomings. First, this approach does not allow to obtain critical values if the actual number of long-run forcing variables has not been considered in the simulations. Second, any attempt to cover a larger range of k inflates the number of regression results that need to be tabulated or stored in a computer program. In the following, we overcome this problem by directly modeling the RS as a function of k. A close look at either the existing RS estimates or those from our Supplementary Appendix reveals that the marginal differences between the quantiles become smaller with increasing k. This suggests to model this diminishing slope with negative powers in the total number of variables $1 + k$. Thus, for each triplet $\{c, d, p\}$, we consider the following regression:

$$Q(k, T, q) = \sum_{i=0}^{r} \sum_{j=0}^{m} \sum_{l=0}^{n} \theta_{i,j,l}(1 + k)^{-i}[N(T, q)]^{-j}[H(q, k)]^{l} + \nu, \tag{7}$$

where $Q(k, T, q)$ refers to the quantiles from the meta-replication EDFs, $N(T, q) = T - \max(q, 1)$ is the effective sample size, $H(q, k) = \max(q - 1, 0) + kq$ denotes the number of unrestricted short-run coefficients in equation (6), and ν is the regression error. The lag order q is uninformative for the asymptotic quantiles which implies the restrictions $\theta_{i,0,l} = 0$ for all $l > 0$. The intercept $\theta_{0,0,0}$ has the interpretation as the asymptotic quantile when both $T \to \infty$ and $k \to \infty$. For a given k, the respective asymptotic quantile

Table 3: Response surface estimates, unrestricted deterministic terms (continued)

	F-statistic, case (iii)						t-statistic, case (iii)					
	$\alpha = 1\%$		$\alpha = 5\%$		$\alpha = 10\%$		$\alpha = 1\%$		$\alpha = 5\%$		$\alpha = 10\%$	
	$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$
$\theta_{0,0,0}$	1.350	2.470	1.277	2.375	1.223	2.301	-3.435	-7.468	-2.864	-6.842	-2.569	-6.499
$\theta_{1,0,0}$	13.398	15.681	8.881	10.044	6.842	7.510	–	26.700	–	26.447	–	26.265
$\theta_{2,0,0}$	-8.848	-18.941	-5.950	-12.381	-4.504	-9.456	–	-81.360	–	-80.567	–	-80.119
$\theta_{3,0,0}$	10.717	23.748	7.242	15.109	5.496	11.500	–	111.526	–	110.499	–	109.953
$\theta_{4,0,0}$	-4.794	-11.014	-3.230	-6.880	-2.447	-5.219	–	-52.870	–	-52.412	–	-52.172
$\theta_{0,1,0}$	44.03	82.07	22.41	43.84	15.25	30.05	-5.17	-4.98	3.98	17.93	7.89	27.14
$\theta_{1,1,0}$	-237.10	-559.08	-125.01	-299.89	-86.71	-209.39	6.22	-87.41	-48.81	-256.41	-73.15	-323.49
$\theta_{2,1,0}$	705.90	1722.88	353.20	914.50	233.62	636.50	4.98	491.89	169.07	1008.33	244.29	1215.15
$\theta_{3,1,0}$	-852.78	-2300.59	-430.58	-1230.48	-281.74	-861.45	-27.27	-849.87	-245.13	-1551.62	-347.31	-1833.77
$\theta_{4,1,0}$	370.54	1067.71	188.68	573.94	122.68	403.54	18.68	449.04	119.55	779.22	167.47	912.20
$\theta_{0,2,0}$	458.0	937.5	243.4	434.7	151.9	263.0	-132.3	-116.8	-51.8	25.0	-26.5	72.4
$\theta_{0,3,0}$	-569	-4086	-1161	-2636	-843	-1731	699	321	319	-553	178	-895
$\theta_{0,1,1}$	-0.38	0.01	-0.41	-0.04	-0.40	-0.04	0.49	1.56	0.53	1.74	0.57	1.86
$\theta_{1,1,1}$	-4.91	-1.84	-3.11	-0.19	-2.44	0.19	-0.27	-8.61	-0.26	-9.03	-0.24	-9.29
$\theta_{2,1,1}$	17.92	23.52	7.35	4.74	4.82	0.25	-3.46	27.14	-1.83	28.84	-1.60	29.72
$\theta_{3,1,1}$	-32.52	-63.72	-12.47	-20.86	-8.01	-9.67	10.25	-32.81	5.50	-37.39	4.40	-39.51
$\theta_{4,1,1}$	16.60	37.60	5.76	12.97	3.48	6.46	-6.34	14.02	-3.39	16.91	-2.64	18.23
$\theta_{0,2,1}$	3.3	51.2	14.7	35.0	13.6	24.8	-12.4	-40.0	-3.9	-28.4	-0.9	-25.3
$\theta_{0,3,1}$	1723	1352	423	264	154	70	-87	166	-60	184	-55	197
$\bar{R}^2$	0.9992	0.9981	0.9995	0.9988	0.9996	0.9990	0.9812	0.9977	0.9733	0.9986	0.9767	0.9986
RMSE	0.0917	0.1235	0.0433	0.0548	0.0294	0.0357	0.0211	0.0328	0.0109	0.0239	0.0086	0.0232

Note: The RS regression model is equation (8). The dependent variable is the simulated α-quantile of the test statistic. $\bar{R}^2$ denotes the adjusted coefficient of determination, and RMSE the root mean square error.

can be computed from the coefficients $\theta_{i,0,0}$. When $k = 0$, it is $\sum_{i=0}^{r} \theta_{i,0,0}$.

Given the 100 meta replications for each feasible combination of k, T, and q, taking into account the restriction on the degrees of freedom, our regressions are performed on 98,000 observations for case (i), 95,500 observations for cases (ii) and (iii), and 93,700 observations for cases (iv) and (v). While these large numbers of observations imply that the estimation uncertainty conditional on the chosen model becomes practically irrelevant, the uncertainty about the correct specification of the RS remains.[25] Regarding the choice of the polynomial orders r, m, and n, there is no clear guidance and the optimal order possibly differs across the many regressions. As emphasized by MacKinnon (1996), it is important to choose the same specification across quantiles in order to avoid discontinuities in the distributions that are inferred from the predicted values. After extensive experimentation, we found that the polynomial orders $r = 4$, $m = 3$, and $n = 1$ yield satisfactory regression fits, as indicated by the adjusted R-squared or the root mean square error (RMSE). In addition, the coefficients of the interaction terms of the variable count with the inverse sample size are often statistically insignificant when the latter is raised to a higher power. We thus set $\theta_{i,j,l} = 0$ when both $i > 0$ and $j > 1$ to obtain a more

[25] The variance of the regression errors is a decreasing function in the effective sample size $N(T, q)$ which could be taken into account with a generalized least squares procedure (MacKinnon, 1991) or a generalized method of moments estimator (MacKinnon, 1994, 1996). However, the numerical differences in the predictions are negligible, in particular in the light of the remaining model uncertainty.

Table 4: Response surface estimates, unrestricted deterministic terms (continued)

	F-statistic, case (v)						t-statistic, case (v)					
	$\alpha = 1\%$		$\alpha = 5\%$		$\alpha = 10\%$		$\alpha = 1\%$		$\alpha = 5\%$		$\alpha = 10\%$	
	$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$
$\theta_{0,0,0}$	1.323	2.484	1.259	2.378	1.208	2.302	-3.964	-7.612	-3.414	-6.985	-3.130	-6.641
$\theta_{1,0,0}$	16.641	17.897	11.876	12.341	9.707	9.756	–	25.239	–	24.831	–	24.559
$\theta_{2,0,0}$	-6.747	-13.778	-4.647	-9.180	-3.621	-6.812	–	-78.352	–	-77.082	–	-76.366
$\theta_{3,0,0}$	7.709	16.603	5.590	11.167	4.487	8.296	–	108.219	–	106.508	–	105.595
$\theta_{4,0,0}$	-3.251	-7.408	-2.428	-5.007	-1.983	-3.718	–	-51.493	–	-50.695	–	-50.278
$\theta_{0,1,0}$	42.71	75.20	21.33	40.54	14.66	27.96	-1.98	-2.97	8.09	19.65	12.51	29.06
$\theta_{1,1,0}$	-166.96	-396.09	-96.37	-228.63	-72.16	-163.75	-47.42	-135.03	-96.70	-285.96	-120.31	-349.46
$\theta_{2,1,0}$	427.49	1081.49	243.34	645.54	178.21	463.61	214.91	675.34	340.16	1109.10	406.41	1297.36
$\theta_{3,1,0}$	-296.67	-1179.41	-204.69	-764.88	-162.47	-563.07	-354.74	-1135.95	-499.93	-1699.38	-583.36	-1948.87
$\theta_{4,1,0}$	53.38	461.64	57.77	322.48	52.23	242.26	185.12	595.01	245.93	852.34	283.04	967.67
$\theta_{0,2,0}$	492.7	959.8	329.8	507.1	222.8	321.5	-171.0	-129.7	-70.4	30.2	-35.3	86.3
$\theta_{0,3,0}$	1789	-1775	-1225	-2687	-1173	-2049	816	209	425	-678	239	-1056
$\theta_{0,1,1}$	0.16	0.60	-0.16	0.20	-0.23	0.11	0.66	1.54	0.77	1.80	0.84	1.95
$\theta_{1,1,1}$	-14.18	-10.45	-8.74	-4.12	-6.79	-2.40	1.01	-6.53	-0.03	-8.20	-0.38	-8.75
$\theta_{2,1,1}$	56.97	63.24	28.92	21.95	20.28	10.89	-10.68	18.82	-4.11	26.70	-2.15	29.03
$\theta_{3,1,1}$	-96.58	-131.54	-48.21	-51.84	-33.75	-29.98	23.05	-19.46	10.04	-34.57	6.01	-39.15
$\theta_{4,1,1}$	49.06	72.15	23.74	28.66	16.32	16.76	-13.18	7.16	-5.91	15.61	-3.61	18.21
$\theta_{0,2,1}$	-22.4	33.3	8.1	33.8	10.7	25.8	-15.8	-44.4	-5.6	-32.4	-1.6	-28.8
$\theta_{0,3,1}$	3273	2911	924	774	399	334	-164	96	-108	159	-96	184
$\bar{R}^2$	0.9991	0.9986	0.9994	0.9991	0.9996	0.9993	0.9836	0.9966	0.9777	0.9976	0.9796	0.9976
RMSE	0.1414	0.1584	0.0730	0.0794	0.0518	0.0558	0.0261	0.0350	0.0149	0.0263	0.0121	0.0255

Note: The RS regression model is equation (8). The dependent variable is the simulated α-quantile of the test statistic. $\bar{R}^2$ denotes the adjusted coefficient of determination, and RMSE the root mean square error.

parsimonious model. Incorporating all the restrictions, equation (7) becomes

$$Q(k,T,q) = \theta_{0,0,0} + \sum_{i=1}^{4} \theta_{i,0,0} \frac{1}{(1+k)^i} + \sum_{j=1}^{3} \theta_{0,j,0} \frac{1}{[N(T,q)]^j} + \sum_{i=1}^{4} \theta_{i,1,0} \frac{1}{(1+k)^i N(T,q)} + \sum_{j=1}^{3} \theta_{0,j,1} \frac{H(q,k)}{[N(T,q)]^j} + \sum_{i=1}^{4} \theta_{i,1,1} \frac{H(q,k)}{(1+k)^i N(T,q)} + \nu. \quad (8)$$

For the t-statistic, as shown by Pesaran et al. (2001), the asymptotic distribution does not depend on k when all variables are $I(0)$. Hence, we further restrict $\theta_{i,0,0} = 0$ for all $i > 0$ in this situation.

The OLS estimates are presented in Tables 2 to 5 for the quantiles corresponding to a nominal size of 1%, 5%, and 10%.[26] For any given k, the fit from equation (8) is expected to be worse than from the tailored regressions in the Supplementary Appendix. However, Figure 4 illustrates that the use of the joint RS model is justified since the differences to the separate RS estimates for each k and the simulated quantiles from our aggregate EDFs are negligible. By contrast, the simple "meta response surface" estimated by Ericsson and MacKinnon (2002) for the asymptotic quantiles as an affine-linear function of k and the number of deterministic model components is only useful as a crude approximation. It does not readily extend to larger models because it ignores the diminishing slope of the

[26] The coefficient estimates for other quantiles are available upon request.

Table 5: Response surface estimates, restricted deterministic terms

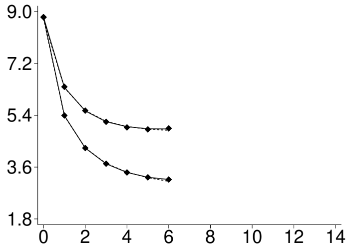
	F-statistic, case (ii)						F-statistic, case (iv)					
	$\alpha = 1\%$		$\alpha = 5\%$		$\alpha = 10\%$		$\alpha = 1\%$		$\alpha = 5\%$		$\alpha = 10\%$	
	$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$
$\theta_{0,0,0}$	1.402	2.528	1.306	2.405	1.244	2.325	1.361	2.527	1.280	2.401	1.224	2.318
$\theta_{1,0,0}$	12.221	12.934	8.078	7.882	6.227	5.588	15.635	15.448	11.133	10.309	9.108	7.967
$\theta_{2,0,0}$	-16.722	-24.920	-11.142	-16.007	-8.568	-11.806	-19.214	-24.208	-13.454	-16.092	-10.848	-12.298
$\theta_{3,0,0}$	16.234	29.080	10.721	18.710	8.205	13.918	17.496	25.719	12.043	16.895	9.599	12.815
$\theta_{4,0,0}$	-6.630	-13.008	-4.324	-8.309	-3.295	-6.186	-6.911	-11.008	-4.700	-7.166	-3.721	-5.413
$\theta_{0,1,0}$	39.76	79.52	21.22	44.06	14.65	30.62	39.35	73.26	21.36	41.49	14.97	29.18
$\theta_{1,1,0}$	-211.81	-555.43	-109.35	-302.87	-74.46	-210.73	-153.90	-402.41	-91.07	-236.86	-67.03	-170.09
$\theta_{2,1,0}$	606.82	1660.56	298.97	909.71	195.96	634.36	376.10	1053.62	230.11	654.85	167.88	477.12
$\theta_{3,1,0}$	-772.21	-2238.76	-372.89	-1231.82	-239.17	-860.62	-343.58	-1227.71	-235.23	-810.13	-174.78	-598.95
$\theta_{4,1,0}$	348.52	1048.84	166.63	578.75	105.58	404.89	107.55	512.69	86.20	355.54	65.80	265.81
$\theta_{0,2,0}$	555.4	997.8	249.9	439.1	147.9	264.0	605.5	1043.4	317.1	499.1	203.9	311.8
$\theta_{0,3,0}$	-1810	-4950	-1216	-2628	-779	-1671	-83	-3304	-1119	-2548	-926	-1817
$\theta_{0,1,1}$	-0.61	-0.16	-0.53	-0.11	-0.48	-0.08	-0.04	0.45	-0.27	0.14	-0.30	0.08
$\theta_{1,1,1}$	-2.98	-0.25	-2.05	0.67	-1.74	0.74	-13.29	-9.71	-7.84	-3.57	-6.09	-2.12
$\theta_{2,1,1}$	9.33	14.77	2.66	0.62	1.49	-2.01	57.45	60.35	28.53	21.05	20.22	11.65
$\theta_{3,1,1}$	-14.82	-46.00	-1.20	-11.00	0.96	-3.23	-92.83	-121.56	-42.51	-45.22	-28.52	-26.29
$\theta_{4,1,1}$	6.97	28.54	-0.43	7.84	-1.49	3.08	47.01	67.56	20.57	25.47	13.36	14.96
$\theta_{0,2,1}$	28.0	72.1	22.2	41.2	17.4	28.0	11.4	64.5	16.9	42.0	14.2	29.5
$\theta_{0,3,1}$	793	400	139	-33	20	-75	1837	1400	526	330	242	135
$\bar{R}^2$	0.9979	0.9931	0.9989	0.9951	0.9991	0.9954	0.9987	0.9967	0.9993	0.9981	0.9995	0.9984
RMSE	0.0728	0.1116	0.0315	0.0455	0.0206	0.0277	0.0816	0.1145	0.0381	0.0506	0.0264	0.0333

Note: The RS regression model is equation (8). The dependent variable is the simulated α-quantile of the test statistic. $\bar{R}^2$ denotes the adjusted coefficient of determination, and RMSE the root mean square error.

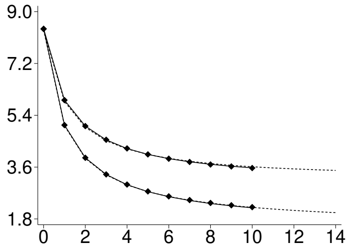
RS with increasing k.

The joint RS model, equation (8), allows us to present the estimates in a more compact way compared to the separate regressions, and to compute the finite-sample critical values for any number k of long-run forcing regressors, effective sample size $N(T, q)$, and number of short-run coefficients $H(q, k)$, as long as there are sufficiently many degrees of freedom. Figure 4 illustrates that for small sample sizes this degrees-of-freedom restriction is often binding. For $T = 30$ and $q = 1$, the EC model can accommodate at most $k = 6$ long-run forcing variables. For larger sample sizes, for example $T = 80$, our procedure allows us to predict critical values beyond the maximum k considered in our simulations and the previous literature.

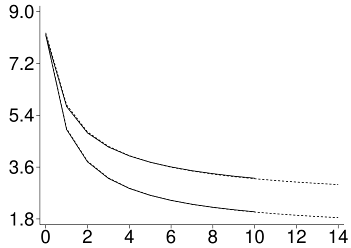
Figure 5 highlights the variation of the RS over the sample size and lag order for selected variable counts. For the F-statistic, the differences across lag orders are more pronounced for the lower-bound critical values that exhibit a slower convergence rate to the respective asymptotic critical value than the upper bounds. Moreover, the convexity of the RS increases with the lag order. While the slope of the RS is negative in q for larger sample sizes, it can become positive for relatively small sample sizes, increasingly so the more long-run forcing variables are in the model. The inconclusive area between the lower and the upper bound widens with increasing lag order. The picture is slightly different for the t-statistic. A larger lag order pulls the critical values closer to zero almost everywhere



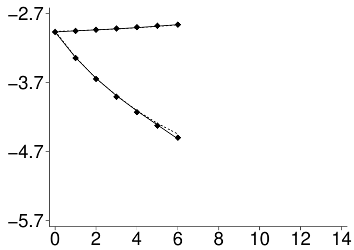
(a) F-statistic, $T = 30$



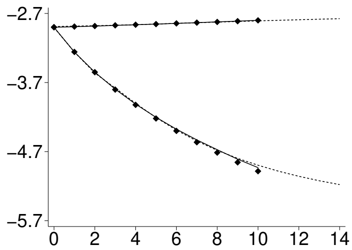
(b) F-statistic, $T = 80$



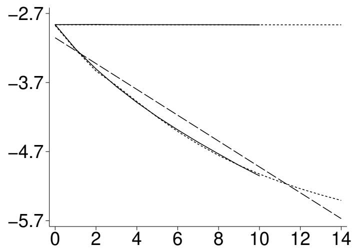
(c) F-statistic, $T = \infty$



(d) t-statistic, $T = 30$



(e) t-statistic, $T = 80$



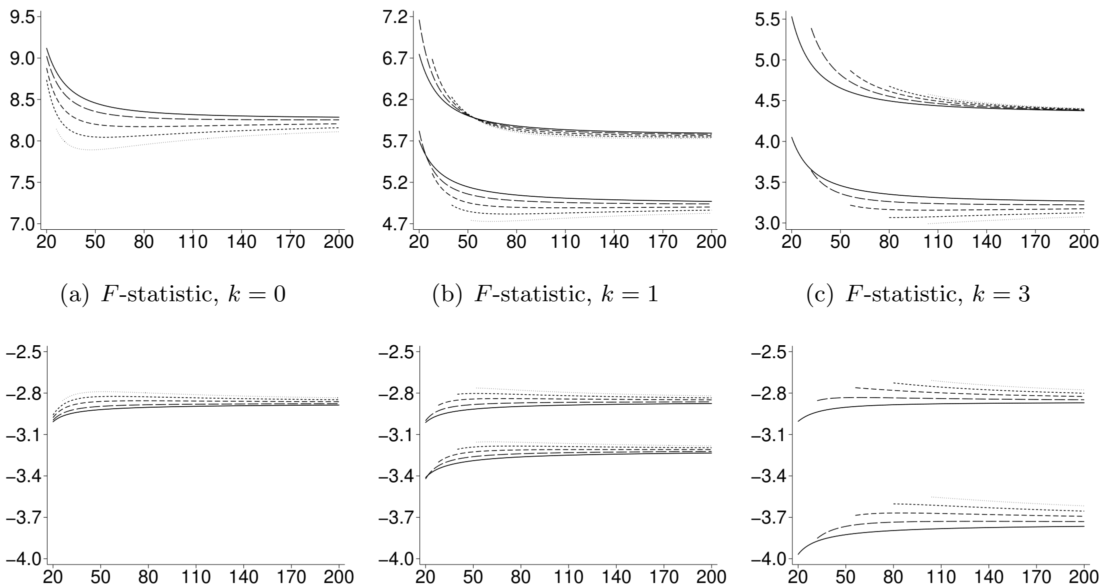
(f) t-statistic, $T = \infty$

Figure 4: RS for the F- and t-statistic in case (iii) at the 5% significance level over a range of variable numbers k with lag order $q = 1$. The solid curves are the combination of the separate RS estimates for each k for the lower bound (closer to zero) and the upper bound, respectively, and the short-dashed curves are the joint RS estimates from equation (8). The diamonds are the critical values directly computed from our aggregate EDFs. The long-dashed line is the "meta response surface" from Ericsson and MacKinnon (2002) for the asymptotic upper-bound critical values.

for both the lower and the upper bound. As seen in Figure 4 before and backed by the asymptotic distributions derived by Pesaran et al. (2001), the lower-bound critical values are fairly stable with respect to the number of variables k.

3.3 Approximate p-values

With the RS regressions from Section 3.2 for a fine grid of quantiles, we can already describe the shape of the finite-sample and asymptotic distributions quite well. To obtain a p-value corresponding to any given value of the test statistic, we still need to interpolate between the two nearest quantiles for which we have obtained predictions. We follow MacKinnon (1996) and Ericsson and MacKinnon (2002) regarding the choice of 221



(a) F-statistic, $k = 0$ (b) F-statistic, $k = 1$ (c) F-statistic, $k = 3$

(d) t-statistic, $k = 0$ (e) t-statistic, $k = 1$ (f) t-statistic, $k = 3$

Figure 5: RS from equation (8) for the F- and t-statistic in case (iii) at the 5% significance level over a range of effective sample sizes $N(T, q)$. The solid curves represent the lower bound (closer to zero) and the upper bound for $q = 0$. With increasing lag order, $q \in \{0, 3, 6, 9, 12\}$, the curves have shorter dashes.

quantiles that we compute for each test statistic:

$$p \in \{0.0001, 0.0002, 0.0005, 0.001, \dots, 0.01, 0.015,$$
$$\dots, 0.99, 0.991, \dots, 0.999, 0.9995, 0.9998, 0.9999\}.$$

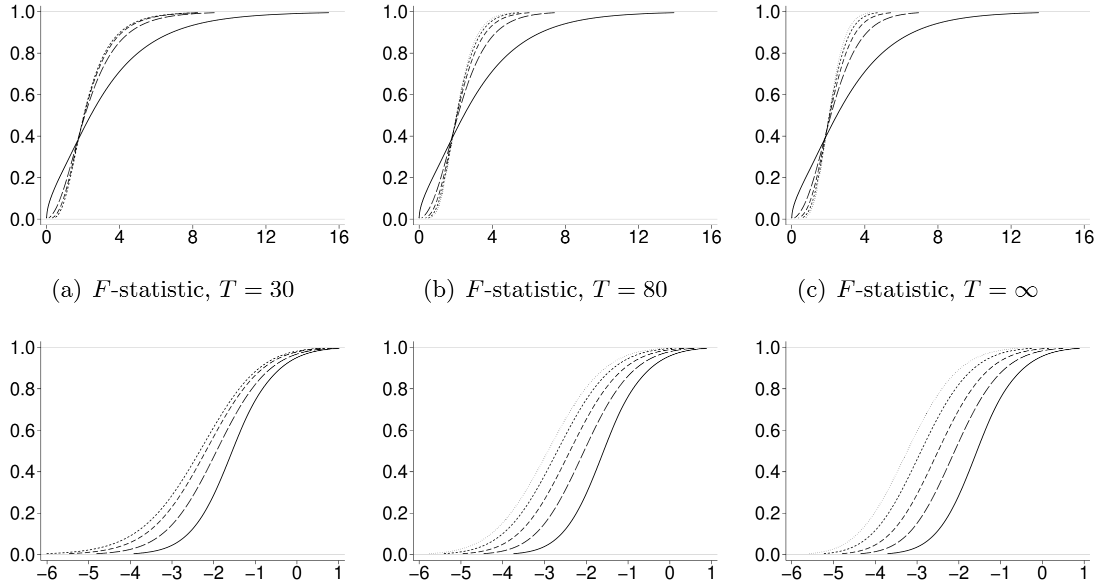
Some of the resulting cumulative distribution functions are shown in Figure 6. It is apparent again that the differences diminish with increasing number of long-run forcing variables, and that the shape of the distributions varies with the sample size.

To obtain p-values, MacKinnon (1994, 1996) suggests a local approximation strategy. Consider the following regression model:

$$F^{-1}(p) = \sum_{i=0}^{n} \phi_i \left[\hat{Q}(p)\right]^i + e, \tag{9}$$

where $F^{-1}(p)$ is the inverse cumulative distribution function of the test statistic that would apply under standard asymptotics,[27] and $\hat{Q}(p)$ is the predicted p-quantile from equation

[27] We use the F-distribution with appropriate degrees of freedom to approximate the shape of the dis-



(a) F-statistic, $T = 30$ (b) F-statistic, $T = 80$ (c) F-statistic, $T = \infty$

(d) t-statistic, $T = 30$ (e) t-statistic, $T = 80$ (f) t-statistic, $T = \infty$

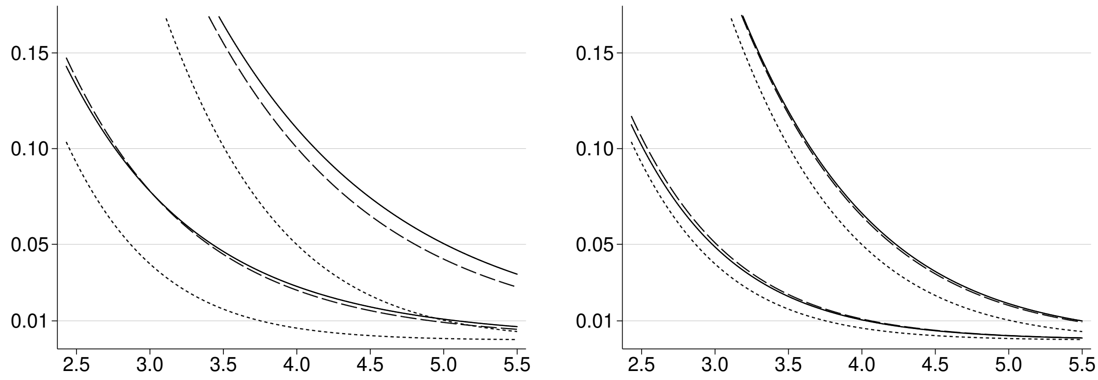
Figure 6: Implied upper-bound cumulative distribution functions from equation (8) for the F- and t-statistic in case (iii) with $k \in \{0, 2, 4, 6, 8\}$ variables with lag order $q = 1$. The solid curve refers to $k = 0$. With increasing k, the curves have shorter dashes.

(8) for a given combination of k, T, and q.[28] If the distributional assumption was correct, then model (9) would be correctly specified with $\phi_1 = 1$ and all other coefficients being zero. $\phi_0 \neq 0$ allows for a shift in the mean and $\phi_1 \neq 1$ for a different variance. Since in our case this regression only serves as an approximation of the unknown shape of the distribution, the higher-order terms potentially help to improve the fit. It turns out that for our purpose a second-order polynomial, $n = 2$, works sufficiently well.

Equation (9) is then estimated for the 9 predicted quantiles that are nearest to the observed value of the test statistic. MacKinnon (1994, 1996) notices that an OLS estimation ignores heteroskedasticity and pairwise correlation of the quantiles, and he suggests to estimate equation (9) by generalized least squares (GLS). However, we do not find that a GLS estimation uniformly improves the fit. For practical purposes, a feasible GLS estimation requires estimates of the variances of the respective quantiles. While the variance estimates can in principle be obtained from the RS regressions, this would require to supply the variance-covariance matrices from all estimations together with the computer

tribution for the F-statistic and the t-distribution for the t-statistic.

[28] For convenience, we are suppressing the arguments k, T, q in favor of p that is variable in this regression.



(a) F-statistic, $T = 30$

(b) F-statistic, $T = 80$

Figure 7: Approximate lower-bound and upper-bound p-value curves from equation (10) for the F-statistic in case (iii) with $k = 4$ variables. The solid curve is obtained accounting for the lag order $q = 1$. The long-dashed curve ignores the presence of the short-run coefficients by setting $q = 0$, and the short-dashed curve relates to the asymptotic distribution.

program that computes the approximate p-values. From our perspective, it seems worth to trade off minor efficiency gains for the convenience of not having to store this larger amount of data, again emphasizing that such efficiency gains are negligible in the light of the remaining model uncertainty.

The approximate p-value corresponding to the observed value of the test statistic τ is finally computed as

$$\hat{p} = F\left(\sum_{i=0}^{n} \hat{\phi}_i \tau^i\right), \tag{10}$$

where $\hat{\phi}_i$ are the coefficient estimates from equation (9). This procedure to approximate p-values, as well as the critical-value predictions from equation (8), is implemented in the *Stata* program described by Kripfganz and Schneider (2018) for both the F-statistic and the t-statistic. Figure 7 illustrates the resulting p-value curves for the right tail of the F-distribution. These p-values can help us to shed some light on the relevance of the differences between the finite-sample and the asymptotic critical values. When we compute a finite-sample p-value for a test statistic τ that equals the asymptotic critical value, we can interpret this p-value as the finite-sample size of the asymptotic test.

For example, consider a situation with $k = 4$ variables, $T = 30$ data points, $q = 1$ lag for each variable, and an unrestricted intercept. Our RS regressions predict an asymptotic upper-bound critical value of 4.00 at a significance level of 5%. The finite-sample upper-bound p-value that corresponds to this value is 0.111 such that we do not even reject the null hypothesis at the 10% significance level. The asymptotic test is substantially oversized in such a small sample. If we ignored the presence of the short-run coefficients, the p-value would slip back by more than one percentage point to 0.100. These differences can be quite relevant in empirical work. With a larger sample size, the asymptotic critical values obviously become better approximations. When we move to $T = 80$ in our example, the correct finite-sample p-value falls to 0.067 which still implies that the test is oversized by a practically relevant magnitude. Because the number of short-run coefficients is now small relative to the sample size, the lag order no longer plays a big role. For higher lag orders, the p-value curves would still be visibly distinct even for moderately large sample sizes.

For the F-statistic, size distortions of more than 5 percentage points are not uncommon, in particular in models with a large number of long-run forcing variables. Furthermore, the distortions tend to be stronger in cases with restricted rather than unrestricted deterministic model components. For the t-statistic, we find less reasons to be overly concerned about the use of the asymptotic critical values. The expected size distortions remain mostly below two percentage points. This is in line with our earlier observation in Figure 5 that the RS for the t-statistic is much flatter than for the F-statistic. More detailed information on the finite-sample size distortions can be found in our Supplementary Appendix.

4 Conclusion

The Pesaran et al. (2001) bounds test for the existence of a level relationship is widely applied in the empirical practice. The current paper provides response surface estimates for the respective lower-bound and upper-bound critical values, corresponding to the situations where all long-run forcing variables are either $I(0)$ or $I(1)$, respectively. Precise finite-sample and asymptotic critical values for various cases of unrestricted or restricted deterministic model components and any number of long-run forcing variables can be computed directly from the regression tables. While such critical values have been reported previously in the literature, they often only cover a rather small subset of the possible model specifications and sample sizes, and they are typically less precise due to a smaller number of replications in the respective Monte Carlo simulations.

With the exception of Cheung and Lai (1995a) for the augmented Dickey-Fuller test that results as a special case of the framework considered here, the previously obtained response surfaces do not account for the lag augmentation in the underlying regression model. With our response surface estimates, accurate finite-sample critical value bounds can be obtained for any number of short-run coefficients. In practice, the correct lag order is usually unknown and possibly different across variables. For the purpose of efficient estimation of the model coefficients, an optimal lag order is often obtained with model selection criteria such as the Akaike or Schwarz information criterion. However, as stressed by Pesaran et al. (2001), for testing purposes it is of primary concern that the error term is free of serial correlation. As long as there are enough degrees of freedom available, additional lags of the variables can help to achieve this aim. Once a conclusion from the test is drawn, a more parsimonious model can be estimated along the lines of the Pesaran and Shin (1998) autoregressive distributed lag (ARDL) modelling approach. In the statistical software *Stata*, the ARDL and EC models can be estimated with the same program that computes the critical values and approximate p-values for the bounds test (Kripfganz and Schneider, 2018).

References

Anderson, E., Z. Bai, C. H. Bischof, S. Blackford, J. W. Demmel, J. J. Dongarra, J. Du Croz, A. Greenbaum, S. J. Hammarling, A. McKenney, and D. C. Sorensen (1999). *LAPACK Users' Guide* (3rd ed.). Philadelphia: Society for Industrial and Applied Mathematics.

Banerjee, A., J. J. Dolado, and R. Mestre (1998). Error-correction mechanism tests for cointegration in a single-equation framework. *Journal of Time Series Analysis 19*(3), 267–283.

Carrion-i-Silvestre, J. L., A. Sansó Rosselló, and M. Artís Ortuño (1999). Response surface estimates for the Dickey-Fuller unit root test with structural breaks. *Economics Letters 63*(3), 279–283.

Cheung, Y.-W. and K. S. Lai (1995a). Lag order and critical values of the augmented Dickey-Fuller test. *Journal of Business & Economic Statistics 13*(3), 277–280.

Cheung, Y.-W. and K. S. Lai (1995b). Lag order and critical values of a modified Dickey-Fuller test. *Oxford Bulleting of Economics and Statistics 57*(3), 411–419.

Cook, S. (2001). Finite-sample critical values of the augmented Dickey-Fuller statistic: A note on lag order. *Economic Issues 6*(2), 31–38.

Dickey, D. A. (1976). *Estimation and hypothesis testing in nonstationary time series*. Ph. D. thesis, Iowa State University.

Dickey, D. A. and W. A. Fuller (1979). Distribution of the estimators for autoregressive time series with a unit root. *Journal of the American Statistical Association 74*(366), 427–431.

Dickey, D. A. and W. A. Fuller (1981). Likelihood ratio statistics for autoregressive time series with a unit root. *Econometrica 49*(4), 1057–1072.

Engle, R. F. and C. W. J. Granger (1987). Co-integration and error correction: representation, estimation, and testing. *Econometrica 55*(2), 251–276.

Ericsson, N. R. and J. G. MacKinnon (2002). Distributions of error correction tests for cointegration. *Econometrics Journal 5*(2), 285–318.

Fuller, W. A. (1976). *Introduction to Statistical Time Series*. New York: Wiley.

Harvey, D. I. and D. van Dijk (2006). Sample size, lag order and critical values of seasonal unit root tests. *Computational Statistics & Data Analysis 50*(10), 2734–2751.

Hassler, U. and J. Wolters (2006). Autoregressive distributed lag models and cointegration. *Allgemeines Statistisches Archiv 90*(1), 59–74.

Kanioura, A. and P. Turner (2005). Critical values for an F-test for cointegration in a multivariate model. *Applied Economics 37*(3), 265–270.

Kripfganz, S. and D. C. Schneider (2018). ardl: Estimating autoregressive distributed lag and equilibrium correction models. Proceedings of the 2018 London Stata Conference.

MacKinnon, J. G. (1991). Critical values for cointegration tests. In R. F. Engle and C. W. J. Granger (Eds.), *Long-Run Economic Relationships: Readings in Cointegration*, Chapter 13, pp. 267–276. Oxford: Oxford University Press.

MacKinnon, J. G. (1994). Approximate asymptotic distribution functions for unit-root and cointegration tests. *Journal of Business & Economic Statistics 12*(2), 167–176.

MacKinnon, J. G. (1996). Numerical distribution functions for unit root and cointegration tests. *Journal of Applied Econometrics 11*(6), 601–618.

MacKinnon, J. G. (2010). Critical values for cointegration tests. QED Working Paper 1227, Queen's University, Department of Economics.

MacKinnon, J. G., A. A. Haug, and L. Michelis (1999). Numerical distribution functions of likelihood ratio tests for cointegration. *Journal of Applied Econometrics 14*(5), 563–577.

McNown, R., C. Y. Sam, and S. K. Goh (2018). Bootstrapping the autoregressive distributed lag test for cointegration. *Applied Economics 50*(13), 1509–1521.

Mills, T. C. and E. J. Pentecost (2001). The real exchange rate and the output response in four EU accession countries. *Emerging Markets Review 2*(4), 418–430.

Narayan, P. K. (2005). The saving and investment nexus for China: evidence from cointegration tests. *Applied Economics 37*(17), 1979–1990.

Narayan, P. K. and R. Smyth (2004). Crime rates, male youth unemployment and real income in Australia: evidence from Granger causality tests. *Applied Economics 36*(18), 2079–2095.

Otero, J. and C. F. Baum (2017). Response surface models for the Elliott, Rothenberg, and Stock unit-root test. *Stata Journal 17*(4), 985–1002.

Otero, J. and J. Smith (2012). Response surface models for the Leybourne unit root tests and lag order dependence. *Computational Statistics 27*(3), 473–486.

Otero, J. and J. Smith (2017). Response surface models for OLS and GLS detrending-based unit-root tests in nonlinear ESTAR models. *Stata Journal 17*(3), 704–722.

Pesaran, M. H. and Y. Shin (1998). An autoregressive distributed-lag modelling approach to cointegration analysis. In S. Strøm (Ed.), *Econometrics and Economic Theory in the 20th Century. The Ragnar Frisch Centennial Symposium*, Chapter 11, pp. 371–413. Cambridge: Cambridge University Press.

Pesaran, M. H., Y. Shin, and R. J. Smith (2000). Structural analysis of vector error correction models with exogenous $I(1)$ variables. *Journal of Econometrics 97*(2), 293–343.

Pesaran, M. H., Y. Shin, and R. J. Smith (2001). Bounds testing approaches to the analysis of level relationships. *Journal of Applied Econometrics 16*(3), 289–326.

Presno, M. J. and A. J. López (2003). Response surface estimates of stationarity tests with a structural break. *Economics Letters 78*(3), 395–399.

Said, S. E. and D. A. Dickey (1984). Testing for unit roots in autoregressive-moving average models of unknown order. *Biometrika 71*(3), 599–607.

Sam, C. Y., R. McNown, and S. K. Goh (2018). An augmented autoregressive distributed lag bounds test for cointegration. *Economic Modelling forthcoming*, 1–12.

Sephton, P. S. (1995). Response surface estimates of the KPSS stationarity test. *Economics Letters 47*(3–4), 255–261.

Sephton, P. S. (2008). Critical values of the augmented fractional Dickey-Fuller test. *Empirical Economics 35*(3), 437–450.

Sephton, P. S. (2017). Finite sample critical values of the generalized KPSS stationarity test. *Computational Economics 50*(1), 161–172.

Turner, P. (2006). Response surfaces for an F-test for cointegration. *Applied Economics Letters 13*(8), 479–482.

Supplementary Appendix

Appendix A Details on the computational methods

In the following, we present some computational aspects about the Monte Carlo simulations in Section 3 of the main paper. All computations are performed in *Stata* 15. The bulk of the computations, the Monte Carlo simulations, are performed in *Stata*'s integrated matrix language, *Mata*. As a byte-compiled language, *Mata* runs about 5 to 6 times slower than a high-performance, compiled language such as *C*. However, most *Mata* functions used in our simulations hook directly into compiled ones, such as *LAPACK* functions (Anderson et al., 1999), which decreases the speed disadvantage substantially. We estimate that our simulation runs about half as fast as pure *C* would. *Mata*, however, is much more user friendly than *C*. For example, an appropriate random number generation mechanism that has a sufficiently large period and that accommodates parallel computations is readily available. For that, we use random number streams based on the Mersenne Twister pseudorandom number generator. Overall, we believe that *Mata* provides a good balance between speed and high-level language features. We run our computations in parallel on 35 cores, each of which running at 2.9 GHz. After the removal of any redundant calculations, such as repeated calculation of the same cross products, the simulations conclude after about three days.

Storing the calculated statistics is a desirable computational aspect of the simulation. One of the advantages is that it isolates sequential steps that are computationally intensive. Once the statistics are saved, any subsequent operations can be done independently, without re-calculating the results from the previous step over and over again, should either bugs or additional research ideas pop up. However, the large number of calculated statistics, roughly 100 billion F-statistics and 60 billion t-statistics, poses several problems, the most serious one being storage. Using floating point numbers with 8 digit precision (4 bytes per number), the (uncompressed) storage requirement is 640 GB. While this is not technically infeasible, it is too much of a hindrance for practical research. Our solution was to round the calculated statistics to three digits after the decimal point. It is important to note that the effect of rounding on the RS regressions is absolutely negligible. We then further transformed the rounded numbers in terms of first differences of sorted statistics and occurrence counts. The transformation is completely reversible, so that the original rounded 10 billion statistics per simulation design can be fully recovered. The resulting storage requirements are 40 GB, which decrease further to 8 GB when adding a conventional compression algorithm. This magnitude is easily manageable.

Appendix B Critical values and approximate p-values

To assess the precision of the empirical distribution functions obtained in our Monte Carlo simulation in Section 3.1 of the main paper, we can compute the coefficient of variation for the quantiles of interest based on the 100 meta replications with 100,000 replications each. For selected simulation designs, they are reported in Tables 6 and 7 in Appendix D. Because the replications for a given design are independent, the coefficient of variation for the quantiles based on 10 million replications is expected to be one-tenth of the one for 100,000 replications.

Besides being useful on their own in an empirical analysis, the approximate p-values computed in Section 3.3 of the main paper can be used to assess the relevance of the differences between asymptotic and finite-sample critical values. Tables 8 and 9 in Appendix D present the approximate finite-sample p-values for a given sample size and variable count that correspond to the respective asymptotic critical values at a specified significance level. These p-values can be interpreted as the expected finite-sample size of the asymptotic test.

Appendix C Separate response surface regressions

In Section 3.2 of the main paper, we have obtained RS estimates for the F- and t-statistic that allow us to predict quantiles of the distributions for any number of long-run forcing variables. In the previous literature, these RS models were estimated separately for each variable count k of interest. In this appendix, we do the same for all $k \in [0, 10]$. While the resulting predictions are expected to be slightly more precise, we have seen in Figure 4 of the main paper that there is hardly any practically relevant difference compared to the joint model for all k.

In the following, we estimate separate RS models for each quadruplet $\{c, k, d, p\}$. Given the 100 meta replications, up to 19 choices of the time horizon T, and 8 different lag orders q, we have between 5,900 and 12,400 observations per estimation, accounting for the restriction that there shall be at least twice as many observations as parameters in equation (6).[29] The RS model is

$$Q_k(T,q) = \sum_{j=0}^{m}\sum_{l=0}^{n} \theta_{j,l}[N(T,q)]^{-j}[H(q,k)]^{l} + u, \tag{11}$$

where $Q_k(T,q)$ is the respective quantile from each meta replication for a given k, $N(T,q)$ is the effective sample size, and $H(q,k)$ the number of unrestricted short-run coefficients. The presence of stationary first-differenced terms in equation (6) when $q > 0$ does not affect the asymptotic properties of the distribution which implies the restrictions $\theta_{0,l} = 0$ for all $l > 0$. The intercept $\theta_{0,0}$ can then be interpreted as the asymptotic quantile when $T \to \infty$. We have chosen the polynomial orders $m = 3$ and $n = 1$. The latter provides a better fit than alternatively setting $n = 3$ together with the restrictions $\theta_{j,l} = 0$ whenever $j \neq l$ for $l > 0$, which has been done by Cheung and Lai (1995a). Equation (11) thus reduces to

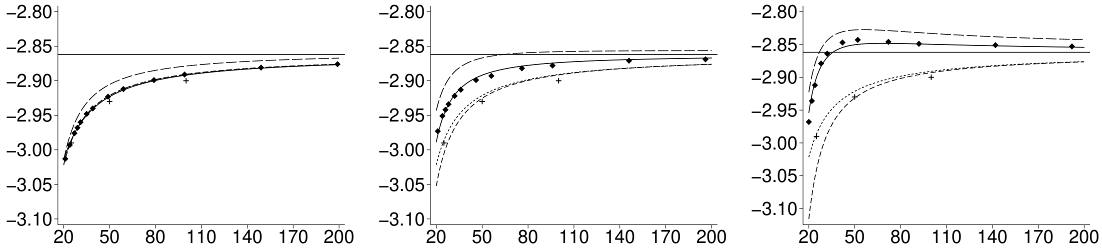
$$Q_k(T,q) = \theta_{0,0} + \sum_{j=1}^{3} \theta_{j,0} \frac{1}{[N(T,q)]^j} + \sum_{j=1}^{3} \theta_{j,1} \frac{H(q,k)}{[N(T,q)]^j} + u. \tag{12}$$

In Appendix D, we report the ordinary least squares results for the quantiles corresponding to a size of 1%, 5%, and 10%.[30] Tables 10 to 17 also contain the standard error (SE) of the intercept, robust to heteroskedasticity, as a measure of uncertainty about the asymptotic quantile. It is always smaller than 0.0041 for the F-statistic and below 0.0011 for the t-statistic. In most experimental designs, the standard error remains far below this magnitude. However, the reported standard errors are too small because they are conditional on the correct specification of the RS model, as emphasized by MacKinnon (1991).

The asymptotic critical values can be read off directly from the RS intercept $\theta_{0,0}$. Our estimates are close to the corresponding near-asymptotic critical values tabulated by Pesaran et al. (2001). The absolute difference is for the most part below 0.05, both for the F-statistic and the t-statistic. However, these asymptotic critical values are less useful in small samples. For a given number of variables in the level relationship, finite-sample

[29] The largest number of observations is available for $k = 1$ in case (i), and the smallest number for $k = 10$ in cases (iv) and (v).

[30] Estimates for other quantiles are available upon request.



(a) $q = 1$

(b) $q = 4$

(c) $q = 8$

Figure 8: RS from equation (12) for the t-statistic in case (iii) with $k = 0$ variables at the 5% significance level for selected lag orders q over a range of effective sample sizes $N(T, q)$. The diamonds are the critical values computed from the aggregate EDFs of the 10^7 t-statistics. The horizontal line represents the respective estimate of $\theta_{0,0}$ in Table 16 and the solid curve the corresponding RS. The long-dashed curve is the RS from Cheung and Lai (1995a), the medium-dashed curve from Ericsson and MacKinnon (2002), and the short-dashed curve from MacKinnon (2010). Crosses are tabulated critical values from Dickey (1976).

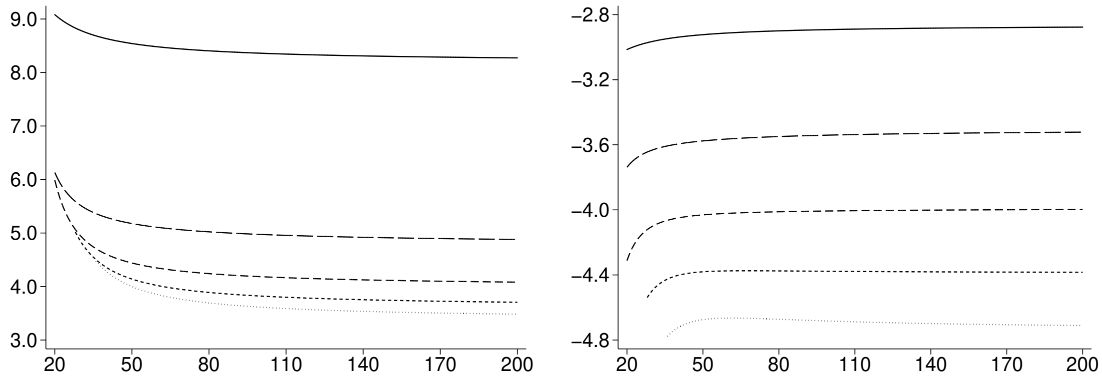
critical values can be computed from the regression coefficients for any combination of the effective sample size and number of short-run coefficients.

Previously reported critical values typically do not take the lag augmentation in equation (6) into account and might thus be inaccurate in many empirically relevant situations, in particular when the sample size is relatively small. Figure 5 in the main paper illustrates the variation across lag orders. For $k = 0$, there is obviously no distinction possible between $I(0)$ and $I(1)$ variables in the level relationship. In this situation, the F-statistic in cases (ii) and (iv) is the one analyzed by Dickey and Fuller (1981). In cases (i), (iii), and (v), it equals the square of the t-statistic. The latter corresponds to the familiar augmented Dickey-Fuller unit-root test statistic. The asymptotic critical values obtained from our RS regressions closely match those reported in the previous literature.[31]

RS estimates for the original Dickey and Fuller (1979) test statistic, $q = 1$, have been previously obtained by MacKinnon (1991, 2010) and Ericsson and MacKinnon (2002).[32] Cheung and Lai (1995a) go one step further by estimating a RS that allows the quantiles of the distribution to vary with the lag order. Figure 8 compares these RS estimates to ours for case (iii) and three different lag orders at a size of 5%. For the test without

[31] See Table 1 in the main paper.

[32] Dickey (1976) obtains his critical values as predictions from RS regressions but he does not report the regression coefficients.



(a) F-statistic

(b) t-statistic

Figure 9: Upper-bound RS from equation (12) for the F- and t-statistic in case (iii) at the 5% significance level with $k \in \{0, 2, 4, 6, 8\}$ variables over a range of effective sample sizes $N(T, q)$ and with a lag order $q = 1$. The solid curve refers to $k = 0$. With increasing k, the curves have shorter dashes.

lag augmentation, $q = 1$, our RS and the ones from MacKinnon (2010) and Ericsson and MacKinnon (2002) are visually indistinguishable and they all fit nicely through the quantiles from the aggregate EDFs obtained in Section 3.1 of the main paper.[33]

The advantage of our approach becomes apparent when we move to higher lag orders. Because the RS from MacKinnon (2010) does not accommodate the lag augmentation, it becomes too conservative. In fact, for higher lag orders the asymptotic critical value would provide a better approximation for most sample sizes than the MacKinnon (2010) surface or the tabulated critical values from Dickey (1976). By contrast, Figure 8 confirms that our RS provides a very good fit to the critical values implied by our simulated aggregate EDFs. It also outperforms the RS from Cheung and Lai (1995a) that is skewed towards zero, possibly due to the smaller number of replications in their simulation and a lower polynomial order in their RS regressions. Ericsson and MacKinnon (2002) indirectly account for the lag order by estimating the RS over the degrees-of-freedom adjusted sample size. However, Figure 8 clearly shows that this strategy is not appropriate for higher lag orders as the fit worsens even compared to MacKinnon (2010).

In the multivariable environment, the order of integration affects the distribution of the test statistic. Banerjee et al. (1998) and Ericsson and MacKinnon (2002) consider the t-statistic for cointegration testing under the assumption that all regressors are individu-

[33] MacKinnon (2010) is an updated version of MacKinnon (1991).

ally $I(1)$, the upper bound for the bounds test, but neither of them account for the lag augmentation. In this situation, when we vary k for a fixed lag order $q = 1$, the spread between the RS curves is largely driven by the asymptotic critical value that now depends on k. This is shown in Figure 9 for both the F- and t-statistic. Importantly, the gap between the curves becomes systematically smaller with increasing k, which justifies our approach in Section 3.2 of the main paper to directly model the variation in k as part of a joint RS.

Appendix D Additional Tables

Table 6: Coefficient of variation, F-statistic

	T	k	1% $I(0)$	1% $I(1)$	5% $I(0)$	5% $I(1)$	10% $I(0)$	10% $I(1)$
Case (i)								
	30	0	0.0082	0.0082	0.0065	0.0065	0.0059	0.0059
		2	0.0067	0.0066	0.0045	0.0042	0.0038	0.0034
		4	0.0064	0.0060	0.0039	0.0035	0.0032	0.0031
	80	0	0.0083	0.0083	0.0070	0.0070	0.0060	0.0060
		2	0.0066	0.0060	0.0037	0.0036	0.0035	0.0030
		4	0.0056	0.0048	0.0033	0.0030	0.0030	0.0024
		8	0.0044	0.0043	0.0030	0.0027	0.0027	0.0021
	1000	0	0.0075	0.0075	0.0062	0.0062	0.0051	0.0051
		2	0.0058	0.0050	0.0038	0.0029	0.0033	0.0027
		4	0.0053	0.0038	0.0035	0.0024	0.0030	0.0024
		8	0.0044	0.0030	0.0028	0.0020	0.0024	0.0015
Case (ii)								
	30	0	0.0065	0.0065	0.0038	0.0038	0.0029	0.0029
		2	0.0056	0.0058	0.0036	0.0033	0.0030	0.0027
		4	0.0064	0.0063	0.0036	0.0033	0.0028	0.0026
	80	0	0.0054	0.0054	0.0036	0.0036	0.0030	0.0030
		2	0.0052	0.0051	0.0031	0.0030	0.0026	0.0027
		4	0.0046	0.0044	0.0027	0.0027	0.0023	0.0021
		8	0.0043	0.0038	0.0024	0.0024	0.0022	0.0019
	1000	0	0.0055	0.0055	0.0034	0.0034	0.0029	0.0029
		2	0.0049	0.0045	0.0027	0.0023	0.0023	0.0022
		4	0.0041	0.0035	0.0025	0.0022	0.0022	0.0020
		8	0.0035	0.0030	0.0020	0.0020	0.0020	0.0015
Case (iii)								
	30	0	0.0069	0.0069	0.0042	0.0042	0.0033	0.0033
		2	0.0068	0.0057	0.0039	0.0035	0.0033	0.0027
		4	0.0060	0.0068	0.0036	0.0034	0.0029	0.0028
	80	0	0.0061	0.0061	0.0042	0.0042	0.0035	0.0035
		2	0.0053	0.0054	0.0032	0.0032	0.0031	0.0030
		4	0.0047	0.0045	0.0028	0.0028	0.0028	0.0023
		8	0.0046	0.0040	0.0024	0.0026	0.0022	0.0020
	1000	0	0.0056	0.0056	0.0035	0.0035	0.0030	0.0030
		2	0.0052	0.0047	0.0029	0.0029	0.0025	0.0026
		4	0.0042	0.0037	0.0026	0.0023	0.0024	0.0020
		8	0.0037	0.0031	0.0023	0.0020	0.0020	0.0016
Case (iv)								
	30	0	0.0061	0.0061	0.0034	0.0034	0.0027	0.0027
		2	0.0060	0.0056	0.0034	0.0032	0.0030	0.0025
		4	0.0054	0.0062	0.0034	0.0034	0.0026	0.0027
	80	0	0.0052	0.0052	0.0035	0.0035	0.0028	0.0028
		2	0.0055	0.0045	0.0030	0.0031	0.0024	0.0022
		4	0.0042	0.0039	0.0026	0.0028	0.0023	0.0021
		8	0.0039	0.0036	0.0025	0.0021	0.0020	0.0017
	1000	0	0.0045	0.0045	0.0027	0.0027	0.0024	0.0024
		2	0.0039	0.0043	0.0025	0.0022	0.0020	0.0020
		4	0.0040	0.0035	0.0025	0.0021	0.0021	0.0017
		8	0.0036	0.0028	0.0022	0.0018	0.0020	0.0014
Case (v)								
	30	0	0.0067	0.0067	0.0034	0.0034	0.0029	0.0029
		2	0.0061	0.0063	0.0037	0.0034	0.0030	0.0027
		4	0.0056	0.0066	0.0036	0.0036	0.0028	0.0027
	80	0	0.0056	0.0056	0.0040	0.0040	0.0029	0.0029
		2	0.0056	0.0043	0.0029	0.0032	0.0026	0.0025
		4	0.0046	0.0039	0.0026	0.0029	0.0023	0.0023
		8	0.0041	0.0039	0.0027	0.0020	0.0021	0.0018
	1000	0	0.0047	0.0047	0.0028	0.0028	0.0024	0.0024
		2	0.0043	0.0040	0.0026	0.0022	0.0021	0.0020
		4	0.0040	0.0035	0.0025	0.0022	0.0021	0.0018
		8	0.0038	0.0027	0.0022	0.0018	0.0020	0.0015

Note: The coefficient of variation is computed as the ratio of the standard deviation to the mean over the 100 meta replications for the empirical quantiles that correspond to the respective significance level and simulation design. Only designs with a lag order $q = 1$ are considered.

Table 7: Coefficient of variation, t-statistic

	T	k	1% $I(0)$	1% $I(1)$	5% $I(0)$	5% $I(1)$	10% $I(0)$	10% $I(1)$
Case (i)								
	30	0	0.0046	0.0046	0.0040	0.0040	0.0031	0.0031
		2	0.0050	0.0036	0.0040	0.0023	0.0036	0.0020
		4	0.0048	0.0037	0.0043	0.0024	0.0036	0.0021
	80	0	0.0044	0.0044	0.0040	0.0040	0.0032	0.0032
		2	0.0041	0.0034	0.0038	0.0020	0.0033	0.0018
		4	0.0048	0.0027	0.0035	0.0019	0.0035	0.0018
		8	0.0051	0.0031	0.0034	0.0016	0.0036	0.0015
	1000	0	0.0042	0.0042	0.0031	0.0031	0.0032	0.0032
		2	0.0043	0.0029	0.0031	0.0021	0.0031	0.0019
		4	0.0044	0.0027	0.0031	0.0017	0.0031	0.0016
		8	0.0044	0.0023	0.0032	0.0013	0.0033	0.0013
Case (iii)								
	30	0	0.0035	0.0035	0.0021	0.0021	0.0017	0.0017
		2	0.0037	0.0036	0.0023	0.0020	0.0019	0.0015
		4	0.0042	0.0032	0.0025	0.0020	0.0021	0.0017
	80	0	0.0031	0.0031	0.0021	0.0021	0.0018	0.0018
		2	0.0030	0.0030	0.0021	0.0019	0.0017	0.0017
		4	0.0033	0.0026	0.0020	0.0017	0.0019	0.0015
		8	0.0035	0.0027	0.0023	0.0016	0.0019	0.0013
	1000	0	0.0028	0.0028	0.0017	0.0017	0.0015	0.0015
		2	0.0030	0.0025	0.0017	0.0016	0.0015	0.0015
		4	0.0029	0.0020	0.0018	0.0016	0.0014	0.0012
		8	0.0030	0.0019	0.0018	0.0012	0.0015	0.0011
Case (v)								
	30	0	0.0033	0.0033	0.0017	0.0017	0.0015	0.0015
		2	0.0034	0.0033	0.0023	0.0018	0.0016	0.0017
		4	0.0036	0.0035	0.0023	0.0020	0.0019	0.0017
	80	0	0.0028	0.0028	0.0020	0.0020	0.0015	0.0015
		2	0.0026	0.0023	0.0020	0.0017	0.0015	0.0015
		4	0.0027	0.0024	0.0019	0.0015	0.0016	0.0014
		8	0.0030	0.0025	0.0019	0.0014	0.0017	0.0013
	1000	0	0.0024	0.0024	0.0014	0.0014	0.0012	0.0012
		2	0.0023	0.0021	0.0014	0.0013	0.0012	0.0011
		4	0.0023	0.0019	0.0014	0.0013	0.0012	0.0012
		8	0.0026	0.0018	0.0014	0.0011	0.0012	0.0009

Note: The coefficient of variation is computed as the ratio of the standard deviation to the absolute value of the mean over the 100 meta replications for the empirical quantiles that correspond to the respective significance level and simulation design. Only designs with a lag order $q = 1$ are considered.

Table 8: Finite-sample p-values, F-statistic

	T	k	1% $I(0)$	1% $I(1)$	5% $I(0)$	5% $I(1)$	10% $I(0)$	10% $I(1)$
Case (i)								
	30	0	0.0141	0.0131	0.0572	0.0554	0.1075	0.1054
		2	0.0219	0.0255	0.0713	0.0765	0.1235	0.1283
		4	0.0325	0.0423	0.0882	0.1018	0.1422	0.1555
	80	0	0.0110	0.0102	0.0517	0.0501	0.1017	0.0997
		2	0.0125	0.0129	0.0550	0.0559	0.1058	0.1064
		4	0.0147	0.0168	0.0595	0.0639	0.1110	0.1159
		8	0.0210	0.0300	0.0700	0.0858	0.1219	0.1402
	1000	0	0.0100	0.0100	0.0501	0.0499	0.1001	0.0999
		2	0.0101	0.0101	0.0502	0.0502	0.1003	0.1002
		4	0.0102	0.0103	0.0505	0.0507	0.1005	0.1009
		8	0.0105	0.0109	0.0510	0.0522	0.1011	0.1027
Case (ii)								
	30	0	0.0187	0.0169	0.0661	0.0633	0.1177	0.1148
		2	0.0280	0.0341	0.0805	0.0919	0.1324	0.1465
		4	0.0402	0.0560	0.0974	0.1223	0.1495	0.1785
	80	0	0.0120	0.0107	0.0540	0.0515	0.1044	0.1016
		2	0.0137	0.0145	0.0573	0.0597	0.1078	0.1115
		4	0.0162	0.0196	0.0617	0.0696	0.1124	0.1231
		8	0.0222	0.0340	0.0707	0.0931	0.1210	0.1490
	1000	0	0.0101	0.0100	0.0502	0.0499	0.1002	0.0999
		2	0.0102	0.0101	0.0503	0.0504	0.1003	0.1005
		4	0.0103	0.0104	0.0506	0.0510	0.1006	0.1013
		8	0.0106	0.0111	0.0510	0.0526	0.1010	0.1032
Case (iii)								
	30	0	0.0170	0.0161	0.0609	0.0594	0.1098	0.1083
		2	0.0243	0.0299	0.0728	0.0828	0.1224	0.1343
		4	0.0358	0.0495	0.0894	0.1107	0.1397	0.1638
	80	0	0.0121	0.0113	0.0531	0.0519	0.1025	0.1012
		2	0.0132	0.0141	0.0553	0.0579	0.1048	0.1084
		4	0.0154	0.0185	0.0593	0.0664	0.1091	0.1183
		8	0.0214	0.0318	0.0685	0.0881	0.1184	0.1422
	1000	0	0.0101	0.0101	0.0502	0.0500	0.1001	0.1000
		2	0.0102	0.0102	0.0503	0.0504	0.1002	0.1004
		4	0.0103	0.0104	0.0505	0.0509	0.1004	0.1010
		8	0.0106	0.0111	0.0510	0.0523	0.1009	0.1028
Case (iv)								
	30	0	0.0237	0.0219	0.0753	0.0727	0.1285	0.1259
		2	0.0334	0.0409	0.0886	0.1019	0.1409	0.1573
		4	0.0468	0.0653	0.1055	0.1342	0.1567	0.1905
	80	0	0.0135	0.0122	0.0571	0.0550	0.1082	0.1059
		2	0.0149	0.0161	0.0595	0.0629	0.1101	0.1153
		4	0.0174	0.0216	0.0633	0.0731	0.1137	0.1269
		8	0.0233	0.0361	0.0717	0.0962	0.1213	0.1524
	1000	0	0.0102	0.0101	0.0504	0.0502	0.1005	0.1002
		2	0.0103	0.0103	0.0505	0.0506	0.1005	0.1008
		4	0.0104	0.0105	0.0507	0.0513	0.1006	0.1016
		8	0.0107	0.0112	0.0511	0.0527	0.1009	0.1034
Case (v)								
	30	0	0.0214	0.0204	0.0686	0.0674	0.1186	0.1174
		2	0.0287	0.0354	0.0792	0.0909	0.1285	0.1427
		4	0.0415	0.0576	0.0958	0.1206	0.1445	0.1734
	80	0	0.0135	0.0128	0.0559	0.0549	0.1058	0.1047
		2	0.0142	0.0154	0.0569	0.0604	0.1063	0.1112
		4	0.0162	0.0201	0.0602	0.0691	0.1094	0.1213
		8	0.0220	0.0335	0.0685	0.0904	0.1172	0.1448
	1000	0	0.0103	0.0102	0.0504	0.0503	0.1004	0.1003
		2	0.0103	0.0103	0.0504	0.0505	0.1003	0.1006
		4	0.0104	0.0105	0.0505	0.0511	0.1003	0.1012
		8	0.0107	0.0112	0.0509	0.0524	0.1006	0.1029

Note: Reported are the approximate finite-sample p-values obtained from equation (10) that are associated with the asymptotic critical value for a given significance level. Only designs with a lag order $q = 1$ are considered.

Table 9: Finite-sample p-values, t-statistic

	T	k	1% $I(0)$	1% $I(1)$	5% $I(0)$	5% $I(1)$	10% $I(0)$	10% $I(1)$
Case (i)								
	30	0	0.0131	0.0116	0.0533	0.0515	0.1003	0.0990
		2	0.0143	0.0169	0.0543	0.0563	0.0995	0.0997
		4	0.0155	0.0203	0.0550	0.0578	0.0986	0.0967
	80	0	0.0107	0.0099	0.0508	0.0505	0.1000	0.1009
		2	0.0110	0.0116	0.0510	0.0516	0.0994	0.0998
		4	0.0113	0.0126	0.0510	0.0515	0.0988	0.0972
		8	0.0119	0.0121	0.0512	0.0438	0.0976	0.0802
	1000	0	0.0100	0.0100	0.0500	0.0501	0.1000	0.1002
		2	0.0100	0.0101	0.0500	0.0501	0.0999	0.1000
		4	0.0101	0.0101	0.0500	0.0500	0.0999	0.0997
		8	0.0101	0.0101	0.0500	0.0492	0.0998	0.0978
Case (iii)								
	30	0	0.0169	0.0153	0.0612	0.0594	0.1106	0.1093
		2	0.0177	0.0200	0.0578	0.0606	0.1009	0.1029
		4	0.0179	0.0234	0.0542	0.0611	0.0920	0.0984
	80	0	0.0117	0.0111	0.0531	0.0536	0.1030	0.1055
		2	0.0118	0.0125	0.0516	0.0532	0.0990	0.1012
		4	0.0118	0.0135	0.0498	0.0527	0.0947	0.0976
		8	0.0114	0.0125	0.0458	0.0435	0.0858	0.0786
	1000	0	0.0101	0.0100	0.0502	0.0503	0.1002	0.1006
		2	0.0101	0.0101	0.0501	0.0502	0.0999	0.1001
		4	0.0101	0.0102	0.0499	0.0501	0.0995	0.0997
		8	0.0100	0.0101	0.0495	0.0491	0.0986	0.0976
Case (v)								
	30	0	0.0208	0.0189	0.0683	0.0666	0.1188	0.1178
		2	0.0208	0.0235	0.0616	0.0654	0.1032	0.1071
		4	0.0205	0.0265	0.0557	0.0644	0.0905	0.1002
	80	0	0.0126	0.0121	0.0553	0.0565	0.1060	0.1094
		2	0.0125	0.0135	0.0525	0.0551	0.0993	0.1029
		4	0.0123	0.0144	0.0496	0.0537	0.0929	0.0979
		8	0.0112	0.0128	0.0432	0.0434	0.0797	0.0773
	1000	0	0.0101	0.0101	0.0503	0.0505	0.1004	0.1009
		2	0.0101	0.0102	0.0501	0.0504	0.0998	0.1003
		4	0.0101	0.0103	0.0498	0.0502	0.0993	0.0997
		8	0.0100	0.0101	0.0492	0.0490	0.0979	0.0973

Note: Reported are the approximate finite-sample p-values obtained from equation (10) that are associated with the asymptotic critical value for a given significance level. Only designs with a lag order $q = 1$ are considered.

Table 10: Response surface estimates, F-statistic, case (i)

	k	α	$\theta_{0,0}$	$\theta_{1,0}$	$\theta_{2,0}$	$\theta_{3,0}$	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	SE($\theta_{0,0}$)	$\bar{R}^2$	RMSE
I(0)												
	0	1%	6.8875	29.187	-48.36	706.4	-0.750	-0.47	813.1	0.0017	0.986	0.091
		5%	4.1053	12.271	-94.58	958.5	-0.627	23.14	-31.7	0.0006	0.983	0.033
		10%	2.9626	7.471	-91.34	862.4	-0.431	20.85	-111.8	0.0004	0.978	0.021
	1	1%	4.7135	22.668	34.02	839.3	-0.302	-11.11	1135.8	0.0011	0.990	0.062
		5%	3.1042	9.650	-0.94	317.0	-0.288	1.26	313.1	0.0004	0.988	0.025
		10%	2.4078	5.744	-9.46	226.0	-0.237	2.52	144.3	0.0003	0.983	0.017
	2	1%	3.8491	21.822	-47.64	3414.6	-0.619	13.15	789.1	0.0009	0.994	0.047
		5%	2.6738	9.348	-4.25	1092.2	-0.470	11.95	142.4	0.0004	0.992	0.021
		10%	2.1503	5.641	-0.33	577.7	-0.382	8.95	34.5	0.0002	0.988	0.014
	3	1%	3.3558	23.794	-132.74	5112.0	-0.491	-10.91	1419.0	0.0007	0.996	0.035
		5%	2.4140	11.339	-62.13	1957.7	-0.400	-4.75	581.3	0.0004	0.994	0.018
		10%	1.9873	7.284	-44.03	1176.8	-0.341	-3.53	362.5	0.0002	0.991	0.013
	4	1%	3.0386	24.475	-149.53	5806.6	-0.395	-34.16	2302.0	0.0007	0.997	0.030
		5%	2.2442	11.792	-51.95	1943.6	-0.377	-15.14	1010.2	0.0004	0.995	0.016
		10%	1.8787	7.765	-35.93	1088.3	-0.338	-10.74	664.7	0.0003	0.992	0.013
	5	1%	2.8243	23.079	-101.35	6237.5	-0.519	-24.09	2469.0	0.0006	0.997	0.025
		5%	2.1241	12.010	-62.03	2502.8	-0.430	-13.81	1247.4	0.0003	0.996	0.015
		10%	1.7999	8.228	-54.36	1575.3	-0.377	-11.00	881.2	0.0003	0.993	0.012
	6	1%	2.6584	23.357	-163.53	8573.9	-0.566	-19.04	2819.3	0.0006	0.998	0.021
		5%	2.0323	12.195	-85.76	3484.8	-0.473	-9.34	1436.7	0.0003	0.997	0.012
		10%	1.7397	8.387	-69.49	2189.8	-0.418	-6.99	1015.9	0.0002	0.996	0.010
	7	1%	2.5221	25.664	-321.24	12416.1	-0.538	-27.69	3450.4	0.0007	0.998	0.022
		5%	1.9556	13.404	-150.84	5045.0	-0.459	-14.47	1814.9	0.0004	0.997	0.013
		10%	1.6885	9.300	-114.41	3206.4	-0.410	-11.00	1310.9	0.0003	0.996	0.010
	8	1%	2.4126	28.117	-527.16	17583.6	-0.563	-27.01	3942.8	0.0006	0.999	0.021
		5%	1.8947	14.143	-209.19	6797.3	-0.478	-13.14	2097.6	0.0003	0.998	0.012
		10%	1.6481	9.658	-141.47	4161.9	-0.428	-9.50	1516.9	0.0003	0.997	0.010
	9	1%	2.3251	28.065	-556.19	20221.0	-0.550	-28.71	4369.8	0.0007	0.998	0.020
		5%	1.8434	14.658	-244.19	8265.1	-0.473	-14.78	2391.1	0.0004	0.997	0.012
		10%	1.6130	10.262	-177.01	5271.3	-0.426	-11.19	1768.8	0.0003	0.996	0.010
	10	1%	2.2538	27.294	-558.99	22795.4	-0.556	-30.81	5322.2	0.0006	0.998	0.017
		5%	1.8014	14.493	-256.90	9683.7	-0.474	-18.00	3180.1	0.0003	0.998	0.011
		10%	1.5848	10.022	-178.29	6106.3	-0.429	-13.61	2425.4	0.0003	0.996	0.009
I(1)												
	0	1%	6.8875	29.187	-48.36	706.4	-0.750	-0.47	813.1	0.0017	0.986	0.091
		5%	4.1053	12.271	-94.58	958.5	-0.627	23.14	-31.7	0.0006	0.983	0.033
		10%	2.9626	7.471	-91.34	862.4	-0.431	20.85	-111.8	0.0004	0.978	0.021
	1	1%	5.8446	27.970	39.91	878.2	-0.073	-5.16	1518.2	0.0012	0.993	0.069
		5%	4.0493	11.417	5.45	320.8	-0.163	7.21	390.9	0.0005	0.993	0.025
		10%	3.2454	6.503	-7.83	272.0	-0.128	6.45	171.3	0.0003	0.992	0.016
	2	1%	5.1368	31.304	-117.99	4609.6	0.008	12.82	1609.5	0.0012	0.995	0.061
		5%	3.7851	13.214	-24.12	1403.0	-0.086	14.90	417.1	0.0005	0.996	0.023
		10%	3.1598	7.680	-11.09	726.4	-0.075	10.86	186.3	0.0003	0.995	0.014
	3	1%	4.7040	29.674	-6.19	5284.2	-0.063	36.43	1218.7	0.0008	0.997	0.043
		5%	3.5887	13.409	25.28	1560.5	-0.061	21.23	348.3	0.0004	0.997	0.017
		10%	3.0652	8.165	16.41	827.4	-0.049	14.00	166.4	0.0002	0.997	0.011
	4	1%	4.3928	31.836	-64.25	7647.1	0.083	18.61	1967.8	0.0010	0.998	0.039
		5%	3.4371	14.924	7.57	2437.9	0.007	14.90	651.6	0.0004	0.998	0.016
		10%	2.9832	9.189	11.98	1278.7	-0.006	10.75	339.8	0.0003	0.998	0.010
	5	1%	4.1779	29.702	86.47	7701.6	-0.045	41.15	1616.5	0.0008	0.998	0.032
		5%	3.3265	14.408	81.75	2306.6	-0.041	24.65	524.6	0.0003	0.998	0.014
		10%	2.9192	9.106	55.87	1216.7	-0.030	16.36	281.1	0.0002	0.998	0.009
	6	1%	3.9941	32.939	-33.92	11544.7	0.062	24.65	2267.8	0.0007	0.999	0.028
		5%	3.2303	16.095	42.64	3757.3	0.015	17.37	821.6	0.0003	0.999	0.012
		10%	2.8616	10.233	38.51	1997.2	0.003	12.49	450.9	0.0002	0.999	0.008
	7	1%	3.8503	35.919	-212.97	17139.0	0.042	26.35	2565.3	0.0008	0.999	0.027
		5%	3.1535	17.427	-3.04	5639.3	0.009	18.55	942.1	0.0003	0.999	0.011
		10%	2.8143	11.150	15.73	3065.6	0.001	13.38	528.9	0.0002	0.999	0.007
	8	1%	3.7253	41.566	-573.99	26110.7	0.045	20.27	3137.6	0.0009	0.999	0.029
		5%	3.0868	19.641	-113.62	8761.4	0.010	17.45	1157.4	0.0003	0.999	0.011
		10%	2.7728	12.491	-39.84	4781.1	0.001	13.15	650.2	0.0002	0.999	0.007
	9	1%	3.6300	41.281	-562.94	29869.2	0.061	19.85	3545.5	0.0009	0.999	0.027
		5%	3.0327	19.932	-108.85	10359.2	0.019	17.32	1364.9	0.0004	0.999	0.011
		10%	2.7379	12.842	-36.82	5757.2	0.009	12.79	795.7	0.0002	0.999	0.007
	10	1%	3.5472	42.192	-587.36	33958.3	0.065	18.73	3494.3	0.0008	0.999	0.022
		5%	2.9848	21.090	-146.81	12623.0	0.021	16.97	1320.1	0.0003	0.999	0.009
		10%	2.7069	13.660	-56.81	7099.3	0.011	12.79	762.8	0.0002	0.999	0.006

Note: The RS regression model is equation (12). The dependent variable is the simulated α-quantile of the test statistic. Separate regressions are run for each number k of individually $I(0)$ or $I(1)$ variables $\mathbf{x}_t$ in equation (6). SE($\theta_{0,0}$) denotes the heteroskedasticity-robust standard error of the intercept, $\bar{R}^2$ the adjusted coefficient of determination, and RMSE the root mean square error.

Table 11: Response surface estimates, F-statistic, case (ii)

	k	α	$\theta_{0,0}$	$\theta_{1,0}$	$\theta_{2,0}$	$\theta_{3,0}$	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	SE($\theta_{0,0}$)	$\bar{R}^2$	RMSE
I(0)												
	0	1%	6.3769	28.932	226.25	-1520.9	-0.906	-60.16	2021.6	0.0012	0.993	0.075
		5%	4.5831	12.625	79.12	-667.0	-1.117	-9.37	574.0	0.0005	0.990	0.032
		10%	3.7792	7.444	42.51	-408.1	-0.987	-2.23	284.6	0.0003	0.984	0.021
	1	1%	4.8785	26.267	69.67	2336.8	-1.172	9.45	1078.3	0.0009	0.995	0.050
		5%	3.5974	11.744	38.59	662.4	-0.989	11.09	279.8	0.0004	0.995	0.020
		10%	3.0150	7.141	19.81	387.0	-0.869	9.11	114.0	0.0002	0.994	0.013
	2	1%	4.0934	26.566	-31.44	4698.6	-0.956	-3.40	1452.7	0.0008	0.997	0.040
		5%	3.0836	12.090	10.41	1514.4	-0.863	5.80	425.2	0.0003	0.997	0.016
		10%	2.6175	7.422	12.95	778.1	-0.767	4.90	214.2	0.0002	0.996	0.011
	3	1%	3.6031	28.947	-198.99	7953.9	-0.816	-19.97	2017.6	0.0008	0.998	0.033
		5%	2.7620	13.307	-38.61	2488.8	-0.748	-5.75	782.8	0.0003	0.998	0.014
		10%	2.3688	8.408	-14.23	1282.1	-0.678	-3.86	480.8	0.0002	0.997	0.010
	4	1%	3.2778	26.433	-85.00	7477.1	-0.738	-25.03	2407.6	0.0007	0.998	0.026
		5%	2.5448	12.919	-6.99	2460.0	-0.690	-9.88	1047.0	0.0003	0.998	0.013
		10%	2.2001	8.479	-1.63	1320.5	-0.635	-7.08	684.2	0.0002	0.997	0.009
	5	1%	3.0379	27.168	-165.03	9944.4	-0.759	-29.36	2963.4	0.0006	0.999	0.022
		5%	2.3851	13.620	-47.51	3562.1	-0.694	-11.30	1338.2	0.0003	0.999	0.011
		10%	2.0766	9.082	-32.18	2074.7	-0.635	-7.74	898.4	0.0002	0.998	0.009
	6	1%	2.8515	28.119	-284.10	13560.4	-0.774	-22.92	3199.6	0.0006	0.999	0.021
		5%	2.2611	14.333	-100.91	5084.5	-0.677	-11.30	1598.1	0.0003	0.999	0.010
		10%	1.9806	9.670	-68.75	3054.5	-0.618	-8.54	1124.7	0.0002	0.998	0.008
	7	1%	2.6974	32.043	-578.21	20047.3	-0.743	-35.30	4044.9	0.0007	0.999	0.022
		5%	2.1606	16.000	-208.71	7526.5	-0.649	-18.00	2071.9	0.0003	0.999	0.011
		10%	1.9028	10.861	-137.52	4553.9	-0.591	-14.14	1508.2	0.0002	0.998	0.009
	8	1%	2.5798	31.021	-580.72	22624.3	-0.735	-29.14	4298.2	0.0006	0.999	0.019
		5%	2.0812	15.912	-223.09	8837.0	-0.641	-15.72	2320.5	0.0003	0.999	0.010
		10%	1.8412	10.958	-152.31	5459.2	-0.585	-12.87	1731.7	0.0002	0.998	0.008
	9	1%	2.4826	29.683	-551.60	24821.1	-0.722	-27.61	4987.9	0.0006	0.999	0.017
		5%	2.0159	15.438	-219.10	10044.9	-0.632	-15.91	2927.5	0.0003	0.999	0.009
		10%	1.7905	10.708	-154.09	6335.3	-0.576	-13.61	2284.5	0.0002	0.998	0.007
	10	1%	2.3987	29.370	-543.71	26930.0	-0.702	-30.75	5412.1	0.0006	0.999	0.016
		5%	1.9586	15.912	-249.88	11602.6	-0.617	-17.75	3179.3	0.0003	0.998	0.010
		10%	1.7458	11.254	-188.26	7592.3	-0.565	-14.73	2458.4	0.0003	0.998	0.008
I(1)												
	0	1%	6.3769	28.932	226.25	-1520.9	-0.906	-60.16	2021.6	0.0012	0.993	0.075
		5%	4.5831	12.625	79.12	-667.0	-1.117	-9.37	574.0	0.0005	0.990	0.032
		10%	3.7792	7.444	42.51	-408.1	-0.987	-2.23	284.6	0.0003	0.984	0.021
	1	1%	5.4618	32.320	43.15	2824.7	-0.345	10.61	1590.0	0.0009	0.997	0.053
		5%	4.1084	15.078	19.97	915.9	-0.369	14.37	450.7	0.0004	0.997	0.020
		10%	3.4855	9.450	6.20	536.4	-0.324	11.02	215.4	0.0002	0.997	0.013
	2	1%	4.9199	34.587	-40.57	5283.9	0.073	-1.69	2360.2	0.0011	0.997	0.052
		5%	3.8155	16.397	11.69	1555.0	-0.088	9.06	788.2	0.0004	0.998	0.020
		10%	3.2969	10.430	8.63	808.8	-0.101	7.50	431.8	0.0003	0.998	0.012
	3	1%	4.5632	37.496	-222.89	10249.9	-0.047	25.14	2073.5	0.0010	0.998	0.042
		5%	3.6167	17.470	-20.18	3027.9	-0.086	20.55	665.5	0.0004	0.999	0.016
		10%	3.1663	11.076	2.00	1538.3	-0.085	15.45	342.9	0.0002	0.999	0.010
	4	1%	4.3109	35.073	-60.22	10305.1	0.061	23.68	2355.8	0.0009	0.998	0.037
		5%	3.4712	17.189	37.78	3212.4	-0.029	21.23	773.1	0.0004	0.999	0.015
		10%	3.0679	11.196	33.82	1685.9	-0.040	15.86	414.8	0.0002	0.998	0.010
	5	1%	4.1121	37.352	-142.31	13805.1	0.089	12.55	2922.8	0.0008	0.999	0.031
		5%	3.3551	18.606	10.03	4568.0	0.005	14.96	1063.2	0.0003	0.999	0.013
		10%	2.9892	12.159	22.13	2444.9	-0.014	12.46	582.7	0.0002	0.999	0.008
	6	1%	3.9571	38.172	-215.56	18262.4	0.004	35.42	2538.4	0.0008	0.999	0.029
		5%	3.2641	18.945	9.01	5997.1	-0.022	25.23	911.7	0.0003	0.999	0.012
		10%	2.9266	12.474	26.64	3249.7	-0.026	18.71	492.5	0.0002	0.999	0.008
	7	1%	3.8218	44.015	-604.70	27864.0	-0.019	29.05	3169.8	0.0010	0.999	0.031
		5%	3.1856	21.334	-117.79	9467.2	-0.033	23.84	1144.8	0.0004	0.999	0.012
		10%	2.8730	13.876	-35.90	5127.8	-0.030	18.20	624.9	0.0002	0.999	0.008
	8	1%	3.7172	43.106	-546.56	31034.3	-0.008	37.79	2981.2	0.0008	0.999	0.027
		5%	3.1216	21.499	-105.36	11043.5	-0.022	27.46	1091.5	0.0003	0.999	0.011
		10%	2.8282	14.157	-28.12	6078.9	-0.022	20.33	609.9	0.0002	0.999	0.007
	9	1%	3.6248	43.962	-570.88	35329.7	0.031	35.17	2827.7	0.0008	0.999	0.023
		5%	3.0651	22.375	-123.66	13046.4	-0.001	26.51	1004.8	0.0003	0.999	0.009
		10%	2.7884	14.939	-44.40	7392.6	-0.007	19.54	561.9	0.0002	0.999	0.006
	10	1%	3.5518	41.779	-364.97	35425.4	0.040	31.98	3421.2	0.0008	0.999	0.019
		5%	3.0187	22.194	-68.96	13851.5	0.008	23.85	1357.8	0.0003	0.999	0.008
		10%	2.7554	15.028	-17.25	8047.5	0.000	17.86	795.0	0.0002	0.999	0.006

Note: The RS regression model is equation (12). The dependent variable is the simulated α-quantile of the test statistic. Separate regressions are run for each number k of individually $I(0)$ or $I(1)$ variables $\mathbf{x}_t$ in equation (6). SE($\theta_{0,0}$) denotes the heteroskedasticity-robust standard error of the intercept, $\bar{R}^2$ the adjusted coefficient of determination, and RMSE the root mean square error.

Table 12: Response surface estimates, F-statistic, case (iii)

	k	α	$\theta_{0,0}$	$\theta_{1,0}$	$\theta_{2,0}$	$\theta_{3,0}$	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	SE($\theta_{0,0}$)	$\bar{R}^2$	RMSE
I(0)												
	0	1%	11.7570	43.861	306.37	-2861.6	-4.037	-28.71	2832.9	0.0024	0.981	0.179
		5%	8.1893	16.491	77.09	-997.0	-3.686	27.36	580.0	0.0011	0.946	0.091
		10%	6.5903	8.444	21.87	-414.0	-3.113	26.22	182.6	0.0008	0.893	0.064
	1	1%	6.8187	33.223	-28.85	4086.1	-2.015	42.84	993.4	0.0012	0.993	0.071
		5%	4.9055	13.345	-14.20	1463.9	-1.586	31.21	115.3	0.0005	0.989	0.033
		10%	4.0346	7.442	-24.28	997.6	-1.356	23.73	-31.1	0.0004	0.980	0.023
	2	1%	5.1280	29.192	-16.16	4569.6	-1.136	-2.71	1783.4	0.0012	0.995	0.053
		5%	3.7841	12.223	17.43	1344.5	-1.009	7.63	521.5	0.0005	0.994	0.022
		10%	3.1638	6.808	22.06	578.7	-0.876	5.98	266.2	0.0003	0.992	0.015
	3	1%	4.2658	29.088	-145.33	7753.9	-1.030	0.92	1706.0	0.0009	0.997	0.037
		5%	3.2112	12.389	-9.60	2302.1	-0.879	7.13	537.1	0.0003	0.997	0.015
		10%	2.7190	7.261	6.75	1141.8	-0.770	5.96	272.7	0.0002	0.997	0.010
	4	1%	3.7410	26.457	-53.15	7531.7	-0.865	-8.31	2081.1	0.0007	0.998	0.029
		5%	2.8601	12.012	18.12	2328.2	-0.760	0.60	789.7	0.0003	0.998	0.012
		10%	2.4460	7.374	20.32	1177.7	-0.679	1.21	458.4	0.0002	0.998	0.008
	5	1%	3.3828	27.150	-138.63	10004.3	-0.826	-17.91	2652.7	0.0006	0.998	0.024
		5%	2.6202	12.815	-22.08	3380.4	-0.720	-4.22	1091.7	0.0003	0.999	0.010
		10%	2.2599	8.123	-9.45	1878.0	-0.644	-2.10	683.5	0.0002	0.998	0.007
	6	1%	3.1213	27.473	-228.84	13186.1	-0.818	-11.21	2791.9	0.0006	0.999	0.022
		5%	2.4453	13.348	-64.85	4740.2	-0.694	-3.27	1274.8	0.0003	0.999	0.009
		10%	2.1240	8.645	-38.20	2733.0	-0.619	-2.31	851.4	0.0002	0.999	0.007
	7	1%	2.9146	31.561	-540.36	19890.5	-0.778	-24.28	3625.6	0.0007	0.999	0.022
		5%	2.3094	15.034	-172.63	7159.2	-0.660	-10.09	1724.9	0.0003	0.999	0.010
		10%	2.0189	9.801	-103.30	4172.4	-0.588	-7.54	1198.4	0.0002	0.999	0.007
	8	1%	2.7598	30.262	-526.09	22151.7	-0.760	-17.91	3778.4	0.0006	0.999	0.019
		5%	2.2043	15.008	-187.17	8435.3	-0.642	-8.63	1946.6	0.0003	0.999	0.009
		10%	1.9372	9.981	-118.54	5049.5	-0.576	-6.84	1394.1	0.0002	0.999	0.007
	9	1%	2.6331	29.426	-528.97	24762.7	-0.732	-19.06	4428.0	0.0006	0.999	0.017
		5%	2.1194	14.693	-186.17	9598.8	-0.626	-9.52	2451.1	0.0003	0.999	0.008
		10%	1.8712	9.926	-125.21	5929.0	-0.563	-8.12	1856.4	0.0002	0.999	0.006
	10	1%	2.5287	28.641	-487.15	26207.1	-0.710	-22.09	4869.5	0.0006	0.999	0.015
		5%	2.0475	15.161	-215.58	11107.1	-0.608	-12.01	2746.5	0.0003	0.999	0.009
		10%	1.8152	10.448	-155.76	7105.5	-0.550	-9.62	2060.5	0.0002	0.998	0.007
I(1)												
	0	1%	11.7570	43.861	306.37	-2861.6	-4.037	-28.71	2832.9	0.0024	0.981	0.179
		5%	8.1893	16.491	77.09	-997.0	-3.686	27.36	580.0	0.0011	0.946	0.091
		10%	6.5903	8.444	21.87	-414.0	-3.113	26.22	182.6	0.0008	0.893	0.064
	1	1%	7.7358	41.914	-47.35	4635.9	-0.976	41.28	1598.1	0.0013	0.996	0.076
		5%	5.7040	18.262	-42.30	1849.9	-0.862	31.91	316.5	0.0006	0.994	0.035
		10%	4.7675	10.770	-48.49	1270.0	-0.755	23.10	88.0	0.0004	0.988	0.026
	2	1%	6.2655	40.712	-65.42	5780.3	0.003	-4.17	2856.5	0.0014	0.997	0.065
		5%	4.7894	18.205	-1.16	1604.2	-0.174	6.49	984.5	0.0006	0.997	0.025
		10%	4.0949	10.958	-2.87	807.4	-0.194	4.77	556.8	0.0004	0.996	0.017
	3	1%	5.4927	41.005	-219.01	10665.3	-0.153	35.87	2145.9	0.0011	0.998	0.049
		5%	4.3026	18.221	-15.94	2996.3	-0.163	24.37	679.9	0.0004	0.998	0.019
		10%	3.7360	10.956	4.11	1462.0	-0.160	17.57	341.6	0.0003	0.998	0.012
	4	1%	5.0052	37.501	-49.78	10659.6	-0.001	32.20	2400.4	0.0010	0.998	0.042
		5%	3.9917	17.658	35.61	3312.6	-0.088	25.69	744.7	0.0004	0.998	0.017
		10%	3.5052	10.932	32.72	1695.6	-0.095	18.64	379.2	0.0003	0.998	0.011
	5	1%	4.6578	39.464	-145.06	14334.9	0.056	17.84	2973.2	0.0009	0.999	0.034
		5%	3.7704	18.852	8.38	4670.2	-0.034	18.36	1027.3	0.0004	0.999	0.014
		10%	3.3408	11.843	19.01	2484.1	-0.048	14.30	550.0	0.0003	0.999	0.009
	6	1%	4.4025	39.981	-225.90	18881.6	-0.016	39.12	2602.0	0.0009	0.999	0.032
		5%	3.6071	19.078	5.16	6137.1	-0.046	27.02	899.1	0.0004	0.999	0.013
		10%	3.2189	12.170	19.32	3340.5	-0.048	19.25	482.8	0.0002	0.999	0.008
	7	1%	4.1964	45.367	-608.38	28419.2	-0.045	34.42	3135.7	0.0010	0.999	0.033
		5%	3.4764	21.309	-120.67	9593.9	-0.055	26.25	1096.2	0.0004	0.999	0.013
		10%	3.1222	13.450	-40.61	5195.8	-0.049	19.23	592.9	0.0003	0.999	0.008
	8	1%	4.0382	44.180	-549.34	31594.4	-0.020	40.53	3012.3	0.0009	0.999	0.028
		5%	3.3727	21.403	-107.32	11133.2	-0.036	28.68	1071.3	0.0004	0.999	0.011
		10%	3.0445	13.787	-39.31	6245.6	-0.035	20.37	601.5	0.0002	0.999	0.008
	9	1%	3.9048	44.723	-568.27	35781.7	0.018	38.66	2789.0	0.0009	0.999	0.024
		5%	3.2853	22.324	-133.27	13251.1	-0.011	26.93	1005.5	0.0004	0.999	0.010
		10%	2.9792	14.447	-50.31	7473.5	-0.018	19.83	536.9	0.0002	0.999	0.007
	10	1%	3.7986	42.582	-375.69	36128.4	0.035	33.27	3466.9	0.0008	0.999	0.020
		5%	3.2145	22.051	-78.77	14078.1	-0.002	24.30	1348.6	0.0003	0.999	0.009
		10%	2.9257	14.493	-23.03	8135.8	-0.009	17.91	778.8	0.0002	0.999	0.006

Note: The RS regression model is equation (12). The dependent variable is the simulated α-quantile of the test statistic. Separate regressions are run for each number k of individually $I(0)$ or $I(1)$ variables $\mathbf{x}_t$ in equation (6). SE($\theta_{0,0}$) denotes the heteroskedasticity-robust standard error of the intercept, $\bar{R}^2$ the adjusted coefficient of determination, and RMSE the root mean square error.

Table 13: Response surface estimates, F-statistic, case (iv)

	k	α	$\theta_{0,0}$	$\theta_{1,0}$	$\theta_{2,0}$	$\theta_{3,0}$	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	SE($\theta_{0,0}$)	$\bar{R}^2$	RMSE
I(0)												
	0	1%	8.2726	45.413	154.08	1124.1	-2.656	4.29	2511.0	0.0017	0.991	0.136
		5%	6.2605	21.046	65.64	69.9	-2.254	31.51	597.1	0.0008	0.985	0.070
		10%	5.3366	13.098	36.97	-82.9	-1.847	25.94	258.5	0.0006	0.975	0.052
	1	1%	6.0697	40.449	-180.48	8097.0	-1.812	42.40	1486.2	0.0013	0.995	0.070
		5%	4.6674	18.888	-72.19	3074.4	-1.501	34.71	312.5	0.0006	0.994	0.035
		10%	4.0162	11.854	-52.40	1862.5	-1.296	25.80	108.9	0.0004	0.990	0.026
	2	1%	4.9692	31.697	51.30	6576.4	-1.337	20.93	1756.0	0.0010	0.996	0.056
		5%	3.8699	14.547	82.80	1696.1	-1.169	19.76	493.0	0.0005	0.995	0.026
		10%	3.3556	8.860	71.23	669.4	-1.053	15.83	212.0	0.0004	0.994	0.019
	3	1%	4.2898	32.093	-71.62	9641.5	-1.183	2.06	2119.0	0.0008	0.998	0.036
		5%	3.3804	14.797	44.85	2841.1	-1.059	10.87	654.3	0.0003	0.998	0.016
		10%	2.9508	9.222	47.72	1375.5	-0.963	9.88	314.9	0.0002	0.997	0.012
	4	1%	3.8394	28.233	48.59	9788.0	-1.121	18.58	1581.3	0.0007	0.998	0.028
		5%	3.0509	13.805	80.96	3052.3	-0.967	13.02	567.5	0.0003	0.999	0.013
		10%	2.6780	8.785	71.64	1461.7	-0.878	9.55	306.0	0.0002	0.998	0.009
	5	1%	3.5043	31.201	-197.13	15048.0	-1.052	-1.08	2486.7	0.0007	0.999	0.025
		5%	2.8099	15.343	-16.90	5094.3	-0.902	1.65	1069.2	0.0003	0.999	0.010
		10%	2.4793	9.939	8.12	2741.5	-0.825	1.94	647.6	0.0002	0.999	0.007
	6	1%	3.2440	34.974	-518.47	22224.5	-0.956	-13.29	3169.2	0.0008	0.999	0.025
		5%	2.6257	16.655	-119.17	7578.0	-0.839	-3.69	1412.4	0.0003	0.999	0.010
		10%	2.3278	10.885	-54.94	4230.4	-0.773	-2.16	918.3	0.0002	0.999	0.007
	7	1%	3.0511	33.952	-524.65	24792.2	-0.936	-16.70	3659.6	0.0006	0.999	0.020
		5%	2.4844	16.891	-154.27	9160.4	-0.818	-6.23	1746.1	0.0003	0.999	0.009
		10%	2.2108	11.225	-82.09	5273.1	-0.751	-4.14	1172.9	0.0002	0.999	0.006
	8	1%	2.8925	33.156	-544.74	27841.4	-0.895	-13.89	3907.1	0.0006	0.999	0.018
		5%	2.3687	17.001	-178.26	10697.4	-0.781	-6.90	1987.5	0.0003	0.999	0.008
		10%	2.1153	11.515	-107.24	6391.1	-0.719	-5.24	1384.4	0.0002	0.999	0.006
	9	1%	2.7640	31.108	-432.76	28269.9	-0.868	-10.38	4082.1	0.0006	0.999	0.016
		5%	2.2744	16.469	-154.84	11461.3	-0.756	-6.82	2314.2	0.0003	0.999	0.008
		10%	2.0373	11.309	-101.55	7072.7	-0.696	-6.06	1736.0	0.0002	0.999	0.006
	10	1%	2.6539	30.619	-409.19	30074.1	-0.825	-18.54	4772.3	0.0006	0.999	0.014
		5%	2.1939	16.723	-171.91	12823.1	-0.728	-10.95	2717.5	0.0003	0.999	0.007
		10%	1.9706	11.816	-135.38	8400.4	-0.672	-9.32	2041.0	0.0002	0.999	0.006
I(1)												
	0	1%	8.2726	45.413	154.08	1124.1	-2.656	4.29	2511.0	0.0017	0.991	0.136
		5%	6.2605	21.046	65.64	69.9	-2.254	31.51	597.1	0.0008	0.985	0.070
		10%	5.3366	13.098	36.97	-82.9	-1.847	25.94	258.5	0.0006	0.975	0.052
	1	1%	6.6057	49.213	-286.87	9705.8	-0.717	36.21	2279.4	0.0014	0.997	0.072
		5%	5.1415	23.731	-115.45	3583.0	-0.609	29.92	690.4	0.0006	0.996	0.035
		10%	4.4554	15.397	-84.94	2174.8	-0.506	20.79	378.8	0.0004	0.994	0.026
	2	1%	5.7472	41.494	13.91	7775.4	-0.121	23.60	2880.7	0.0012	0.997	0.064
		5%	4.5616	20.327	57.22	2120.1	-0.185	20.73	1028.2	0.0005	0.997	0.027
		10%	3.9993	13.130	46.79	903.7	-0.173	14.56	599.0	0.0004	0.997	0.018
	3	1%	5.2013	42.313	-108.26	12225.4	-0.141	35.73	2780.6	0.0009	0.998	0.047
		5%	4.1945	20.458	37.75	3664.3	-0.167	30.29	900.6	0.0004	0.998	0.021
		10%	3.7116	13.335	35.17	1883.5	-0.153	22.65	478.8	0.0003	0.998	0.014
	4	1%	4.8255	39.532	7.21	13681.4	-0.109	60.74	1974.7	0.0008	0.999	0.036
		5%	3.9383	20.035	73.80	4469.8	-0.115	39.43	596.0	0.0004	0.999	0.016
		10%	3.5111	13.189	59.54	2384.0	-0.106	28.44	282.2	0.0003	0.999	0.011
	5	1%	4.5396	42.973	-233.57	20394.7	-0.079	46.84	2749.5	0.0009	0.999	0.035
		5%	3.7464	21.604	-2.07	6891.7	-0.079	32.41	955.1	0.0004	0.999	0.015
		10%	3.3616	14.197	22.15	3730.1	-0.074	24.21	497.3	0.0003	0.999	0.010
	6	1%	4.3068	49.285	-669.32	30910.4	0.007	32.96	3384.3	0.0011	0.999	0.035
		5%	3.5944	23.756	-123.77	10370.3	-0.033	27.73	1188.4	0.0004	0.999	0.014
		10%	3.2438	15.485	-38.89	5652.1	-0.042	22.04	618.0	0.0003	0.999	0.009
	7	1%	4.1366	49.091	-683.47	35272.2	0.009	27.49	3816.6	0.0009	0.999	0.029
		5%	3.4769	24.468	-154.95	12634.5	-0.028	24.97	1401.5	0.0004	0.999	0.012
		10%	3.1520	16.038	-53.45	6967.6	-0.034	19.97	758.1	0.0002	0.999	0.008
	8	1%	3.9994	46.572	-572.10	38234.9	-0.029	47.98	3430.1	0.0009	0.999	0.027
		5%	3.3825	23.668	-104.39	13951.7	-0.041	34.42	1246.2	0.0004	0.999	0.011
		10%	3.0778	15.633	-18.33	7709.4	-0.039	25.81	678.8	0.0002	0.999	0.008
	9	1%	3.8817	44.795	-359.86	37839.1	0.003	51.35	2725.5	0.0008	0.999	0.022
		5%	3.3007	23.597	-41.90	14429.5	-0.018	34.84	969.8	0.0004	0.999	0.009
		10%	3.0132	16.091	-6.92	8541.1	-0.020	25.40	526.2	0.0002	0.999	0.007
	10	1%	3.7827	43.554	-234.69	39555.6	0.022	43.32	3603.2	0.0008	0.999	0.019
		5%	3.2321	23.516	2.38	15498.7	-0.004	30.06	1443.9	0.0004	0.999	0.009
		10%	2.9597	16.097	26.02	9120.7	-0.010	22.36	832.8	0.0002	0.999	0.006

Note: The RS regression model is equation (12). The dependent variable is the simulated α-quantile of the test statistic. Separate regressions are run for each number k of individually $I(0)$ or $I(1)$ variables $\mathbf{x}_t$ in equation (6). SE($\theta_{0,0}$) denotes the heteroskedasticity-robust standard error of the intercept, $\bar{R}^2$ the adjusted coefficient of determination, and RMSE the root mean square error.

Table 14: Response surface estimates, F-statistic, case (v)

	k	α	$\theta_{0,0}$	$\theta_{1,0}$	$\theta_{2,0}$	$\theta_{3,0}$	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	SE($\theta_{0,0}$)	$\bar{R}^2$	RMSE
I(0)												
	0	1%	15.6672	74.372	185.49	1121.9	-8.703	87.05	3197.9	0.0041	0.976	0.327
		5%	11.6378	31.535	31.60	229.5	-7.003	98.25	346.7	0.0021	0.942	0.174
		10%	9.7837	17.880	-1.41	123.6	-5.744	68.01	-4.3	0.0015	0.894	0.125
	1	1%	8.6578	53.977	-386.80	11734.0	-3.184	72.38	1673.8	0.0021	0.992	0.115
		5%	6.5535	23.744	-188.44	4752.8	-2.584	51.94	271.9	0.0010	0.985	0.061
		10%	5.5742	14.234	-145.42	3046.8	-2.227	35.98	73.9	0.0007	0.972	0.046
	2	1%	6.3327	35.190	126.81	6172.7	-1.852	30.34	1984.7	0.0015	0.994	0.077
		5%	4.8627	14.956	118.92	1272.0	-1.599	25.70	524.5	0.0007	0.991	0.039
		10%	4.1747	8.269	97.39	290.0	-1.435	20.04	208.0	0.0005	0.986	0.028
	3	1%	5.1477	33.218	-7.67	9634.1	-1.585	25.23	1833.6	0.0010	0.997	0.048
		5%	4.0046	14.191	82.85	2586.3	-1.356	24.63	412.6	0.0004	0.996	0.023
		10%	3.4649	8.117	75.53	1137.6	-1.215	20.17	111.4	0.0003	0.994	0.017
	4	1%	4.4331	28.756	100.06	9858.5	-1.374	38.23	1147.5	0.0008	0.998	0.034
		5%	3.4833	13.035	118.45	2787.8	-1.151	24.95	241.6	0.0004	0.998	0.016
		10%	3.0333	7.755	98.15	1222.3	-1.029	18.30	40.4	0.0003	0.997	0.011
	5	1%	3.9439	31.261	-148.35	15056.8	-1.226	14.56	2086.4	0.0008	0.999	0.029
		5%	3.1298	14.459	22.61	4762.9	-1.022	11.20	751.8	0.0003	0.999	0.012
		10%	2.7420	8.810	40.12	2417.6	-0.922	9.44	379.2	0.0002	0.999	0.009
	6	1%	3.5836	34.980	-478.71	22258.5	-1.072	-1.42	2768.6	0.0009	0.999	0.027
		5%	2.8731	15.844	-86.21	7309.6	-0.921	4.56	1078.1	0.0003	0.999	0.011
		10%	2.5309	9.862	-25.60	3913.4	-0.836	4.13	638.2	0.0002	0.999	0.007
	7	1%	3.3228	34.048	-499.60	24964.6	-1.016	-8.14	3293.5	0.0007	0.999	0.022
		5%	2.6822	16.125	-121.73	8835.6	-0.868	-0.70	1450.6	0.0003	0.999	0.009
		10%	2.3730	10.338	-57.36	4993.3	-0.790	0.58	903.9	0.0002	0.999	0.006
	8	1%	3.1167	32.784	-499.57	27638.0	-0.968	-2.71	3415.5	0.0007	0.999	0.019
		5%	2.5318	16.062	-140.03	10303.2	-0.823	0.13	1630.4	0.0003	0.999	0.008
		10%	2.2488	10.504	-74.45	5983.0	-0.749	0.19	1082.3	0.0002	0.999	0.006
	9	1%	2.9520	30.647	-381.70	27841.5	-0.918	-1.10	3521.8	0.0006	0.999	0.016
		5%	2.4111	15.672	-118.12	10965.9	-0.785	-0.25	1845.2	0.0003	0.999	0.007
		10%	2.1491	10.421	-67.38	6526.9	-0.716	-0.88	1330.0	0.0002	0.999	0.005
	10	1%	2.8148	29.849	-334.35	29037.9	-0.867	-9.39	4222.0	0.0006	0.999	0.014
		5%	2.3108	15.822	-126.64	12126.2	-0.749	-4.91	2283.5	0.0003	0.999	0.007
		10%	2.0663	10.804	-93.09	7703.5	-0.686	-3.89	1639.1	0.0002	0.999	0.005
I(1)												
	0	1%	15.6672	74.372	185.49	1121.9	-8.703	87.05	3197.9	0.0041	0.976	0.327
		5%	11.6378	31.535	31.60	229.5	-7.003	98.25	346.7	0.0021	0.942	0.174
		10%	9.7837	17.880	-1.41	123.6	-5.744	68.01	-4.3	0.0015	0.894	0.125
	1	1%	9.4757	66.489	-508.98	13578.6	-1.643	54.31	2765.4	0.0021	0.995	0.116
		5%	7.2736	31.008	-258.62	5500.5	-1.410	38.54	804.6	0.0010	0.991	0.062
		10%	6.2405	19.323	-195.54	3484.2	-1.229	22.90	471.9	0.0007	0.982	0.049
	2	1%	7.3794	49.423	14.60	8348.6	-0.346	20.79	3530.4	0.0016	0.997	0.080
		5%	5.7921	23.183	50.86	2084.2	-0.427	16.13	1328.6	0.0007	0.996	0.036
		10%	5.0387	14.418	30.44	884.8	-0.417	8.50	828.5	0.0005	0.994	0.026
	3	1%	6.2934	47.128	-98.49	12772.6	-0.322	43.34	3073.3	0.0011	0.998	0.055
		5%	5.0280	22.005	30.71	3780.0	-0.343	33.73	991.0	0.0005	0.998	0.024
		10%	4.4209	13.666	28.22	1876.3	-0.319	23.69	545.6	0.0004	0.997	0.017
	4	1%	5.6231	43.203	-2.49	14546.8	-0.220	67.63	2136.5	0.0010	0.998	0.041
		5%	4.5535	21.223	53.59	4805.8	-0.222	41.27	675.8	0.0004	0.999	0.018
		10%	4.0376	13.548	34.80	2644.2	-0.210	28.11	360.6	0.0003	0.998	0.012
	5	1%	5.1587	45.567	-235.69	21158.0	-0.168	54.40	2814.2	0.0010	0.999	0.039
		5%	4.2277	22.329	-19.03	7209.9	-0.156	34.58	994.8	0.0004	0.999	0.016
		10%	3.7757	14.209	3.89	3939.3	-0.149	24.74	534.9	0.0003	0.999	0.011
	6	1%	4.8057	51.615	-691.45	31924.2	-0.058	39.12	3419.8	0.0011	0.999	0.038
		5%	3.9860	24.135	-135.46	10634.7	-0.091	30.11	1191.8	0.0004	0.999	0.015
		10%	3.5826	15.234	-50.44	5789.7	-0.100	22.94	629.4	0.0003	0.999	0.010
	7	1%	4.5507	51.076	-714.13	36444.1	-0.034	30.83	3916.8	0.0010	0.999	0.031
		5%	3.8043	24.729	-167.90	12906.1	-0.068	25.95	1427.3	0.0004	0.999	0.012
		10%	3.4363	15.807	-70.96	7194.4	-0.074	19.74	792.0	0.0003	0.999	0.008
	8	1%	4.3518	48.107	-597.71	39355.8	-0.058	49.73	3573.6	0.0009	0.999	0.028
		5%	3.6625	23.853	-122.07	14288.0	-0.071	34.41	1301.0	0.0004	0.999	0.012
		10%	3.3218	15.342	-35.71	7944.8	-0.070	25.04	722.4	0.0003	0.999	0.008
	9	1%	4.1866	46.029	-378.60	38771.0	-0.021	53.53	2803.7	0.0009	0.999	0.023
		5%	3.5444	23.693	-62.05	14837.0	-0.043	34.68	1031.3	0.0004	0.999	0.010
		10%	3.2264	15.744	-27.02	8844.7	-0.045	24.54	586.4	0.0003	0.999	0.007
	10	1%	4.0495	45.024	-287.62	41126.0	0.005	43.67	3743.9	0.0008	0.999	0.020
		5%	3.4469	23.628	-27.22	16086.0	-0.025	29.75	1506.3	0.0004	0.999	0.009
		10%	3.1484	15.668	8.66	9388.1	-0.031	21.73	873.8	0.0003	0.999	0.006

Note: The RS regression model is equation (12). The dependent variable is the simulated α-quantile of the test statistic. Separate regressions are run for each number k of individually $I(0)$ or $I(1)$ variables $\mathbf{x}_t$ in equation (6). SE($\theta_{0,0}$) denotes the heteroskedasticity-robust standard error of the intercept, $\bar{R}^2$ the adjusted coefficient of determination, and RMSE the root mean square error.

Table 15: Response surface estimates, t-statistic, case (i)

	k	α	$\theta_{0,0}$	$\theta_{1,0}$	$\theta_{2,0}$	$\theta_{3,0}$	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	SE($\theta_{0,0}$)	$\bar{R}^2$	RMSE
I(0)												
	0	1%	-2.5570	-4.228	17.84	-177.5	0.536	-12.54	1.3	0.0003	0.977	0.015
		5%	-1.9356	-1.794	27.57	-257.0	0.477	-11.72	63.6	0.0002	0.919	0.008
		10%	-1.6133	-0.889	28.87	-263.2	0.427	-10.13	63.4	0.0001	0.812	0.006
	1	1%	-2.5601	-3.931	13.74	-347.2	0.267	-14.04	12.1	0.0003	0.983	0.014
		5%	-1.9372	-1.621	25.65	-335.6	0.243	-9.16	52.5	0.0001	0.947	0.007
		10%	-1.6145	-0.751	29.32	-332.2	0.237	-6.68	47.1	0.0001	0.758	0.006
	2	1%	-2.5597	-4.083	26.05	-691.1	0.162	-12.93	-11.4	0.0003	0.984	0.014
		5%	-1.9373	-1.682	32.07	-480.2	0.163	-7.05	28.6	0.0002	0.948	0.007
		10%	-1.6145	-0.773	34.74	-425.3	0.170	-4.09	17.9	0.0001	0.773	0.006
	3	1%	-2.5618	-3.574	2.53	-629.1	0.136	-14.66	35.5	0.0003	0.981	0.013
		5%	-1.9386	-1.385	19.98	-401.7	0.137	-6.99	40.5	0.0002	0.939	0.007
		10%	-1.6157	-0.498	26.13	-351.6	0.150	-3.60	20.5	0.0001	0.781	0.005
	4	1%	-2.5614	-3.616	0.82	-770.8	0.092	-11.73	-39.5	0.0003	0.981	0.013
		5%	-1.9387	-1.409	22.33	-486.7	0.115	-5.63	15.0	0.0002	0.938	0.007
		10%	-1.6160	-0.499	32.12	-464.8	0.139	-2.94	10.3	0.0001	0.809	0.005
	5	1%	-2.5637	-3.065	-31.75	-536.9	0.094	-13.35	8.0	0.0004	0.979	0.012
		5%	-1.9393	-1.264	14.86	-443.1	0.106	-5.17	18.0	0.0002	0.927	0.007
		10%	-1.6162	-0.472	34.07	-507.5	0.128	-1.80	-3.5	0.0002	0.797	0.005
	6	1%	-2.5630	-3.175	-30.45	-776.8	0.072	-11.72	-30.6	0.0004	0.980	0.012
		5%	-1.9396	-1.196	12.88	-486.8	0.097	-4.65	14.3	0.0002	0.933	0.007
		10%	-1.6168	-0.354	33.32	-533.1	0.126	-1.74	5.1	0.0002	0.828	0.005
	7	1%	-2.5635	-3.220	-27.84	-1090.6	0.074	-12.67	-12.1	0.0004	0.982	0.012
		5%	-1.9402	-1.073	11.31	-571.5	0.098	-5.69	50.1	0.0002	0.938	0.007
		10%	-1.6173	-0.202	31.79	-557.9	0.124	-2.02	25.2	0.0002	0.832	0.005
	8	1%	-2.5631	-3.528	-7.83	-1696.0	0.067	-11.43	-68.4	0.0004	0.984	0.013
		5%	-1.9402	-1.173	20.72	-803.3	0.091	-4.42	15.7	0.0002	0.946	0.007
		10%	-1.6173	-0.196	37.97	-689.4	0.118	-0.96	3.5	0.0002	0.845	0.005
	9	1%	-2.5637	-3.277	-22.08	-1847.4	0.064	-12.25	-50.4	0.0004	0.981	0.012
		5%	-1.9407	-1.082	19.38	-892.4	0.091	-4.62	26.0	0.0002	0.935	0.007
		10%	-1.6177	-0.179	45.41	-855.5	0.120	-0.99	3.4	0.0002	0.854	0.005
	10	1%	-2.5639	-3.316	-18.55	-2274.8	0.062	-12.64	7.4	0.0005	0.977	0.012
		5%	-1.9410	-0.999	18.75	-1000.6	0.090	-5.32	81.1	0.0002	0.918	0.007
		10%	-1.6179	-0.151	52.67	-1031.8	0.120	-1.32	38.4	0.0002	0.838	0.005
I(1)												
	0	1%	-2.5570	-4.228	17.84	-177.5	0.536	-12.54	1.3	0.0003	0.977	0.015
		5%	-1.9356	-1.794	27.57	-257.0	0.477	-11.72	63.6	0.0002	0.919	0.008
		10%	-1.6133	-0.889	28.87	-263.2	0.427	-10.13	63.4	0.0001	0.812	0.006
	1	1%	-3.2084	-6.088	13.10	-388.0	0.341	-13.90	-53.9	0.0003	0.989	0.016
		5%	-2.5919	-2.736	22.11	-338.2	0.336	-9.88	44.5	0.0002	0.971	0.009
		10%	-2.2631	-1.555	23.72	-314.9	0.333	-7.78	61.0	0.0001	0.905	0.008
	2	1%	-3.6158	-8.125	50.03	-994.4	0.263	-6.47	-294.8	0.0003	0.992	0.016
		5%	-3.0024	-3.498	39.12	-541.5	0.317	-2.75	-127.3	0.0002	0.974	0.010
		10%	-2.6728	-1.854	35.56	-401.7	0.352	-0.78	-92.8	0.0002	0.924	0.010
	3	1%	-3.9436	-7.563	-15.28	-449.8	0.361	-11.84	-294.7	0.0003	0.993	0.015
		5%	-3.3268	-2.848	6.46	-130.7	0.416	-3.31	-206.9	0.0002	0.975	0.010
		10%	-2.9950	-1.143	16.79	-73.3	0.461	0.17	-202.6	0.0002	0.946	0.010
	4	1%	-4.2179	-8.454	14.11	-1172.9	0.455	-14.42	-383.7	0.0004	0.994	0.015
		5%	-3.6006	-2.813	28.79	-528.0	0.550	-7.95	-209.3	0.0002	0.976	0.011
		10%	-3.2672	-0.772	39.91	-411.0	0.613	-5.05	-183.6	0.0002	0.956	0.012
	5	1%	-4.4577	-9.614	60.84	-2104.3	0.507	-10.49	-613.0	0.0004	0.992	0.016
		5%	-3.8367	-3.803	109.24	-1648.5	0.599	-1.01	-499.2	0.0003	0.962	0.013
		10%	-3.5015	-1.583	130.40	-1590.4	0.665	2.85	-498.5	0.0004	0.947	0.014
	6	1%	-4.6757	-10.385	116.71	-3354.8	0.587	-10.28	-829.7	0.0005	0.992	0.017
		5%	-4.0551	-3.525	150.65	-2381.0	0.721	-4.64	-600.1	0.0004	0.965	0.014
		10%	-3.7195	-0.854	167.29	-2184.4	0.806	-2.55	-554.3	0.0004	0.959	0.015
	7	1%	-4.8779	-10.502	156.73	-4534.7	0.679	-13.75	-980.6	0.0005	0.992	0.017
		5%	-4.2556	-2.905	183.40	-3046.5	0.815	-7.00	-733.8	0.0004	0.962	0.016
		10%	-3.9191	0.112	198.56	-2717.8	0.904	-4.72	-686.2	0.0004	0.963	0.016
	8	1%	-5.0635	-11.403	249.18	-6523.7	0.731	-7.50	-1453.6	0.0006	0.992	0.019
		5%	-4.4408	-2.487	231.22	-3900.0	0.875	-1.77	-1128.6	0.0005	0.960	0.017
		10%	-4.1036	0.922	240.35	-3336.5	0.972	0.05	-1065.4	0.0005	0.965	0.019
	9	1%	-5.2413	-11.169	303.31	-8191.6	0.831	-12.18	-1646.7	0.0006	0.990	0.020
		5%	-4.6169	-1.830	292.46	-5127.8	0.992	-7.77	-1198.8	0.0006	0.961	0.019
		10%	-4.2789	1.662	315.56	-4623.3	1.098	-6.40	-1112.7	0.0007	0.968	0.021
	10	1%	-5.4088	-10.274	331.14	-9654.3	0.883	-7.08	-2554.5	0.0007	0.986	0.020
		5%	-4.7837	-0.179	321.40	-6140.7	1.058	-4.90	-2101.8	0.0007	0.962	0.020
		10%	-4.4453	3.781	337.64	-5412.7	1.173	-5.17	-2005.1	0.0008	0.971	0.021

Note: The RS regression model is equation (12). The dependent variable is the simulated α-quantile of the test statistic. Separate regressions are run for each number k of individually $I(0)$ or $I(1)$ variables $\mathbf{x}_t$ in equation (6). SE($\theta_{0,0}$) denotes the heteroskedasticity-robust standard error of the intercept, $\bar{R}^2$ the adjusted coefficient of determination, and RMSE the root mean square error.

Table 16: Response surface estimates, t-statistic, case (iii)

	k	α	$\theta_{0,0}$	$\theta_{1,0}$	$\theta_{2,0}$	$\theta_{3,0}$	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	SE($\theta_{0,0}$)	$\bar{R}^2$	RMSE
I(0)												
	0	1%	-3.4298	-6.418	-32.65	332.8	0.683	-3.74	-270.9	0.0003	0.982	0.023
		5%	-2.8619	-2.902	-10.94	158.3	0.671	-6.05	-77.9	0.0002	0.948	0.015
		10%	-2.5672	-1.666	-3.56	77.2	0.625	-5.09	-31.3	0.0001	0.900	0.012
	1	1%	-3.4290	-6.987	24.00	-853.0	0.539	-17.17	-52.3	0.0003	0.989	0.018
		5%	-2.8609	-3.076	24.74	-531.0	0.540	-10.99	36.4	0.0002	0.965	0.012
		10%	-2.5663	-1.667	25.81	-441.9	0.544	-7.87	45.4	0.0002	0.932	0.010
	2	1%	-3.4266	-6.721	1.31	-559.3	0.343	-2.29	-388.9	0.0004	0.989	0.017
		5%	-2.8595	-2.620	3.01	-158.8	0.411	0.27	-191.6	0.0002	0.972	0.010
		10%	-2.5653	-1.147	8.09	-104.4	0.454	1.56	-138.8	0.0002	0.960	0.008
	3	1%	-3.4302	-5.918	-8.97	-921.4	0.410	-13.32	-146.2	0.0003	0.991	0.014
		5%	-2.8623	-1.795	-9.54	-204.4	0.461	-6.22	-42.5	0.0002	0.979	0.008
		10%	-2.5679	-0.265	-5.45	-53.7	0.502	-3.39	-21.3	0.0001	0.976	0.007
	4	1%	-3.4308	-5.528	-11.93	-1222.7	0.402	-13.59	-152.3	0.0003	0.990	0.014
		5%	-2.8630	-1.488	-0.16	-475.9	0.472	-7.04	-20.5	0.0002	0.982	0.008
		10%	-2.5687	0.087	5.42	-277.9	0.521	-4.10	1.0	0.0002	0.985	0.006
	5	1%	-3.4302	-5.498	-2.50	-1689.0	0.388	-11.04	-195.9	0.0004	0.992	0.012
		5%	-2.8625	-1.280	8.71	-711.6	0.463	-5.29	-34.5	0.0002	0.984	0.007
		10%	-2.5683	0.442	12.29	-427.6	0.514	-2.84	2.6	0.0002	0.986	0.006
	6	1%	-3.4304	-5.296	1.52	-2132.2	0.387	-10.69	-190.7	0.0004	0.992	0.012
		5%	-2.8626	-1.066	17.75	-948.2	0.461	-3.33	-66.5	0.0002	0.987	0.007
		10%	-2.5684	0.673	24.53	-619.3	0.514	-0.28	-44.6	0.0002	0.989	0.006
	7	1%	-3.4299	-5.292	27.71	-3022.0	0.387	-10.41	-239.7	0.0004	0.993	0.012
		5%	-2.8626	-0.622	20.17	-1159.7	0.457	-2.94	-74.4	0.0002	0.986	0.007
		10%	-2.5685	1.295	21.42	-665.1	0.511	-0.26	-34.8	0.0002	0.989	0.005
	8	1%	-3.4301	-5.097	37.53	-3660.4	0.382	-8.30	-293.3	0.0004	0.991	0.012
		5%	-2.8627	-0.419	32.93	-1475.8	0.456	0.07	-168.2	0.0002	0.986	0.007
		10%	-2.5686	1.533	36.16	-905.1	0.510	3.29	-140.4	0.0002	0.990	0.006
	9	1%	-3.4300	-5.032	53.67	-4395.3	0.381	-6.21	-369.7	0.0004	0.990	0.012
		5%	-2.8633	-0.064	42.72	-1811.6	0.463	1.77	-284.3	0.0002	0.986	0.007
		10%	-2.5694	1.994	43.55	-1081.3	0.520	4.52	-230.9	0.0002	0.990	0.006
	10	1%	-3.4314	-4.262	22.34	-4335.8	0.388	-7.73	-299.3	0.0005	0.987	0.012
		5%	-2.8644	0.640	28.19	-1730.2	0.475	-1.75	-81.1	0.0003	0.984	0.007
		10%	-2.5701	2.596	41.47	-1115.0	0.533	1.45	-55.3	0.0002	0.990	0.006
I(1)												
	0	1%	-3.4298	-6.418	-32.65	332.8	0.683	-3.74	-270.9	0.0003	0.982	0.023
		5%	-2.8619	-2.902	-10.94	158.3	0.671	-6.05	-77.9	0.0002	0.948	0.015
		10%	-2.5672	-1.666	-3.56	77.2	0.625	-5.09	-31.3	0.0001	0.900	0.012
	1	1%	-3.7946	-8.954	39.61	-1093.4	0.527	-19.61	-62.8	0.0004	0.990	0.020
		5%	-3.2140	-4.244	36.61	-684.5	0.517	-12.19	36.5	0.0002	0.968	0.014
		10%	-2.9080	-2.556	38.55	-594.4	0.523	-8.70	47.6	0.0002	0.915	0.013
	2	1%	-4.0902	-10.288	25.15	-835.1	0.336	0.21	-588.7	0.0004	0.993	0.019
		5%	-3.5031	-4.818	30.89	-401.7	0.420	3.00	-332.0	0.0002	0.976	0.014
		10%	-3.1906	-2.804	36.87	-327.2	0.474	4.65	-266.7	0.0002	0.936	0.013
	3	1%	-4.3540	-10.586	19.00	-1163.3	0.471	-9.57	-522.6	0.0004	0.995	0.017
		5%	-3.7596	-4.441	19.67	-283.1	0.541	-1.04	-357.6	0.0003	0.979	0.013
		10%	-3.4423	-2.066	23.42	-65.9	0.598	2.57	-333.3	0.0003	0.950	0.013
	4	1%	-4.5868	-11.348	56.77	-2131.2	0.565	-13.23	-622.0	0.0004	0.995	0.016
		5%	-3.9877	-4.855	77.50	-1176.9	0.674	-5.09	-421.7	0.0003	0.979	0.013
		10%	-3.6664	-2.453	97.35	-1068.5	0.748	-1.39	-397.2	0.0003	0.961	0.014
	5	1%	-4.7974	-12.275	112.58	-3378.9	0.633	-13.40	-803.0	0.0004	0.995	0.017
		5%	-4.1950	-4.786	125.13	-2009.3	0.766	-8.44	-493.3	0.0004	0.974	0.014
		10%	-3.8713	-1.912	140.27	-1752.3	0.856	-6.58	-420.6	0.0004	0.954	0.015
	6	1%	-4.9901	-13.760	212.04	-5299.6	0.681	-5.99	-1243.5	0.0005	0.994	0.018
		5%	-4.3843	-5.767	230.97	-3542.9	0.816	2.52	-1013.7	0.0005	0.970	0.017
		10%	-4.0581	-2.706	253.14	-3243.9	0.909	5.70	-978.1	0.0005	0.960	0.018
	7	1%	-5.1719	-14.050	276.98	-7038.5	0.747	-6.57	-1545.9	0.0005	0.994	0.018
		5%	-4.5637	-4.831	256.85	-4194.2	0.880	1.42	-1232.6	0.0005	0.963	0.017
		10%	-4.2357	-1.279	267.61	-3585.7	0.973	4.34	-1176.7	0.0005	0.957	0.018
	8	1%	-5.3435	-14.516	356.92	-9008.6	0.798	2.77	-2236.6	0.0006	0.991	0.020
		5%	-4.7320	-5.272	374.13	-6160.3	0.944	13.00	-2025.8	0.0006	0.963	0.019
		10%	-4.4025	-1.489	389.40	-5457.4	1.046	15.96	-1990.3	0.0006	0.965	0.021
	9	1%	-5.5100	-13.404	385.52	-10621.9	0.889	3.11	-3208.2	0.0007	0.990	0.020
		5%	-4.8970	-3.385	399.12	-7200.6	1.054	12.52	-3104.8	0.0007	0.969	0.019
		10%	-4.5665	0.748	417.04	-6421.4	1.166	15.13	-3144.6	0.0007	0.973	0.021
	10	1%	-5.6635	-14.155	537.48	-14372.1	0.970	-2.52	-3136.1	0.0009	0.985	0.022
		5%	-5.0495	-3.069	521.85	-9850.3	1.154	1.38	-2695.3	0.0008	0.962	0.022
		10%	-4.7175	1.126	560.01	-9287.0	1.275	1.94	-2595.8	0.0009	0.970	0.023

Note: The RS regression model is equation (12). The dependent variable is the simulated α-quantile of the test statistic. Separate regressions are run for each number k of individually $I(0)$ or $I(1)$ variables $\mathbf{x}_t$ in equation (6). SE($\theta_{0,0}$) denotes the heteroskedasticity-robust standard error of the intercept, $\bar{R}^2$ the adjusted coefficient of determination, and RMSE the root mean square error.

Table 17: Response surface estimates, t-statistic, case (v)

	k	α	$\theta_{0,0}$	$\theta_{1,0}$	$\theta_{2,0}$	$\theta_{3,0}$	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	SE($\theta_{0,0}$)	$\bar{R}^2$	RMSE
I(0)												
	0	1%	-3.9594	-9.262	-16.02	-72.9	1.187	-20.52	-220.4	0.0005	0.978	0.036
		5%	-3.4117	-4.616	-2.02	-19.4	1.050	-16.42	-22.2	0.0003	0.944	0.024
		10%	-3.1280	-2.862	0.71	-3.5	0.929	-11.23	1.2	0.0002	0.898	0.019
	1	1%	-3.9525	-10.789	91.02	-1992.6	0.763	-19.87	-192.9	0.0004	0.987	0.025
		5%	-3.4064	-5.322	66.45	-1137.3	0.743	-11.24	-60.1	0.0003	0.965	0.018
		10%	-3.1237	-3.233	57.56	-856.0	0.733	-6.57	-44.6	0.0002	0.935	0.015
	2	1%	-3.9567	-8.224	-33.20	-563.7	0.588	-9.72	-426.3	0.0004	0.990	0.021
		5%	-3.4091	-3.358	-17.09	-37.8	0.635	-2.92	-227.2	0.0002	0.974	0.014
		10%	-3.1259	-1.510	-7.47	62.7	0.668	0.63	-185.5	0.0002	0.963	0.012
	3	1%	-3.9600	-7.446	-30.31	-1196.7	0.659	-20.37	-201.9	0.0004	0.992	0.017
		5%	-3.4121	-2.419	-18.03	-307.4	0.706	-11.26	-43.7	0.0002	0.982	0.011
		10%	-3.1288	-0.458	-9.78	-96.7	0.743	-7.16	-13.2	0.0002	0.977	0.009
	4	1%	-3.9589	-7.344	-9.52	-1960.8	0.636	-20.39	-109.1	0.0004	0.993	0.015
		5%	-3.4112	-2.317	9.20	-834.6	0.699	-9.10	-38.7	0.0002	0.989	0.009
		10%	-3.1281	-0.368	23.37	-607.6	0.747	-4.28	-40.6	0.0002	0.990	0.008
	5	1%	-3.9589	-7.155	11.82	-2778.2	0.638	-18.94	-156.5	0.0004	0.994	0.014
		5%	-3.4119	-1.604	8.85	-1031.5	0.704	-9.23	-8.6	0.0002	0.990	0.008
		10%	-3.1287	0.492	19.96	-661.3	0.754	-4.65	3.8	0.0002	0.991	0.007
	6	1%	-3.9584	-7.034	38.21	-3730.7	0.630	-17.16	-186.0	0.0004	0.995	0.013
		5%	-3.4121	-1.180	24.21	-1435.6	0.711	-8.13	-18.0	0.0002	0.993	0.007
		10%	-3.1295	1.118	29.16	-861.8	0.769	-4.06	6.1	0.0002	0.993	0.006
	7	1%	-3.9575	-6.939	63.56	-4671.5	0.607	-11.15	-349.3	0.0004	0.994	0.013
		5%	-3.4113	-0.744	35.01	-1754.3	0.691	-3.64	-109.3	0.0002	0.991	0.007
		10%	-3.1285	1.662	39.30	-1050.0	0.747	0.37	-76.8	0.0002	0.993	0.006
	8	1%	-3.9581	-6.365	69.77	-5435.3	0.608	-9.92	-416.4	0.0004	0.993	0.012
		5%	-3.4114	-0.427	60.01	-2328.1	0.692	0.17	-246.5	0.0002	0.992	0.007
		10%	-3.1286	2.044	64.10	-1472.5	0.750	4.62	-223.6	0.0002	0.994	0.006
	9	1%	-3.9587	-5.852	73.44	-6090.6	0.613	-9.47	-403.1	0.0005	0.992	0.012
		5%	-3.4122	0.126	75.17	-2792.7	0.703	1.26	-328.0	0.0003	0.992	0.007
		10%	-3.1295	2.642	83.23	-1877.5	0.764	6.09	-348.5	0.0002	0.994	0.007
	10	1%	-3.9602	-4.887	48.23	-6285.7	0.623	-11.39	-298.2	0.0005	0.989	0.012
		5%	-3.4131	0.980	70.39	-2937.1	0.718	-1.93	-134.1	0.0003	0.991	0.007
		10%	-3.1305	3.549	82.88	-1981.9	0.783	2.01	-106.7	0.0003	0.994	0.007
I(1)												
	0	1%	-3.9594	-9.262	-16.02	-72.9	1.187	-20.52	-220.4	0.0005	0.978	0.036
		5%	-3.4117	-4.616	-2.02	-19.4	1.050	-16.42	-22.2	0.0003	0.944	0.024
		10%	-3.1280	-2.862	0.71	-3.5	0.929	-11.23	1.2	0.0002	0.898	0.019
	1	1%	-4.2423	-12.982	107.28	-2216.0	0.673	-17.64	-275.4	0.0004	0.989	0.027
		5%	-3.6829	-6.773	80.14	-1266.2	0.654	-8.56	-123.6	0.0003	0.970	0.019
		10%	-3.3893	-4.445	73.88	-999.1	0.655	-3.71	-107.6	0.0002	0.932	0.017
	2	1%	-4.4946	-12.270	4.22	-983.3	0.496	-2.92	-719.4	0.0004	0.993	0.022
		5%	-3.9239	-6.180	23.99	-360.5	0.559	5.01	-486.2	0.0003	0.979	0.016
		10%	-3.6222	-3.862	37.59	-269.6	0.611	8.67	-435.5	0.0002	0.951	0.016
	3	1%	-4.7214	-12.803	22.32	-1718.9	0.623	-12.39	-699.5	0.0004	0.995	0.019
		5%	-4.1427	-6.012	46.11	-792.1	0.702	-3.23	-464.3	0.0003	0.983	0.014
		10%	-3.8352	-3.368	59.42	-570.4	0.764	1.06	-421.5	0.0003	0.958	0.014
	4	1%	-4.9232	-14.522	117.59	-3548.8	0.663	-9.17	-957.2	0.0004	0.995	0.017
		5%	-4.3380	-7.476	163.07	-2451.3	0.767	3.86	-847.6	0.0003	0.982	0.015
		10%	-4.0260	-4.703	188.60	-2259.9	0.845	9.50	-861.9	0.0004	0.969	0.015
	5	1%	-5.1123	-15.288	181.92	-5110.2	0.735	-9.81	-1176.4	0.0005	0.995	0.018
		5%	-4.5230	-7.181	207.53	-3285.3	0.861	-0.09	-919.4	0.0004	0.979	0.016
		10%	-4.2074	-3.989	229.31	-2918.5	0.948	4.18	-886.2	0.0004	0.960	0.017
	6	1%	-5.2898	-16.099	275.37	-7277.1	0.817	-10.54	-1474.8	0.0005	0.996	0.018
		5%	-4.6979	-6.718	262.66	-4341.6	0.969	-2.54	-1149.5	0.0005	0.979	0.017
		10%	-4.3801	-3.056	275.64	-3679.5	1.071	0.78	-1096.3	0.0005	0.967	0.018
	7	1%	-5.4562	-16.433	347.30	-9217.5	0.862	-7.31	-1871.5	0.0006	0.994	0.019
		5%	-4.8601	-6.246	326.82	-5622.3	1.020	-1.69	-1432.3	0.0005	0.968	0.018
		10%	-4.5397	-2.227	339.39	-4803.6	1.125	0.88	-1354.9	0.0006	0.961	0.019
	8	1%	-5.6130	-17.509	469.67	-11915.9	0.893	6.31	-2687.0	0.0007	0.990	0.021
		5%	-5.0123	-7.505	495.74	-8385.3	1.048	17.95	-2426.2	0.0007	0.964	0.022
		10%	-4.6894	-3.462	524.69	-7615.2	1.157	21.92	-2387.6	0.0007	0.965	0.023
	9	1%	-5.7677	-16.761	551.99	-14837.3	0.981	7.39	-3806.7	0.0008	0.989	0.021
		5%	-5.1644	-6.340	607.68	-11312.1	1.152	20.63	-3819.9	0.0008	0.971	0.021
		10%	-4.8399	-2.040	648.28	-10633.6	1.269	24.58	-3881.2	0.0008	0.973	0.023
	10	1%	-5.9121	-17.343	717.56	-19266.1	1.059	2.13	-3774.9	0.0010	0.984	0.023
		5%	-5.3060	-6.425	791.19	-15498.8	1.250	9.72	-3433.1	0.0010	0.964	0.024
		10%	-4.9800	-1.800	834.06	-14708.0	1.378	10.12	-3284.1	0.0011	0.969	0.025

Note: The RS regression model is equation (12). The dependent variable is the simulated α-quantile of the test statistic. Separate regressions are run for each number k of individually $I(0)$ or $I(1)$ variables $\mathbf{x}_t$ in equation (6). SE($\theta_{0,0}$) denotes the heteroskedasticity-robust standard error of the intercept, $\bar{R}^2$ the adjusted coefficient of determination, and RMSE the root mean square error.