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Measuring unilateral and multilateral gains from tackling current economic inefficiencies in CO_2 reductions: Theory and evidence

Working Paper Version

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ABSTRACT

Measuring unilateral and multilateral gains from tackling current economic inefficiencies in CO_2 reductions: Theory and evidence

We develop a methodology for (a) constructing unilateral profit (producer surplus)-increasing and emission-decreasing policy reforms and (b) measuring marginal abatement cost (MAC), when countries operate inefficiently in meeting their self-imposed emission caps and when instantaneous radical jumps from their inefficient status-quos to their emission-constrained optima are infeasible due to existing institutional and political constraints. Data from 118 countries combined with the theoretical methodology developed reveals that (a) allocative inefficiencies are pervasive, (b) our proposed unilateral-efficiency increasing reform can result in more than 8% increase in global profit and 30% reduction in net global emission of CO_2 – the biggest gainers being USA, China, Japan, Russia, India, and several countries from western European, and (c) MACs range from zero to 3,000 USD per ton of carbon (USDptc) in 94% of countries in our sample. MAC is more than (resp., less than) 1,000 USDptc in 80% of OECD (resp., 61% of non-OECD) countries. While MACs are zero for many countries in the former Soviet block, they are more than 2,000 USDptc for countries in western Europe. These differences in MACs imply considerable scope for multilateral efficiency improvements in meeting voluntary emission-reduction targets through international emission trading and other international climate initiatives.

JEL classification codes: Q5, Q54, Q58
Keywords: allocative inefficiencies under an emission cap, marginal and non-marginal efficiency-improving policy reforms, marginal abatement costs, ability to abate, reduction in profit.

Measuring unilateral and multilateral gains from tackling current economic inefficiencies in CO_2 reductions: Theory and evidence

1. Introduction.

Current emission levels of countries are a result of domestic choices made after due (mostly unilateral) deliberations on the costs and benefits they entail. Given their sovereign status, countries tend not to succumb to heavy international pressures in the wake of climate change concerns to reduce their self-imposed caps on emissions. While reducing country-level and, hence, global emissions of green house gases is a prime objective of international environmental organisations, ensuring maximum growth in objectives such as output, profit, and material well being (standard of living) of their populations are the major concerns of individual countries. Our intuition and economic theory generally perceive these two objectives to be quite in conflict with each other. However, one must appreciate the fact that this trade-off in objectives is, in theory, strictly binding only in situations where countries actually maximise their individual economic interests subject to various institutional and self-imposed environmental constraints. In this regards, there seems to be scanty empirical evidence that rules out non-optimising behaviour (economic inefficiencies) in the way countries manage their resources. In the presence of such inefficiencies, the trade-off between the two objectives mentioned above could weaken considerably.

In this paper, we build a model and develop a theory of reforms that allow for (allocative) inefficiencies in the way countries operate.[1] In the face of such inefficiencies, the trade-off mentioned above is non-binding and *unilateral*-efficiency improving reforms exist in the current economic environment and at the existing levels of technological development. In this paper, we restrict ourselves to studying the profit motive, *i.e.,* the objective of countries to increase their producer surpluses. Under this objective, at one extreme, there are reforms that can promote a country's profit motive while maintaining the current level of emission and, at the other extreme, there are reforms that make it possible for it to reduce its self-imposed emission cap while maintaining its current level of profit. In between these two extremes are reforms that not only increase the country's profit but can also simultaneously reduce its emission. In this paper, we provide a methodology to construct some unilateral reforms with these properties.

One may hold the view that the efficiency improvements induced by such reforms would be negligible and of second-order importance. However, taking the theoretical

[1] We would like to acknowledge that our theory of efficiency-improving reforms is inspired by the classic literature on optimal taxation and tax-policy reforms visualised, developed, and studied by Diamond and Mirrlees (1971, I and II), Feldstein (1975), Guesnerie (1977, 1995), Weymark (1978, 1979), Ahmad and Stern (1984), Murty and Ray (1989), Myles (1995), Blackorby and Brett (2000), and Murty and Russell (2005).

methodology developed in this paper to the data we could collect, we find that (a) allocative inefficiencies are pervasive – none of the 118 countries in our sample maximises its profit subject to its self-imposed emission cap and (b) the magnitudes of efficiency improvements due to the reforms we propose seem quite significant. The reform that we construct for every country is estimated to result in more than 8% increase in global profit and more than 30% reduction in global emission of CO_2. Increases in profits and reductions in emission levels vary widely across countries depending upon their current usage of fossil fuels and cleaner sources of energy and carbon sequestration efforts.

Conventional methods to measure marginal abatement cost (MAC) too assume optimising behaviour by countries. MAC is usually obtained as a result of a comparative static exercise, which measures the impact of reducing the emission cap on the maximised value of an objective function.[2] Such measures of MAC implicitly assume big radical adjustments in policies that make a country instantaneously jump to the constrained optimum corresponding to its self-imposed emission cap, when this optimum is significantly different from the current status-quo (which will be true if the country is operating inefficiently). However, in reality, such radical instantaneous adjustments may be infeasible due to institutional and political constraints operating at the status-quo. Hence, while measuring MAC, it may be more realistic to confine the analysis to a *local/small neighbourhood* of policies around the current status-quo, as these may be more practical and easier to attain in the short-run given current institutional and political rigidities. In this framework, the MAC captures the impact of a unit reduction in the emission cap on the maximum profit that can be attained by policies that lie in this local neighbourhood. This is the definition of MAC that we adopt in this paper and it is measured in the framework of marginal (local) reforms in policies that affect both emission generation and the level of profit.[3]

Our empirical analysis shows significant variations in marginal abatement costs (MACs) across countries and, hence also, considerable scope for multilateral efficiency improvements in the way many countries can realise their self-chosen or internationally-negotiated reductions in emission targets with least loss in profit by buying abatement more cheaply from countries with lower MACs.

In Section 2, we lay out the model we will employ in this paper. Section 3 is an intuitive exposition of (a) the nature of allocative inefficiencies studied in this paper, (b)

[2] See, for example, the computable general equilibrium models of Ellerman and Decaux (1998), EPPA model of Paltsev et al (2005), and Eyckmans et al (2001).

[3] This approach is inspired by the tax reform literature which takes the view that the current status-quo may not be second-best optimal and radical jumps to the second-best may not be institutionally and politically feasible. So analysis is confined to marginal tax reforms that lead to a local neighbourhood of status-quo tax rates.
We show in the next section that, in the special case where a country operates efficiently, our measure of MAC will correspond to the conventional measure.

profit increasing and environmentally-permissible marginal (local) policy reforms that are available at an allocatively-inefficient status-quo and the local reform that maximises the increase in profit without increasing the emission level at the status-quo, and (c) a measure of MAC that is defined in the context of marginal policy reforms.

In Section 4, we focus on the differential changes in profit and emission levels induced by marginal policy reforms and provide formulae to compute MAC, which are based entirely on data that is available at the status-quo. We derive conditions (in terms of observable data) under which MAC can be expected to be zero.

However, we find that, since this measure of MAC is based on reforms in the *absolute levels* of policy variables, it is not independent of the units in which input policies are measured. Hence, in Section 5, we switch to a framework of local (small) *proportional* changes in policy variables, and derive another measure of MAC that does not have this drawback. We show that the optimal vector of local proportional changes in policies at an inefficient status-quo recommends increases in the usage of an input (*e.g.,* a fossil fuel) if and only if the marginal proportional benefit from the input (which is based on its marginal profitability) is greater than its marginal proportional cost (which is associated with the fact that the use of the input contributes to emission generation and, hence, impinges on the emission-cap). We also show that MAC is a ratio of what we call the "reduction in profit" (RIP) and the "ability to abate" (ATA) of a country when the reform that results in the maximum proportional reduction in emission is adopted at the status-quo.[4]

While the proportional changes in policy variables that are studied in Section 5 are marginal (small), the changes (reforms) in the actual levels of the policy variables they induce may be non-marginal (large). In Section 6, we distinguish between (a) the vector of local proportional changes in policy variables that maximises the proportional increase in profit subject to no proportional increase in the emission level and (b) the vector of local proportional changes in policy variables that solves the *dual problem* of maximising the proportional *reduction* in the emission level subject to no proportional *decrease* in profit. We show, using a novel application of the fixed-point theory, that these two vectors of local proportional changes in policies can be fruitfully combined to construct a vector of reforms (usually non-marginal) in the *levels* of policy variables that leads to a *non-differential* increase in profit and a reduction in the emission level. An algorithm is also provided that aids in numerically solving for such a reform vector for every country in our sample.

In Section 7, we derive a rich set of empirical results, of which we present a snapshot below.[5] Our empirical results seem to be consistent with our intuitive understanding of

[4] More precisely, this holds when RIP is non-negative. If RIP is negative, then MAC is zero.

[5] For a complete set of results for all the 118 countries in our sample, please see the appendix of the working-paper version.

the global picture. The countries that figure prominently in our results, are USA, emerging economies such as China and India, Russia, Japan, and western European countries such as Germany, UK, France, and Spain. Efficiency improvements from the unilateral reforms we propose are the greatest for these countries. In our sample, profit increases from these reforms range from 57.7 million USD to 777,113.5 million USD, while emission reductions they induce range from 0.006 million tons to 1,016.4 million tons. The biggest gainer in profit is USA, while the country that can reduce emission of CO_2 the most is China. Moreover, the efficiency-improving reforms we propose recommend decreases in the usage of fossil fuels such as coal and gas and increases in afforestation efforts in a majority of countries in our sample.

MACs vary widely across countries – in 94% of countries in our sample, it ranges between zero and 3,000 USD per ton of carbon (USDptc). At one extreme of the global spectrum are nine countries, mostly from the former Soviet block (including Russia), with zero MACs and negative reductions in profits (RIPs). These are countries whose self-imposed emission constraint is non-binding when profit is maximised using marginal (local) reforms starting from their respective inefficient status-quos. This is explained primarily by the fact that marginal profits of fossil fuels such as coal and gas are significantly negative in these countries (there is a significant over-utilization of coal and gas in these countries). Hence, the optimal local reform in these countries recommends reducing usage of these inputs, which, while increasing their profit levels, also reduces their emission levels below their self-imposed emission caps (*i.e.,* below the emission levels prevailing at their status-quos.) At the other end of the global spectrum are several western European countries whose MACs are more than 2,000 USDptc. In these countries, marginal profits from fossil fuels are positive and high, which implies that their RIPs from reducing usage of these fossil fuels are very high. These tend to dominate their abilities to abate (ATAs) leading to high MACs. China, with its very high ATA, has a low MAC (119.34 USDptc), while USA with both a high ATA and a high RIP has a MAC of 1,059.56 USDptc.

The data employed and the estimation procedures are discussed in Section 8, and we conclude in Section 9. Most of the proofs of the theorems and lemmas can be found in the appendix.

2. The model.

Data considerations were crucial for the design of the model presented below, which can be extended and improved further as richer and more disaggregated country-level data becomes available.

2.1. By-production approach to modelling emission-generating technology.

The by-production approach in MRL and Murty (2015) is employed to model the emission-generating technology. This approach models the overall technology as an intersection of (i) a standard neo-classical *intended-output technology* of human design that describes how all inputs are transformed into desirable (intended) outputs of the producing unit and (ii) a *nature's emission-generating set* that is governed by basic physical laws such as the material balance conditions that capture how the emission-generating inputs (such as fossil fuels) transform into emissions in nature and how sequestration efforts (human and natural) help in abating them. In this paper we assume away technical inefficiencies, *i.e.,* producing units are assumed to be on the frontiers of both the intended-output technology and nature's emission-generating set. Rather, the focus of this paper is on allocative inefficiencies.

Energy generated by coal, gas, and renewables, denoted by c, g, and r, respectively, is mainly employed in the stationary energy sector and can be electrical or non-electrical. Energy generated from oil, denoted by o, is mainly required by the non-stationary (transportation) sector. Hence, we treat these two forms of energy separately. In the data we find that although individual entries for energy from coal, gas, and renewables could be zero for some countries, the aggregate energy from these sources is positive for every country.[6] The total (stationary) energy generated by coal, gas, and renewables is denoted by

$$e := c + g + r. \tag{2.1}$$

In our data set, we find also that oil energy is consumed in positive amounts by all countries. These empirical observations support the intuition that both stationary energy e and non-stationary energy o are *essential inputs* in production.[7] Hence, in our empirical analysis, we would like to employ a functional form for the production function that reflects this. The production function that represents the intended-output technology is given by a mapping $F : \mathbf{R}^4_+ \longrightarrow \mathbf{R}_+$ with image

$$y = F(k, l, e, o), \tag{2.2}$$

where y is the level of the country's intended output and k and l denote, respectively, the levels of capital and labour.[8]

Forests sequester atmospheric carbon, which leads to an increase in their biomass (*e.g.,* sequestering carbon every year translates into increase in the wood content of trees).

[6] Twenty three countries (mainly African and middle eastern) in our sample do not use coal, twenty seven countries do not use gas, while three countries do not use any form of renewable energy.

[7] *I.e.,* output is zero whenever one or both forms of these energies are zero.

[8] Since energy is an intermediate input of production in a country, y is taken to be the sum of GDP and the value of energy. See, for instance, Van der Werf (2008).

Global Forest Resources Assessment (FRA) of the FAO provides guidelines to compute the amount of carbon sequestered by forests in a year from the *increase* in the stock (volume) of forest (biomass) during that year. The stock of forest changes every year due to natural growth of existing forests, afforestation, and deforestation activities. Following the Intergovernmental Panel on Climate Change's good practice guidelines (see IPCC (2006)), net emission is the difference between the gross emission produced (carbon released) by the combustion of emission-causing fossil fuels and the carbon sequestered by forests reflected in the net change in the stock of forests. In line with the BP approach, nature's *net* emission generating mechanism is represented by the function $Z : \mathbf{R}_+^4 \longrightarrow \mathbf{R}_+$ with image

$$Z(c, g, o, a - d) := \alpha_c c + \alpha_g g + \alpha_o o - s[a - d], \tag{2.3}$$

which captures the physical laws of emission generation in nature. For $i = c, g, o$, $\alpha_i > 0$ is the emission factor of fossil fuel i, *i.e.,* it is the amount of carbon released by the generation of one unit of energy by combusting fossil fuel i; $s > 0$ is the sequestration of carbon per unit increase in the volume of forest stock; while a and d denote, respectively, afforestation and deforestation levels planned for the current period.[9] This linear specification of gross emission-generation by fossil fuel combustion and sequestration by afforestation is supported by linear emission and sequestration factors recommended in IPCC (2006). Note that net emission can be negative (*i.e.,* gross emission can be more than offset by carbon sequestration), in which case we have positive net sequestration. There are four countries in our sample, namely, Bhutan, Gambia, Kyrghyz Republic, and Latvia, which currently sequester more than they emit. In what follows, we will use the terms emission and net emission interchangeably.

Government incurs expenditure on forests. This could be on maintenance of the total stock of forest and also on afforestation efforts to increase the stock of forests. Government's expenditure function is given by the mapping $G : \mathbf{R}_+^2 \longrightarrow \mathbf{R}_+$ with image

$$\zeta = G\,(f + a - d, a)\,, \tag{2.4}$$

where f denotes the country's inherited stock of forest in the current period. Hence, $f + a - d$ is the stock of forest at the end of the planning period if no further changes (reforms) in afforestation and deforestation policies take place.

[9] d is assumed to be deforestation net of natural growth of existing forests.

2.2. Policy variables – active and sporadic

Let w denote the wage rate. The prices in the energy sector are summarised by the vector $p = \langle p_c, p_g, p_r, p_o \rangle \in \mathbf{R}^4_{++}$ where, for $i = c, g, r, o$, p_i is the price (per-unit cost) of generating energy from the i^{th} source. Our focus is on improving the status-quo situation, *i.e.,* on finding if we can do better (make more profit and, at the same time, reduce emission) given the status-quo economic environment and level of technological development. Hence, this is like a short-run analysis, where we will hold prices of all inputs, the stocks of capital and forest, and the deforestation level fixed.[10] Existence of any scope for improvement under these conditions is an indicator and a measure of the extent of allocative inefficiency prevailing at the status-quo. The vector that summarises the levels of economic variables that are fixed in the short-run is denoted by $\Theta = \langle w, p, k, f, d \rangle \in \mathbf{R}^8_+$.

Our policy reform variables are quantity based.[11] We distinguish between two types of policy variables: (i) an emission cap, denoted by $z \in \mathbf{R}_+$, also called a sporadic policy, reforms in which are exogenously determined and (ii) a vector of active policy variables which adjust to meet the change in the sporadic policy. These comprise the levels of labour, energy inputs, and afforestation. An active policy vector is denoted by $\nu = \langle l, c, g, r, o, a \rangle \in \mathbf{R}^6_+$. A policy vector comprises of both active and sporadic policies: $\langle \nu, z \rangle \in \mathbf{R}^7_+$.[12]

2.3. The profit function and its properties.

The profit function (the value of output net of all the input costs) of the economy, conditional on an environment that is fixed in the short-run, Θ, is defined by the function $\Pi : \mathbf{R}^{14}_+ \longrightarrow \mathbf{R}_+$ with image

$$\pi = \Pi(l, c, g, r, o, a, \Theta) = F(k, l, c{+}g{+}r, o) - wl - p_c c - p_g g - p_r r - p_o o - G(f{+}a{-}d, a). \quad (2.5)$$

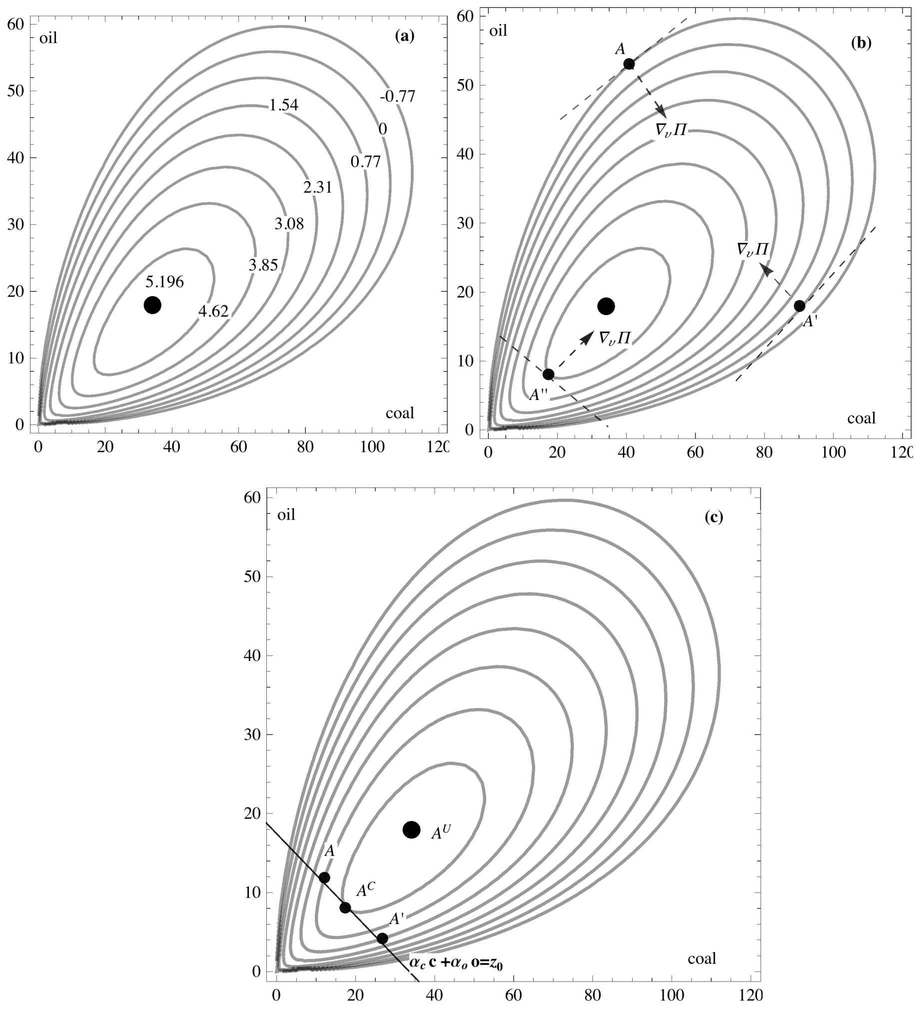
We will maintain the following assumptions throughout the analysis:

Assumption 1: Π is continuously differentiable in the interior of the domain of its definition. It is strictly concave in ν and has a global maximum with respect to ν.

[10] Deforestation is assumed to be fixed as reliable data on this variable is not available for most countries. Rather, in this paper, we focus on afforestation reforms.

[11] We assume that, at an overall economic planning stage, the government first chooses feasible changes in its inherited input and output targets that best facilitate its economic and environmental objectives. Once these changes have been identified, they can be decentralised using price (and/or direct quantity) instruments in the second step. Our analysis in this paper focuses on the first step of planning. For example, the government might decide on the level of reduction in emission and the abatement strategy (*e.g.,* the extent of changes in fossil-fuel and alternative energy usage) considered appropriate to achieve this reduction. Later, these targets can be decentralised by tax/subsidy schemes to limit fossil-fuel usage or increase usage of cleaner sources of energy.

[12] Modelling emission policy as a sporadic policy facilitates defining the marginal abatement cost in a policy reform framework. Please see Sections 3.3 and 4.



(a)
oil
coal
5.196
4.62
3.85
3.08
2.31
1.54
0.77
0
-0.77
(b)
A
A'
A''
$\nabla_v \Pi$
(c)
A^U
A
A^C
A'
α_c c + α_o o=z_0

Figure 1

To illustrate Assumption 1 and many other concepts in this paper we will take help of diagrams drawn in a two-dimensional space with only two inputs (active policies) coal and gas. Suppose production function F in (2.2) is Cobb-Douglas in the two inputs, and takes the form: $y = 2c^{0.5}o^{0.4}$. Suppose $p_c = .4$ and $p_o = .5$. In this case, the profit function in (2.5) becomes $\pi = 2c^{0.5}o^{0.4} - .4c - .5c$. It can be verified that this profit function is strictly concave and has a global maximum when coal and oil usage are 32.48 and 15.59, respectively. The maximum profit is equal to 5.196. Some level curves of this profit function are shown in panel (a) of Figure 1. Note that profit can be negative, in which case it reflects a positive loss. There are fifteen countries in our sample with negative profits at the status-quo.

The gradient of the profit function is the vector of marginal profits associated with the active policies. Evaluated at a given vector of active policies $\nu \in \mathbf{R}^6_+$ and a fixed economic environment $\Theta \in \mathbf{R}^8_+$, it is given by

$$\begin{aligned} \nabla_\nu \Pi(\nu, \Theta) &= \Big\langle \Pi_l,\ \Pi_c,\ \Pi_g,\ \Pi_r,\ \Pi_o,\ \Pi_a \Big\rangle \\ &= \Big\langle F_l - w,\ F_e - p_c,\ F_e - p_g,\ F_e - p_r,\ F_o - p_o,\ -[G_f + G_a] \Big\rangle. \end{aligned} \tag{2.6}$$

For $i = l, c, g, r, o$, the marginal profit Π_i is the marginal product of the i^{th} input, F_i, minus its marginal cost.[13] This can be positive or negative. In panel (b) of Figure 1, at point A, which lies on the upward sloping part of an iso-profit line, the marginal profit of coal is positive, while the marginal profit of oil is negative. So profit increases with increase in coal usage and decrease in oil usage. The reverse is true at point A', which also lies on the upward sloping part of an iso-profit line. At point A'', where the iso-profit line is downward sloping, marginal profits of both inputs are positive, so that profit increases with increase in usage of both inputs.

2.4. Permissible policies.

Recalling (2.3), define the function $\mathcal{Z} : \mathbf{R}^{15}_+ \longrightarrow \mathbf{R}_+$ with image

$$\mathcal{Z}(l, c, g, r, o, a, z, \Theta) = Z(c, g, o, a - d) - z. \tag{2.7}$$

Given a sporadic policy level z and an environment Θ that is fixed in the short-run, an active policy vector $\nu \in \mathbf{R}^6_+$ is *permissible* if $\mathcal{Z}(\nu, z, \Theta) \leq 0 \iff Z(c, g, o, a-d) \leq z$, *i.e.*, if the emission actually generated under the given vector of active policies, $\nu = \langle l, c, g, r, o, a \rangle$, is less than or equal to the existing emission-cap, z. It is *tightly permissible* if $\mathcal{Z}(\nu, z, \Theta) = 0 \iff Z(c, g, o, a - d) = z$, *i.e.*, if the actual emission generated under ν is exactly equal to the emission cap.

[13] Π_i denotes that partial derivative of the profit function with respect to $i = l, c, g, r, o, a$. In (2.6) it is implicitly assumed that these derivatives are evaluated at $\langle \nu, \Theta \rangle$. Similarly, we define F_i for $i = l, c, g, r, o$.

The gradient of $\mathcal{Z}$ with respect to active policies evaluated at a given policy vector $\langle \nu, z \rangle \in \mathbf{R}^7_+$ and a fixed economic environment $\Theta \in \mathbf{R}^8_+$ is given by[14]

$$\nabla_\nu \mathcal{Z} = \Big\langle 0,\ \alpha_c,\ \alpha_g\ 0,\ \alpha_o,\ -s \Big\rangle. \tag{2.8}$$

Assumption 2: The vector $\langle \alpha_c, \alpha_g, \alpha_o, s \rangle \gg 0^T_4$, *i.e.,* $\nabla_v \mathcal{Z} \neq 0_6$.[15]

2.5. The status-quo.

A *status-quo* is an inherited policy environment within which reforms are designed. It consists of the inherited economic environment that is fixed in the short run as well as inherited levels of active and sporadic policy variables such that the active policy vector is permissible. Suppose vector $\langle \nu_0, z_0, \Theta_0 \rangle \in \mathbf{R}^{12}_+$ is the status-quo. Then, given z_0 and Θ_0, the active policy vector ν_0 is permissible. The status-quo is called a tight status-quo if ν_0 is tightly permissible given z_0 and Θ_0.

3. Profit increasing, permissible, emission non-increasing marginalmarginal reforms, and measuring marginal abatement cost (MAC): An intuitive exposition.

In this section, we provide an intuitive exposition of profit increasing and emission non-increasing input reforms and our measure of the marginal abatement cost (MAC).

3.1. Policy reforms, profit increasing and emission non-increasing policy reforms.

Given a status-quo, $\langle \nu_0, z_0, \Theta_0 \rangle \in \mathbf{R}^{15}_+$, a reform in active policies (riap), denoted by $\delta = \langle \delta_l, \delta_c, \delta_g, \delta_r, \delta_o, \delta_a \rangle \in \mathbf{R}^6$, is a vector of changes in active policies starting from the status-quo. A reform in the sporadic policy (risp) starting from the status-quo, is a change in the emission cap, denoted by $\delta_z \in \mathbf{R}$.

It is clear that, starting from status-quo, a riap δ increases the profit if

$$\Pi(\nu_0 + \delta, \Theta_0) - \Pi(\nu_0, \Theta_0) > 0. \tag{3.1}$$

Suppose the status-quo is tight. Starting from it and given a risp δ_z, a riap δ is a *permissible* riap if the new active policy vector $\nu_0 + \delta$ is permissible given the new emission

[14] Since labour and renewables do not cause emission, the derivative of functions Z and $\mathcal{Z}$ with respect to these inputs are zero.

[15] 0_n denotes a zero vector in $\mathbf{R}^n$. If $x \in \mathbf{R}^n$, then $x \gg 0$ if and only if every element of vector x is greater than zero.

cap $z_0+\delta_z$ or, equivalently, recalling (2.3), if the change in net emission induced by reform δ is less than or equal to the change in the sporadic policy δ_z, *i.e.,* if[16]

$$\begin{aligned}\mathcal{Z}(\nu_0+\delta,\ z_0+\delta_z,\Theta_0)\leq 0 \iff \mathcal{Z}(\nu_0+\delta,z_0+\delta_z,\Theta_0)-\mathcal{Z}(\nu_0,z_0,\Theta_0)\leq 0\\ \iff Z(c_0+\delta_c,g_0+\delta_g,o_0+\delta_o,a_0+\delta_a-d_0)-Z(c_0,g_0,o_0,a_0-d_0)\leq\delta_z.\end{aligned} \tag{3.2}$$

A special case of interest arises when $\delta_z=0$. Starting from a the tight status-quo, any riap δ that is permissible when the $\delta_z=0$ is called an *emission non-increasing* riap.

The magnitude of a riap δ is given by $\|\delta\|$.[17] In general, reform vectors can vary with respect to both their magnitudes and directions. Given a status-quo $\langle\nu_0,z_0,\Theta_0\rangle\in\mathbf{R}^{15}_+$, we will be interested in reforms that lead to active policies lying in a local neighbourhood of it. Let the set of active policies that lie in a local neighbourhood of radius one around the status-quo be denoted by $\bar{N}_1(\nu_0)$.[18] We will call a riap having magnitude less than or equal to one a *marginal/local reform.* Starting from the status-quo, marginal reforms lead to active policy vectors in the local neighbourhood $\bar{N}_1(\nu_0)$.

3.2. The optimal profit-increasing and emission non-increasing marginal policy reforms.

We will maintain the following assumption throughout.

Assumption 3. The status-quo, denoted by $S_0=\langle\nu_0,z_0,\Theta_0\rangle$, is a tight status-quo.

The following programme searches in the set $\bar{N}_1(\nu_0)$ for a non-emission increasing marginal reform that leads to the maximum increase in profit.

$$\max_{\delta\in\mathbf{R}^6}\left\{\Pi(\nu_0+\delta,\Theta_0)-\Pi(\nu_0,\Theta_0)\ \middle|\ \mathcal{Z}(\nu_0+\delta,z_0,\Theta_0)\leq 0,\ \|\delta\|\leq 1\right\}. \tag{3.3}$$

Suppose $\overset{*}{\delta}\in\mathbf{R}^6$ is the solution to problem (3.3) and the maximum increase in profit is positive, *i.e.,* $\Pi(\nu_0+\overset{*}{\delta},\Theta_0)-\Pi(\nu_0,\Theta_0)>0$. Then this implies that, at the status-quo, profit is not maximised when the emission cap is z_0 (*i.e.,* there are allocative inefficiencies). There exist marginal reforms that increase profit without increasing the emission level. The active policy vector that maximises profit in the local neighbourhood around ν_0 is $\nu_0+\overset{*}{\delta}$, and the maximum profit is $\Pi(\nu_0+\overset{*}{\delta},\Theta_0)$.

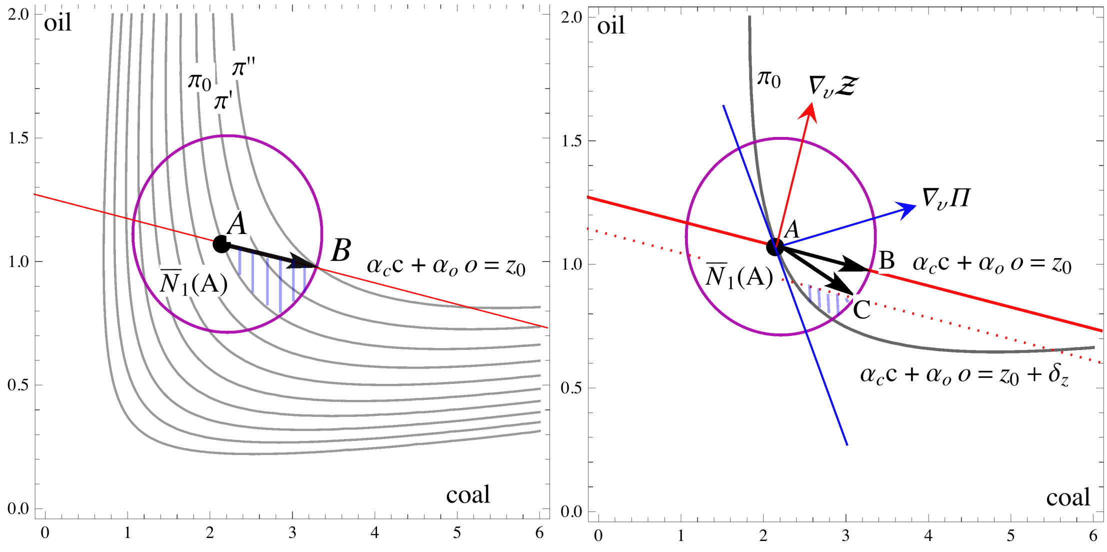
Panel (c) of Figure 1 illustrates situations with allocative inefficiencies. In the absence of an emission cap, (unconstrained) profit maximisation occurs at point A^U. However, suppose due to environmental regulation, the status-quo emission level is z_0.[19] Recalling

[16] Recall, by the definition of a tight status-quo, we have $\mathcal{Z}(\nu_0,z_0,\Theta_0)=0$.

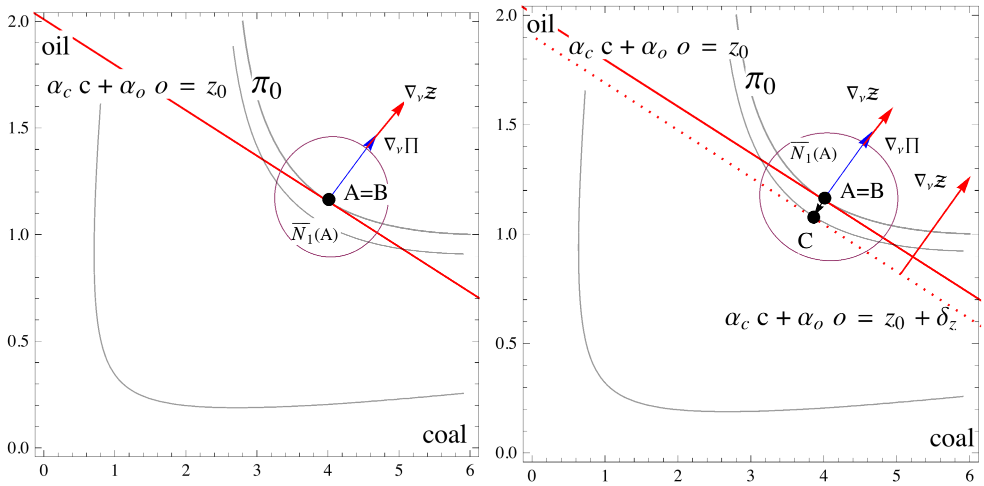
[17] Given a vector $x\in\mathbf{R}^n$, $\|x\|=\sqrt{\sum_{i=1}^n x_i^2}$ is its Euclidean norm.

[18] $\bar{N}_1(\nu_0):=\{\nu'\in\mathbf{R^6}\mid\|\nu'-\nu_0\|\leq 1\}$.

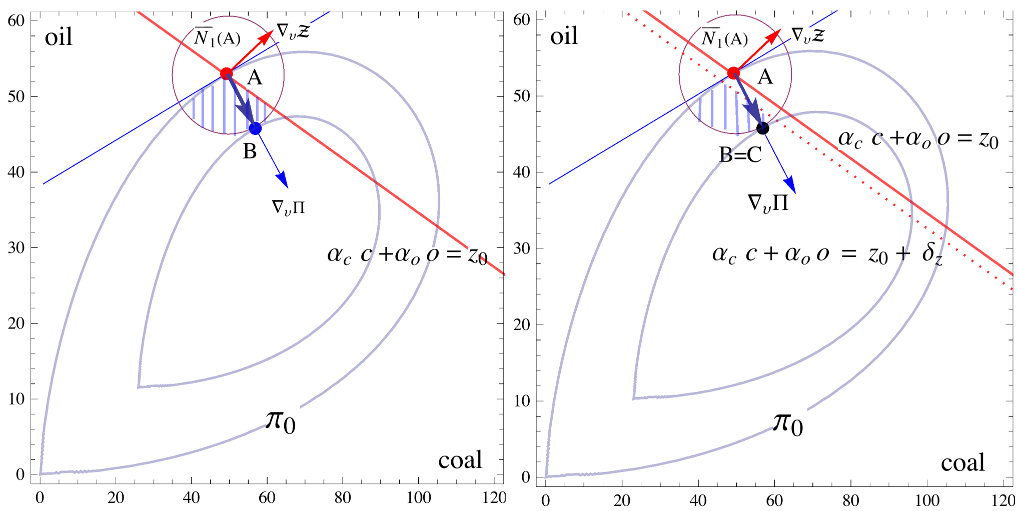
[19] This can be interpreted as the amount of emission a country is permitting itself currently, *i.e.,* z_0 can be interpreted as a self-imposed cap.



(a) Figure 2 (b)



(a) Figure 3 (b)



(a) Figure 4 (b)

(2.3) in the context of this two-inputs case, the line with equation $z_0 = \alpha_c c + \alpha_o o$ depicts combinations of coal and oil inputs that are exactly equal to z_0. All points on or below this line are permissible under the existing emission cap. Given the emission cap, the constrained profit maximum occurs at point A^C. However, allocative inefficiencies may imply that the status-quo does not coincide with A^C. Indeed, we find that all countries in our sample operate with allocative inefficiencies, *e.g.,* the status-quo could be a point like A or A'.

Panel (a) of Figures 2, 3, and 4 illustrate the existence of profit increasing and emission non-increasing marginal reforms at a tight status-quo represented by point A. It is clear that A is permissible given emission cap z_0. The status-quo level of profit is π_0. Active policies that lie in the local neighbourhood of A are indicated in the figure by the disc $\bar{N}_1(A)$. The set of all profit increasing and emission non-increasing marginal reforms is indicated by the shaded region in each of these figures. In particular, it is empty in the case depicted in Figure 3, where the status-quo is the emission constrained profit maximum.

Starting from the status-quo, the marginal reform that leads to the maximum increase in profit without increasing emission (*i.e.,* the solution $\overset{*}{\delta}$ to problem (3.3)) is $\overrightarrow{AB}$. In particular, note that in Figure 3, where there exist no marginal reforms that are profit increasing and emission non-increasing, $\overrightarrow{AB} = 0_2$. Note also that in Figures 2 and 3, the emission constraint of problem (3.3) is binding at the optimal reform $\overrightarrow{AB}$, while it is not in Figure 4. Figures 2 and 3 correspond to situations where the marginal profits of coal and oil are both positive at the status-quo.[20] In Figure 4, the marginal profit of oil is negative and that of coal is positive at the status-quo. Intuitively, this represents a situation where oil is over-utilized at A (with reference to (2.6), this means that the marginal product of oil is less than the marginal cost of oil). Profit can be increased by *decreasing* the usage of oil and increasing the usage of coal. Indeed, in Figure 4, the best local reform $\overrightarrow{AB}$ requires this. But a decrease in oil usage implies also a reduction in emission. In Figure 4, the increase in emission due to increase in usage of coal implied by $\overrightarrow{AB}$ is offset by the decrease in emission due to the decrease in usage of oil. Hence, the status-quo is such that the maximum local increase in profit goes hand-in-hand with decrease in emission, so that the emission constraint is not binding at the optimum of problem (3.3).

3.3. An intuitive definition of MAC with policy reforms.

Suppose the emission constraint is tightened at S_0 by a risp $\delta_z < 0$, which reduces the emission cap to $z_0 + \delta_z$. Recalling (3.2), the following programme searches in the set

[20] Both elements of the gradient $\nabla_\nu \Pi$, evaluated at the status-quo, are positive. The iso-profit curve is downward sloping at A.

of all permissible marginal reforms under the new emission cap for the reform that leads to the maximum increase in profit.

$$\Delta\Pi(\nu_0, z_0, \Theta_0, \delta_z) := \max_{\delta \in \mathbf{R}^6} \left\{ \Pi(\nu_0 + \delta, \Theta_0) - \Pi(\nu_0, \Theta_0) \;\middle|\; \mathcal{Z}(\nu_0 + \delta, z_0 + \delta_z, \Theta_0) \leq 0, \;\; \|\delta\| \leq 1 \right\}. \tag{3.4}$$

Suppose the optimal marginal reform corresponding to the above problem is $\hat{\delta}(\nu_0, z_0, \Theta_0, \delta_z)$. This means that the active policy vector that maximises profit in the local neighbourhood around ν_0 is $\nu_0 + \hat{\delta}(\nu_0, z_0, \Theta_0, \delta_z)$ and the maximum profit is $\Pi(\nu_0 + \hat{\delta}(\nu_0, z_0, \Theta_0, \delta_z), \Theta_0)$. The maximum increase in the profit relative to the status-quo is given by $\Delta\Pi(\nu_0, z_0, \Theta_0, \delta_z) = \Pi(\nu_0 + \hat{\delta}(\nu_0, z_0, \Theta_0, \delta_z), \Theta_0) - \Pi(\nu_0, \Theta_0)$.

In particular, note that programme (3.3) is a special case of programme (3.4), where $\delta_z = 0$, *i.e.*, $\overset{*}{\delta} = \hat{\delta}(\nu_0, z_0, \Theta_0, 0)$. It is intuitive that the tightening of the emission constraint from z_0 to $z_0 + \delta_z$, where $\delta_z < 0$ will shrink the set of permissible marginal reforms. So the best that the producing unit can do in the local neighbourhood of its status-quo when the emission-cap is reduced can be no bigger than the best that it can do in that local neighbourhood with the existing emission cap, *i.e.*,

$$\Pi(\nu_0 + \hat{\delta}(\nu_0, z_0, \Theta_0, 0), \Theta_0) \geq \Pi(\nu_0 + \hat{\delta}(\nu_0, z_0, \Theta_0, \delta_z), \Theta_0).$$

If the above inequality is a strict inequality, then the reduction in the emission cap imposes a potential cost on the producing unit as it reduces the maximum potential profit that can be achieved in the local neighbourhood of the status-quo. This leads to the following intuitive definition of the MAC as the change in the local maximum profit per unit change in the emission cap:

$$m(\nu_0, z_0, \Theta_0) = \lim_{\delta_z \longrightarrow 0} \frac{\Pi(\nu_0 + \hat{\delta}(\nu_0, z_0, \Theta_0, \delta_z), \Theta_0) - \Pi(\nu_0 + \hat{\delta}(\nu_0, z_0, \Theta_0, 0), \Theta_0)}{\delta_z}. \tag{3.5}$$

Adding and subtracting $\Pi(\nu_0, \Theta_0)$ to the numerator of the above and recalling the value functions of problems (3.3) and (3.4), the MAC can be re-written as

$$m(\nu_0, z_0, \Theta_0) = \lim_{\delta_z \longrightarrow 0} \frac{\Delta\Pi(\nu_0, z_0, \Theta_0, \delta_z) - \Delta\Pi(\nu_0, z_0, \Theta_0, 0)}{\delta_z}. \tag{3.6}$$

Thus, the MAC can be measured as the change in the maximum possible *increase* in profit in a local neighbourhood of the status-quo per unit change in the emission cap.

This measure of MAC is illustrated in Panel (b) of Figures 2, 3, and 4. A sporadic reform $\delta_z < 0$ decreases the emission cap. All points on or below the dashed line that satisfy equation $z_0 + \delta_z = \alpha_c c + \alpha_o o$ are permissible under the emission cap $z_0 + \delta_z$. The intersection of set $\bar{N}_1(A)$ with the set of all combinations of coal and oil that are permissible under emission cap $z_0 + \delta_z$ is the constraint set of problem (3.4). Note that the constraint set of problem (3.4) is a subset of the constraint set of problem (3.3). Hence, the maximum increase in profit under the reduced cap can be no bigger than the maximum increase in profit under the existing cap. In Panel (b) of Figures 2, 3, and 4, starting from the local neighbourhood of A, reform $\overrightarrow{AB}$ results in the maximum possible increase in profit under the existing emission cap. Profit increases from π_0 to π''. Under the reduced emission cap, reform $\overrightarrow{AC}$ leads to the maximum possible increase in profit in the local neighbourhood of A. Profit increases to π'.

In Figures 2 and 3, $\pi'' > \pi'$, while in Figure 4, where the emission constraint is non-binding at the optimum under the existing cap, reduction in the cap leads to no change in the maximum profit: $\pi' = \pi''$ as $\overrightarrow{AC} = \overrightarrow{AB}$. Our intuitive definition of MAC is based on the difference $\pi' - \pi''$, which is the numerator of (3.5) and is non-positive. This difference can be re-written as $(\pi' - \pi_0) - (\pi'' - \pi_0)$, which is the maximum increase in profit computed by problem (3.4) minus the maximum increase in profit computed by problem (3.3). This corresponds to the numerator of (3.6). Recalling that $\delta_z < 0$, the measure of MAC defined in (3.5) and (3.6) is non-negative. In particular, in Figures 2 and 3, MAC is positive, while in Figure 4, it is zero. Note that when the MAC is positive, the gradients of the profit and emission functions form a positive dot product, which implies that the angle between the gradients of the profit and emission function is less than 90°. On the other hand, when the MAC is zero, the dot product $\nabla\Pi \cdot \nabla\mathcal{Z}$ is nonpositive, which corresponds to the case when the angle between the two gradients is obtuse (more than 90°).[21] A necessary (but not sufficient) condition for this is that the marginal profit of at least one input is negative, *e.g.,* this is true in the case represented by Figure 4, where this leads to a non-binding emission constraint at the optimum of problem (3.3).

4. Deriving formulae to compute MAC under policy reforms: A differential approach.

The formulae for computing the MAC using data at the status-quo are derived in this section using the differential versions of problems (3.3) and (3.4). To do so, we first present the differential characterisations of profit increasing, permissible, and emission non-increasing input reforms.

4.1. Differentially profit increasing, permissible, and emission non-increasing input reforms.

Employing the gradient of the profit function evaluated at the tight status-quo S_0, a marginal reform δ is *differentially profit increasing* (*i.e.,* results in a differential increase in profit) if

$$\nabla_\nu \Pi(\nu_0, \Theta_0) \cdot \delta > 0. \tag{4.1}$$

[21] The dot product of the two vectors x and $\bar{x}$ in $\mathbf{R}^n$ leads to a scalar given by $x \cdot \bar{x} = \|x\|\|\bar{x}\| \cos\theta$, where θ is the angle between vectors x and $\bar{x}$. Note, $\|x\| = 1 \iff x \cdot x = 1$.

Similarly, given a risp δ_z, employing the gradient of the emission function evaluated at the status-quo, a marginal reform δ is *differentially permissible* if[22]

$$\nabla_\nu \mathcal{Z}(\nu_0, z_0, \Theta_0) \cdot \delta + \nabla_z \mathcal{Z}(\nu_0, z_0, \Theta_0)\delta_z \leq 0 \iff \nabla_\nu \mathcal{Z}(\nu_0, z_0, \Theta_0) \cdot \delta - \delta_z \leq 0. \quad (4.2)$$

If there is no change in the sporadic policy, *i.e.,* $\delta_z = 0$, then the differentially permissible marginal reform δ is *differentially emission non-increasing*, *i.e.,*

$$\nabla_\nu \mathcal{Z}(\nu_0, \Theta_0) \cdot \delta \leq 0, \quad (4.3)$$

In what follows, we will use the notation $\dot{\nu}$ and δ and the notation $\dot{z}$ and δ_z interchangeably.[23]

4.2. MAC: A formal definition, computation, and possible signs.

4.2.1. A formal definition.

Let $\dot{z}$ be a risp. The following programme (the differential analogue of programme (3.4)) searches in the set of all differentially permissible marginal reforms starting from the tight status-quo S_0 for the riap that leads to the greatest differential increase in profit.

$$\dot{\mathbf{\Pi}}(\nu_0, z_0, \Theta_0, \dot{z}) := \max_{\dot{\nu}} \left\{ \nabla_\nu \Pi(\nu_0, \Theta_0) \cdot \dot{\nu} \;\middle|\; \nabla_\nu Z(\nu_0, z_0, \Theta_0) \cdot \dot{\nu} \leq \dot{z} \;\wedge\; \dot{\nu} \cdot \dot{\nu} \leq 1 \right\}. \quad (4.4)$$

Define the Lagrangian of problem (4.4) as

$$L(\nu_0, z_0, \Theta_0, \dot{z};\; \mu, \lambda) = \nabla_\nu \Pi(\nu_0, \Theta_0) \cdot \dot{\nu} - \mu \left[\nabla_\nu \mathcal{Z}(\nu_0, z_0, \Theta_0) \cdot \dot{\nu} - \dot{z}\right] - \lambda[\dot{\nu} \cdot \dot{\nu} - 1], \quad (4.5)$$

where μ and λ are the Lagrange multipliers for the two constraints of problem (4.4). A special case of problem (4.4) is one where there is no change in the emission cap at the status-quo, *i.e.,* when risp $\dot{z} = 0$.

$$\dot{\mathbf{\Pi}}(\nu_0, z_0, \Theta_0, 0) = \max_{\dot{\nu}} \left\{ \nabla_\nu \Pi(\nu_0, \Theta_0) \cdot \dot{\nu} \;\middle|\; \nabla_\nu \mathcal{Z}(\nu_0, z_0, \Theta_0) \cdot \dot{\nu} \leq 0 \;\wedge\; \dot{\nu} \cdot \dot{\nu} \leq 1 \right\}. \quad (4.6)$$

Suppose the solution mapping and the mappings of the Lagrange multipliers evaluated at the optimum of problem (4.4) are[24]

$$\langle \dot{\nu}, \mu, \lambda \rangle \in \left\langle \omega(\nu_0, z_0, \Theta_0, \dot{z}),\; \mu(\nu_0, z_0, \Theta_0, \dot{z}),\; \lambda(\nu_0, z_0, \Theta_0, \dot{z}) \right\rangle. \quad (4.7)$$

[22] Note, from (2.7), $\nabla_z \mathcal{Z}(\nu_0, z_0, \Theta_0) = -1$.

[23] This is because the differential change in profit due to a marginal reform δ depicted in the left-side of (4.1) can be interpreted more rigorously as the derivative of the profit function along a linear path of active policies starting from ν_0 that is parametrized by a variable $t \in \mathbf{R}_+$ and given by $\nu(t) = \nu_0 + t\delta$. This implies that the gradient of $\nu(t)$ evaluated at $t = 0$ is $\dot{\nu} := \nabla_t \nu(0) = \delta$. Similarly, the differential change in the emission function $\mathcal{Z}$ due to $\langle \delta, \delta_z \rangle$ in (4.2) can be interpreted as the derivative of the emission function along a linear path of active and sporadic policies starting from $\langle \nu_0, z_0 \rangle$ that is parametrized by a variable $t \in \mathbf{R}_+$ and given by $\langle \nu(t), z(t) \rangle = \langle \nu_0 + t\delta,\; z_0 + t\delta_z \rangle$. This implies, $\dot{z} := \frac{dz(0)}{dt} = \delta_z$.

[24] The solution mapping could be set-valued, *i.e.,* multiple solutions can exist. See Theorem 1 below.

This implies that the maximum differential increase in profit at the status-quo is

$$\dot{\mathbf{\Pi}}(\nu_0, z_0, \Theta_0, \dot{z}) = \nabla_\nu \Pi(\nu_0, \Theta_0) \cdot \dot{\nu}, \qquad \dot{\nu} \in \omega\left(\nu_0, z_0, \Theta_0, \dot{z}\right). \tag{4.8}$$

In particular, when $\dot{z} = 0$, a solution vector of problem (4.6) and the Lagrange multipliers evaluated at the optimum will be denoted by

$$\langle \overset{*}{\dot{\nu}}, \overset{*}{\mu}, \overset{*}{\lambda} \rangle \in \left\langle \omega\left(\nu_0, z_0, \Theta_0, 0\right),\ \mu\left(\nu_0, z_0, \Theta_0, 0\right),\ \lambda\left(\nu_0, z_0, \Theta_0, 0\right) \right\rangle. \tag{4.9}$$

The differential analogue of the intuitive definition of the MAC derived in (3.6) is

$$\mathcal{MAC}(\nu_0, z_0, \Theta_0) = \left.\frac{\partial \dot{\mathbf{\Pi}}(\nu_0, z_0, \Theta_0, \dot{z})}{\partial \dot{z}}\right|_{\dot{z}=0}.$$

Employing the envelope theorem after recalling (4.5) and (4.9), we obtain

$$\begin{aligned} \mathcal{MAC}(\nu_0, z_0, \Theta_0) = \left.\frac{\partial \dot{\mathbf{\Pi}}(\nu_0, z_0, \Theta_0, \dot{z})}{\partial \dot{z}}\right|_{\dot{z}=0} &= \left.\frac{\partial L\left(\nu_0, z_0, \Theta_0, \dot{z};\ \overset{*}{\mu}, \overset{*}{\lambda}\right)}{\partial \dot{z}}\right|_{\dot{z}=0} \\ &= \overset{*}{\mu} \equiv \mu\left(\nu_0, z_0, \Theta_0, 0\right). \end{aligned} \tag{4.10}$$

4.2.2. Using status-quo data to compute MAC and its possible signs.

To ease notation, in what follows, we will denote the gradients of the profit and emission functions evaluated at the tight status-quo S_0 employing status-quo data on all inputs, afforestation, prices, and stocks of capital and forests as

$$\nabla_\nu \Pi := \nabla_\nu \Pi(\nu_0, \Theta_0) \quad \text{and} \quad \nabla_\nu \mathcal{Z} := \nabla_\nu \mathcal{Z}(\nu_0, z_0, \Theta_0). \tag{4.11}$$

The Kuhn-Tucker first-order conditions of problem (4.6) are

$$\begin{gathered} \nabla_\nu \Pi - \overset{*}{\mu} \nabla_\nu \mathcal{Z} - 2\overset{*}{\lambda}\overset{*}{\dot{\nu}} = 0_6, \\ \nabla_\nu \mathcal{Z} \cdot \overset{*}{\dot{\nu}} \leq 0, \quad \overset{*}{\mu} \geq 0, \quad \overset{*}{\mu}\left[\nabla_\nu \mathcal{Z} \cdot \overset{*}{\dot{\nu}}\right] = 0, \\ \overset{*}{\dot{\nu}} \cdot \overset{*}{\dot{\nu}} \leq 1, \quad \overset{*}{\lambda} \geq 0, \quad \overset{*}{\lambda}[\overset{*}{\dot{\nu}} \cdot \overset{*}{\dot{\nu}} - 1] = 0. \end{gathered} \tag{4.12}$$

Theorem 1, below, derives the formulae for computing MAC defined in (4.10) in terms of the gradients, $\nabla_\nu \Pi$ and $\nabla_\nu \mathcal{Z}$, which are evaluated at the status-quo using status-quo data. In addition, part (1) of the theorem provides formulae for the optimal marginal reform in the case when the emission constraint is binding at the optimum. It distinguishes between two cases: (i) when the status-quo is not profit maximising subject to the existing emission cap implying that the optimal reform leads to a strict increase in profit (*e.g.,* see Figure 2) and (ii) when the status-quo is profit maximising as in Figure 3, where the gradients of the profit and emission function are collinear. In the latter case, the optimal reform is not unique, and each such reform leads to *no* increase in profit. In particular, making no changes in active policies (*i.e.,* choosing $\overset{*}{\dot{\nu}} = 0_6$) is an optimal reform. Part (2) of the theorem gives the formula for the optimal reform in the case when the emission constraint is not binding at the optimum of problem (4.6) (*e.g.,* see Figure 4). The theorem states that the MAC is zero in this case.

Theorem 1: *For any* $\overset{*}{\dot{\nu}} \in \omega(\nu_0, z_0, \Theta_0, 0)$, *we have* $\nabla_\nu \Pi \cdot \overset{*}{\dot{\nu}} \geq 0$ *and*

$$\begin{aligned} \mathcal{MAC}(\nu_0, z_0, \Theta_0) = \overset{*}{\mu} &= (\nabla_\nu \mathcal{Z} \cdot \nabla_\nu \mathcal{Z})^{-1} (\nabla_\nu \mathcal{Z} \cdot \nabla_\nu \Pi) && \textit{if} \quad \nabla_\nu \mathcal{Z} \cdot \overset{*}{\dot{\nu}} = 0 \\ &= 0 && \textit{if} \quad \nabla_\nu \mathcal{Z} \cdot \overset{*}{\dot{\nu}} < 0. \end{aligned}$$

(1) *Suppose* $\nabla_\nu \mathcal{Z} \cdot \overset{*}{\dot{\nu}} = 0$.

(i) If $\nabla_\nu \Pi - \overset{*}{\mu} \nabla_\nu \mathcal{Z} \neq 0$, *then the following are true.*

$$\overset{*}{\lambda} = \frac{1}{2}\sqrt{\left[\nabla_\nu \Pi - \overset{*}{\mu}\nabla_\nu \mathcal{Z}\right] \cdot \left[\nabla_\nu \Pi - \overset{*}{\mu}\nabla_\nu \mathcal{Z}\right]} \quad \wedge \quad \overset{*}{\dot{\nu}} = \frac{\nabla_\nu \Pi - \overset{*}{\mu}\nabla_\nu \mathcal{Z}}{\sqrt{\left[\nabla_\nu \Pi - \overset{*}{\mu}\nabla_\nu \mathcal{Z}\right] \cdot \left[\nabla_\nu \Pi - \overset{*}{\mu}\nabla_\nu \mathcal{Z}\right]}}$$

$$\wedge \quad \overset{*}{\mu} \geq 0 \quad \wedge \quad \dot{\mathbf{\Pi}}(\nu_0, z_0, \Theta_0, 0) = \nabla_\nu \Pi \cdot \overset{*}{\dot{\nu}} > 0.$$

(ii) If $\nabla_\nu \Pi - \overset{*}{\mu} \nabla_\nu \mathcal{Z} = 0$, *then the following are true.*

$$\overset{*}{\mu} > 0 \quad \wedge \quad \dot{\mathbf{\Pi}}(\nu_0, z_0, \Theta_0, 0) = 0 \quad \wedge \quad \|\dot{\nu}\| \leq 1 \textit{ and } \nabla_\nu \Pi \cdot \dot{\nu} = 0 \implies \dot{\nu} \in \omega(\nu_0, z_0, \Theta_0, 0)$$

$$\wedge \quad 0_6 \in \omega(\nu_0, z_0, \Theta_0, 0) \quad \wedge \quad \overset{*}{\lambda} = 0.$$

(2) *Suppose* $\nabla_\nu \mathcal{Z} \cdot \overset{*}{\dot{\nu}} < 0$. *Then* $\nabla_\nu \Pi - \overset{*}{\mu} \nabla_\nu \mathcal{Z} \neq 0_6$ *and the following are true:*

$$\overset{*}{\lambda} = \frac{1}{2}\sqrt{\nabla_\nu \Pi \cdot \nabla_\nu \Pi} \quad \wedge \quad \overset{*}{\dot{\nu}} = \frac{\nabla_\nu \Pi}{\sqrt{\nabla_\nu \Pi \cdot \nabla_\nu \Pi}}, \quad \wedge \quad \dot{\mathbf{\Pi}}(\nu_0, z_0, \Theta_0, 0) = \nabla_\nu \Pi \cdot \overset{*}{\dot{\nu}} > 0.$$

Theorem 2, below, provides a characterisation of Part (2) of Theorem 1 in terms of data. It shows that the emission constraint is non-binding at the optimum if and only if the dot product of the gradients of the emission and profit function evaluated at the status-quo is negative.[25] Given the formula of the MAC in Theorem 1, Theorem 2 relates MAC to this dot product.

Theorem 2: (1) $\nabla_\nu \mathcal{Z} \cdot \overset{*}{\dot{\nu}} < 0$ *if and only if* $\nabla_\nu \mathcal{Z} \cdot \nabla_\nu \Pi < 0$ *and*

$$(2) \qquad \begin{aligned} \mathcal{MAC}(\nu_0, z_0, \Theta_0) &= 0 && \textit{if} \quad \nabla_\nu \mathcal{Z} \cdot \nabla_\nu \Pi \leq 0 \\ &> 0 && \textit{if} \quad \nabla_\nu \mathcal{Z} \cdot \nabla_\nu \Pi > 0. \end{aligned}$$

[25] *E.g.,* see Figure 4, where the angle between these two gradients lies between 90° and 180°.

5. A measure of MAC that is independent of the units of measurement of inputs.

Consider the case when, at the tight status-quo S_0, $\nabla_\nu \Pi \cdot \nabla_\nu \mathcal{Z} > 0$. Recalling (2.6) and (2.8), employing conclusions of Theorems 1 and 2 it can be shown that

$$\mathcal{MAC}(\nu_0, z_0, \Theta_0) = \frac{[F_e - p_c]\alpha_c + [F_e - p_g]\alpha_g + [F_o - p_o]\alpha_o - [G_f + G_a]s}{\alpha_c^2 + \alpha_g^2 + \alpha_o^2 + s^2}. \tag{5.1}$$

This measure of MAC is not free of units in which the inputs (the active policy variables) are measured.[26] We now derive a measure of MAC that is independent of the units of measurement of all inputs, which will be used in our empirical analysis.

5.1. Proportional changes in active policies and the elasticity analogue of problem (4.4).

We turn to the elasticity analogue of the analysis in the previous sections. To do so, suppose at the tight status-quo S_0, $\nu_0 \gg 0_6$. Starting from S_0, denote a vector of proportional changes in active policies by $q = \langle q_l, q_c, q_g, q_r, q_o, q_a \rangle \in \mathbf{R}^6$. Then there exists a riap $\delta \equiv \dot{\nu} = \langle \dot{l}, \dot{c}, \dot{g}, \dot{r}, \dot{o}, \dot{a} \rangle \in \mathbf{R}^6$ such that[27]

$$q = \left\langle \frac{\dot{l}}{l_0}, \frac{\dot{c}}{c_0}, \frac{\dot{g}}{g_0}, \frac{\dot{r}}{r_0}, \frac{\dot{o}}{o_0}, \frac{\dot{a}}{a_0} \right\rangle \iff \delta \equiv \dot{\nu} = \langle q_l l_0,\ q_c c_0,\ q_g g_0,\ q_r r_0,\ q_o o_0,\ q_a a_0 \rangle. \tag{5.2}$$

We will be interested in local proportional changes in active policies (lpcap), *i.e.,* with vectors of proportional changes in active policies with length no bigger than one ($\|q\| \leq 1$). Let the profit evaluated at S_0 be denoted by π_0 and assume that $\pi_0 \neq 0$. Evaluated at S_0, define the vector of (partial) elasticities of the profit function with respect to various inputs:

$$\varepsilon := \langle \varepsilon_l, \varepsilon_c, \varepsilon_g, \varepsilon_r, \varepsilon_o, \varepsilon_a \rangle,$$

where, *e.g.,* $\varepsilon_l = \Pi_l \frac{l_0}{|\pi_0|}$ is the partial elasticity of profit with respect to labour. Starting from S_0, the proportional increase (resp., decrease) in profit (resp., loss) when $\pi_0 > 0$ (resp., $\pi_0 < 0$) due to any lpcap q is given by

$$\frac{\dot{\pi}}{|\pi_0|} = \varepsilon \cdot q \iff \dot{\pi} = |\pi_0|\ \varepsilon \cdot q = \nabla_\nu \Pi \cdot \dot{\nu}, \tag{5.3}$$

where $\dot{\nu}$ is the riap defined in (5.2).[28] Hence, the term $\dot{\pi}$ in (5.3) is the differential increase in profit (or decrease in loss when $\pi_0 < 0$) at the status-quo due to riap $\dot{\nu}$ corresponding to lpcap q.

[26] *E.g.,* the value of MAC obtained if afforestation is measured in meter-cube will be different from the value obtained if afforestation is measured in kilometer-cube.

[27] Recall, we use notation δ and $\dot{\nu}$ for a riap interchangeably.

[28] When $\pi_0 > 0$, then $\frac{\dot{\pi}}{|\pi_0|} = \frac{\dot{\pi}}{\pi_0}$ is the proportional increase in profit. But, if $\pi_0 < 0$ then $-\frac{\dot{\pi}}{|\pi_0|} = \frac{-\dot{\pi}}{-\pi_0} = \frac{\dot{\pi}}{\pi_0}$ is the proportional increase in loss. Hence, $\frac{\dot{\pi}}{|\pi_0|}$ is the proportional decrease in loss.

Similarly, evaluated at S_0, define the vector of the elasticities of the emission function with respect to all the active policies

$$\Psi := \langle \Psi_l, \Psi_c, \Psi_g, \Psi_r, \Psi_o, \Psi_a \rangle,$$

where for $i = l, c, g, o, r$, $\Psi_i = \mathcal{Z}_i \frac{i_0}{|z_0|} = \frac{\alpha_i i_0}{|z_0|}$ is the elasticity of emission with respect to input i and $\Psi_a = \mathcal{Z}_a \frac{a_0}{|z_0|} = \frac{-sa_0}{|z_0|}$ is the elasticity of emission with respect to afforestation, evaluated at the status-quo. Since labour and renewables do not cause emission, $\Psi_l = \Psi_r = 0$. Thus, starting from S_0, the proportional increase (resp., decrease) in net emission (resp., net sequestration) when $z_0 > 0$ (resp., $z_0 < 0$) due to lpcap q is given by[29]

$$\frac{\dot{z}}{|z_0|} = \Psi \cdot q \iff \dot{z} = |z_0| \; \Psi \cdot q = \nabla_\nu \mathcal{Z} \cdot \dot{\nu}, \tag{5.4}$$

i.e., $\dot{z}$ is the (differential) change in emission due to riap $\dot{\nu}$ corresponding to lpcap q.

Remark 1. *The derivatives $\dot{\pi}$ and $\dot{z}$ defined in (5.3) and (5.4) are independent of the units of measurement of all inputs as they are functions of profit and emission elasticities and the proportionate changes in inputs.*

The problem, below, is the elasticity analogue of problem (4.4) that identifies, at the tight status-quo, the lpcap that results in the maximal proportionate increase in profit, given a risp $\dot{z}$ and, hence, given a proportional change $\frac{\dot{z}}{|z_0|}$ in the net emission. The problem is written for the special case $\dot{z} = 0$.

$$\mathcal{V}(\nu_0, z_0, \Theta_0, \dot{z} = 0) = \max_{q \in \mathbf{R}^6} \left\{ \varepsilon \cdot q \;\middle|\; \Psi \cdot q \leq \frac{\dot{z}}{|z_0|} \;\wedge\; q \cdot q \leq 1 \right\} \Bigg|_{\dot{z}=0}. \tag{5.5}$$

5.2. A measure of MAC that is independent of the units of measurement of all inputs.

The Lagrangian of problem (5.5) evaluated at $\dot{z} = 0$ with the required Lagrange multipliers is

$$\tilde{L}\left(\nu_0, z_0, \Theta_0, \dot{z};\; \tilde{\mu}, \tilde{\lambda}\right)\Bigg|_{\dot{z}=0} = \left[\varepsilon \cdot q - \tilde{\mu}\left[\Psi \cdot q - \frac{\dot{z}}{|z_0|}\right] - \tilde{\lambda}[q \cdot q - 1]\right]\Bigg|_{\dot{z}=0}. \tag{5.6}$$

[29] When net emission $z_0 > 0$, then $\frac{\dot{z}}{|z_0|} = \frac{\dot{z}}{z_0}$ is the proportional increase in net emission. But, if $z_0 < 0$ then $-\frac{\dot{z}}{|z_0|} = \frac{-\dot{z}}{-z_0} = \frac{\dot{z}}{z_0}$ is the proportional increase in net sequestration. Hence, $\frac{\dot{z}}{|z_0|}$ is the proportional decrease in net sequestration.

We will denote an optimal vector of choice variables corresponding to $\dot{z} = 0$ as $\langle \bar{q}, \bar{\bar{\mu}}, \bar{\bar{\lambda}} \rangle$. Then the maximum proportionate increase in profit (or maximum proportionate decrease in loss) at the status-quo with no change in emission policy is given by

$$\frac{\dot{\pi}_{\bar{q}}}{|\pi_0|} := \mathcal{V}(\nu_0, z_0, \Theta_0, \dot{z})\Big|_{\dot{z}=0} = \varepsilon \cdot \bar{q} \iff \dot{\pi}_{\bar{q}} = |\pi_0|\varepsilon \cdot \bar{q} = \nabla_\nu \Pi \cdot \dot{\nu}_{\bar{q}},$$

where $\dot{\nu}_{\bar{q}}$ denotes the riap corresponding to the lpcap $\bar{q}$ (as defined in (5.2)). As in Section 4.2.1, by employing the envelope theorem, we obtain a measure of MAC, evaluated at the status-quo, as

$$\begin{aligned} MAC_\Psi(\nu_0, z_0, \Theta_0) := \frac{\partial \dot{\pi}_{\bar{q}}}{\partial \dot{z}}\Big|_{\dot{z}=0} &= |\pi_0| \frac{\partial \mathcal{V}(\nu_0, z_0, \Theta_0, \dot{z})}{\partial \dot{z}}\Big|_{\dot{z}=0} \\ &= |\pi_0| \frac{\partial \tilde{L}\left(\nu_0, z_0, \Theta_0, \dot{z};\ \bar{\bar{\mu}}, \bar{\bar{\lambda}}\right)}{\partial \dot{z}}\Big|_{\dot{z}=0} = \frac{|\pi_0|}{|z_0|}\bar{\bar{\mu}}. \end{aligned} \tag{5.7}$$

A theorem analogous to Theorem 1 can be stated that computes the Lagrange multiplier $\bar{\bar{\mu}}$ as well as the optimal lpcap $\bar{q}$ employing data available at the status-quo.[30] Since, it is repetitive, we refrain from doing so here.[31] But, it is helpful to note that the MAC is given by

$$\begin{aligned} MAC_\Psi(\nu_0, z_0, \Theta_0) &= \frac{|\pi_0|}{|z_0|}(\Psi \cdot \Psi)^{-1}(\Psi \cdot \varepsilon) \quad \text{if} \quad \Psi \cdot \bar{q} = 0 \\ &= 0 \quad \text{if} \quad \Psi \cdot \bar{q} < 0 \end{aligned} \tag{5.8}$$

For future reference, we also provide Remark 2, below, that distinguishes between three cases based on the sign of the dot product $\Psi \cdot \varepsilon$.

Remark 2.

(i) If ε and Ψ are proportional then $\bar{q} = 0_6$ solves (5.5)[32]
(this is the case where there are no lpcaps that lead to a proportionate increase in profit with no proportionate increase in emission).

(ii) If ε and Ψ are not proportional then $\varepsilon \cdot \bar{q} > 0$ and
(a) $\varepsilon \cdot \Psi \geq 0$ if and only if $\Psi \cdot \bar{q} = 0$ (the case where the emission constraint in (5.5) is binding at the optimum).
(b) $\varepsilon \cdot \Psi < 0$ if and only if $\Psi \cdot \bar{q} < 0$ (the case where the emission constraint in (5.5) is non-binding at the optimum).

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[30] Just replace $\nabla_\nu \Pi$ and $\nabla_\nu \mathcal{Z}$ in Theorem 1 with ε and Ψ and $\overset{*}{\nu}$, $\overset{*}{\mu}$, and $\overset{*}{\lambda}$ with $\bar{q}$, $\bar{\bar{\mu}}$, and $\bar{\bar{\lambda}}$, resp.

[31] But see Section 2A of the appendix, which characterises, in detail, the solutions of problem (5.5) and its dual problem that is defined and employed later in Section 6.

[32] ε and Ψ are proportional if there exists a scalar $\kappa > 0$ such that $\varepsilon = \kappa\Psi$. They are not proportional if no such scalar exists.

5.3. Structure of optimal lpcap $\bar{q}$ and riap $\bar{\delta}$.

We now show that lpcap $\bar{q}$ that solves problem (5.5) has an important interpretation. Since (5.5) is the elasticity analogue of problem (4.6), replacing $\nabla_\nu \Pi$ and $\nabla_\nu \mathcal{Z}$ with ε and Ψ, resp., and $\overset{*}{\mu}$ by $\bar{\tilde{\mu}}$, parts (1) and (2) of Theorem 1 yield[33]

$$\bar{q} = \frac{\varepsilon - \bar{\tilde{\mu}}\Psi}{\sqrt{[\varepsilon - \bar{\tilde{\mu}}\Psi] \cdot [\varepsilon - \bar{\tilde{\mu}}\Psi]}} =: \frac{\varepsilon - \bar{\tilde{\mu}}\Psi}{\Lambda}. \tag{5.9}$$

Since ε is the vector of partial elasticities of the profit function, elements of vector ε indicate the proportionate increases in profit per unit proportionate increases in various active policy inputs. In that sense, it can be interpreted as the vector of marginal proportional benefits from using these inputs in production. It is clear that ε and, hence, marginal proportional benefits of inputs depend on the marginal profits of inputs. So it is possible for some inputs to yield negative marginal proportional benefits, *e.g.,* marginal proportional benefit of afforestation is always negative as marginal profit of afforestation is always negative. But the signs of marginal profits and, hence, marginal proportional benefits, of other inputs are ambiguous and depend on the size of their marginal products relative to their prices (see (2.6)).

Since Ψ is the vector of partial elasticities of the emission function, its elements indicate the proportionate increases in emission per unit proportionate increases in various inputs. Since, $\bar{\tilde{\mu}}$ is the Lagrange multiplier of the emission constraint in problem (5.5) (*i.e.,* it can be interpreted as the proportional reduction in profit per unit proportional tightening of the cap on emission), $\bar{\tilde{\mu}}\Psi$ can be interpreted as the vector of marginal proportional costs of using these inputs.

Given these interpretations of Ψ and ε, Remark 3 follows:

Remark 3. *From (5.9) it follows that optimal lpcap $\bar{q}$ recommends proportionally increasing (resp., decreasing) usage of any input if the marginal proportional benefit from it exceeds (resp., falls short of) its marginal proportional cost, i.e., for $i = l, c, g, o, r, a$, $\bar{q}_i > 0$ if and only if $\varepsilon_i > \bar{\tilde{\mu}}\Psi_i$. In particular, the following qualifications are true:*

(i) Since renewables and labour do not cause emissions, their marginal proportional costs are zero (as $\Psi_r = \Psi_l = 0$). Hence, $\bar{q}$ recommends proportionally increasing usage of labour or renewable inputs if and only if their marginal proportional benefits are greater than zero.

(ii) As afforestation leads to a reduction in emission, its marginal proportional cost is non-positive (as $\Psi_a < 0$). Since government incurs expenditure on afforestation, its marginal proportional benefit is negative (as $\varepsilon_a < 0$).

[33] In particular, part (2) corresponds to the case where $\bar{\tilde{\mu}} = 0$.

(iii) *When $\Psi \cdot \varepsilon < 0$, Remark 2 implies $\bar{\bar{\mu}} = 0$. Hence, in this case, for $i = c, g, o, a$, the sign of $\bar{q}_i$ depends only on the marginal proportional benefit ε_i of input i. In particular, since $\varepsilon_a < 0$, we have $\bar{q}_a < 0$*

(iv) *For $i = l, c, g, o, r$, the sign of marginal proportional benefit ε_i depends on the sign of marginal profit of i at the status-quo.*

Riap $\bar{\delta}$ is derived from $\bar{q}$ by employing (5.2). (5.9) and Remark 3 imply that it has the following structure:

$$\begin{aligned}
\bar{\delta}_l &= l_0 \bar{q}_l = \frac{l_0 \varepsilon_l}{\Lambda} = \frac{l_0}{\Lambda}\left(\frac{\Pi_l l_0}{|\pi_0|}\right) & \bar{\delta}_c &= c_0 \bar{q}_c = \frac{c_0[\varepsilon_c - \bar{\bar{\mu}}]\Psi_c}{\Lambda} = \frac{c_0}{\Lambda}\left(\frac{\Pi_c c_0}{|\pi_0|} - \frac{\bar{\bar{\mu}}\alpha_c c_0}{|z_0|}\right) \\
\bar{\delta}_g &= g_0 \bar{q}_g = \frac{g_0[\varepsilon_g - \bar{\bar{\mu}}\Psi_g]}{\Lambda} = \frac{g_0}{\Lambda}\left(\frac{\Pi_g g_0}{|\pi_0|} - \frac{\bar{\bar{\mu}}\alpha_g g_0}{|z_0|}\right) & \bar{\delta}_o &= o_0 \bar{q}_o = \frac{o_0[\varepsilon_o - \bar{\bar{\mu}}\Psi_o]}{\Lambda} = \frac{o_0}{\Lambda}\left(\frac{\Pi_0 o_0}{|\pi_0|} - \frac{\bar{\bar{\mu}}\alpha_o o_0}{|z_0|}\right) \\
\bar{\delta}_r &= r_0 \bar{q}_r = \frac{r_0 \varepsilon_r}{\Lambda} = \frac{r_0}{\Lambda}\left(\frac{\Pi_r r_0}{|\pi_0|}\right) & \bar{\delta}_a &= a_0 \bar{q}_a = \frac{a_0[\varepsilon_a - \bar{\bar{\mu}}\Psi_a]}{\Lambda} = \frac{a_0}{\Lambda}\left(\frac{\Pi_a a_0}{|\pi_0|} + \frac{\bar{\bar{\mu}} s a_0}{|z_0|}\right).
\end{aligned} \tag{5.10}$$

5.4. Decomposition of MAC_Ψ into measures of ability to abate (ATA) and reduction in profit (RIP).

A marginal reform $\dot{\nu}$ is an *abatement reform* at the tight status-quo S_0 if it leads to a (differential) decrease in emission starting from S_0, *i.e.*, if

$$\nabla_\nu \mathcal{Z} \cdot \dot{\nu} < 0. \tag{5.11}$$

As demonstrated by Figures 2 to 4, several abatement reforms can exist at the status-quo.[34] Intuitively, each such abatement reform has two consequences for the economic unit: (i) it affects its profit (generally reducing it) and (ii) it reduces its emission.

Suppose at the tight status-quo S_0, $z_0 \neq 0$, $\nu_0 \gg 0_6$, and $\pi_0 \neq 0$. It can be shown that, starting from S_0, the lpcap that results in the maximum proportionate decrease in emission is $q_\Psi := -\frac{\Psi}{\|\Psi\|}$.[35] Suppose the riap corresponding to q_Ψ (as defined in (5.2)) is denoted by $\dot{\nu}_\Psi$. Employing (5.3) and (5.4), define the ability to abate (ATA) (denoted by $ATA_\Psi(\nu_0, z_0, \Theta_0)$) and the reduction in profit (RIP) (denoted by $RIP_\Psi(\nu_0, z_0, \Theta_0)$) as the (differential) reductions in emission and profit, resp., at the status-quo, due to the adoption of abatement riap $\dot{\nu}_\Psi$ corresponding to lpcap q_Ψ:[36]

$$RIP_\Psi = |\pi_0|\ \varepsilon \cdot \frac{\Psi}{\|\Psi\|} = -\nabla_\nu \Pi \cdot \dot{\nu}_\Psi \qquad \text{and} \qquad ATA_\Psi = |z_0|\Psi \cdot \frac{\Psi}{\|\Psi\|} = -\nabla_\nu \mathcal{Z} \cdot \dot{\nu}_\Psi. \tag{5.12}$$

[34] There are several ways in which fossil fuels and afforestation efforts can be combined to reduce emission.

[35] q_Ψ solves $\max_q \{-\Psi \cdot q \mid q \cdot q \leq 1\}$, as $-\Psi \cdot q$ is the proportionate *reduction* in emission due to a lpcap q and this is greatest (given the definition of a dot product) when $q = -\frac{\Psi}{\|\Psi\|}$.

[36] From (5.3), $RIP_\Psi := -|\pi_0|\ \varepsilon \cdot q_\Psi = -\nabla_\nu \Pi \cdot \dot{\nu}_\Psi$. Hence, while $\nabla_\nu \Pi \cdot \dot{\nu}_\Psi$ is the differential increase in profit due to reform $\dot{\nu}_\Psi$, $-\nabla_\nu \Pi \cdot \dot{\nu}_\Psi$ is the differential *reduction* in profit. If the increase is negative, then the reduction is positive and vice-versa. Similarly, we can interpret $ATA_\Psi := -|z_0|\Psi \cdot q_\Psi = -\nabla_\nu \mathcal{Z} \cdot \dot{\nu}_\Psi$.

Theorem 3, which follows from (5.8), Remark 2, and the definitions of RIP_Ψ and ATA_Ψ, decomposes the MAC into the ATA and the RIP at the status-quo.

Theorem 3: *Suppose $z_0 \neq 0$, $\nu_0 \gg 0_6$, and $\pi_0 \neq 0$.*

$$\begin{aligned} MAC_\Psi(\nu_0, z_0, \Theta_0) &= \frac{RIP_\Psi}{ATA_\Psi} \qquad \text{if} \quad RIP_\Psi > 0 \\ &= 0 \qquad\qquad \text{if} \quad RIP_\Psi \leq 0. \end{aligned} \tag{5.13}$$

Remark 4. *When $RIP_\Psi > 0$, then MAC can be interpreted as the reduction in profit per unit reduction in emission when the abatement reform $\dot{\nu}_\Psi$ (corresponding to the lpcap q_Ψ that leads to the maximum proportionate reduction in emission at the status-quo) is adopted.*

5.5. Factors affecting ATA_Ψ and RIP_Ψ.

It can be shown using (5.2) that the lpcap q_Ψ and the riap $\dot{\nu}_\Psi$ corresponding to it are

$$\begin{aligned} q_\Psi &\equiv \left\langle q_{l_\Psi},\ q_{c_\Psi},\ q_{g_\Psi},\ q_{r_\Psi},\ q_{o_\Psi},\ q_{a_\Psi} \right\rangle = -\frac{\Psi}{\|\Psi\|} = \frac{-1}{\|\Psi\|}\langle 0, \psi_{zc}, \psi_{zg}, 0, \psi_{zo}, \psi_{za}\rangle \\ &= \frac{-1}{\sqrt{\alpha_c^2 c_0^2 + \alpha_g^2 g_0^2 + \alpha_o^2 o_0^2 + s^2 a_0^2}}\langle 0,\ \alpha_c c_0,\ \alpha_g g_0,\ 0,\ \alpha_o o_0,\ -s a_0\rangle \\ \dot{\nu}_\Psi &:= \left\langle \dot{l}_\Psi,\ \dot{c}_\Psi,\ \dot{g}_\Psi,\ \dot{r}_\Psi,\ \dot{o}_\Psi, \dot{a}_\Psi \right\rangle = \left\langle q_{l_\Psi} l_0,\ q_{c_\Psi} c_0,\ q_{g_\Psi} g_0,\ q_{r_\Psi} r_0,\ q_{o_\Psi} o_0,\ q_{a_\Psi} a_0 \right\rangle \\ &= \frac{1}{\sqrt{\alpha_c^2 c_0^2 + \alpha_g^2 g_0^2 + \alpha_o^2 o_0^2 + s^2 a_0^2}}\left\langle 0,\ -\alpha_c c_0^2,\ -\alpha_g g_0^2,\ 0,\ -\alpha_o o_0^2,\ s a_0^2\right\rangle. \end{aligned} \tag{5.14}$$

The structure of vector $\dot{\nu}_\Psi$ in (5.14) implies that the changes in fossil-fuel usage implied by q_Ψ are negative and the change in afforestation is positive. Thus, q_Ψ discourages the use of fossil fuels and encourages afforestation. It can be verified that, for $i = c, g, o$, the derivative of $-\dot{i}_\Psi$ with respect to i_0 is positive and the derivative of $\dot{a}_\Psi$ with respect to a_0 is positive.[37] This leads to the following remark:

Remark 5. *The reduction in any fossil-fuel implied by lpcap q_Ψ is higher, the higher is the consumption of the fossil fuel at the status-quo. The increase in afforestation implied by q_ψ is higher, the higher is the status-quo level of afforestation.*

From (5.12) and (5.14) it follows that ATA_Ψ is given by:

$$ATA_\Psi = [\alpha_c(-\dot{c}_\Psi) + \alpha_g(-\dot{g}_\Psi) + \alpha_o(-\dot{o}_\Psi) + s\dot{a}_\Psi]\,. \tag{5.15}$$

[37] For example, employing (5.14), it can be verified that the derivative $-\frac{\partial \dot{c}_\Psi}{\partial c_0} > 0$.

Remark 6: *(5.15) implies that the ATA_Ψ will be higher, the higher are the reductions of fossil fuels and the greater is the increase in afforestation implied by q_Ψ at the status-quo. Remark 5 implies that this will be true the higher are the consumptions of the fossil fuels or the higher is the afforestation at the status-quo.*

Empirically, ATA_Ψ is computed by the formula below that follows from (5.15) and (5.14).

$$ATA_\Psi = \sqrt{\alpha_c^2 c_0^2 + \alpha_g^2 g_0^2 + \alpha_o^2 o_0^2 + s^2 a_0^2}. \tag{5.16}$$

For all $i = l,\ c,\ g,\ r,\ o$, we will call $i_0 \Pi_i = i_0[F_i - p_i]$ as the *profitability of a one-per cent increase in input i* and $-a_0\Pi_a = a_0[G_f + G_a]$ as the *increase in cost due to a one-percent increase in afforestation.*

Employing the definition of RIP_Ψ in (5.12) it can be show that

$$RIP_\Psi = c_0\Pi_c(-q_{c_\Psi}) + g_0\Pi_g(-q_{g_\Psi}) + o_0\Pi_o(-q_{o_\Psi}) - a_0\Pi_a q_{a_\Psi} \tag{5.17}$$

It was noted in Section 2.3 that for $i = c,\ g,\ o$ the sign of Π_i (the marginal profit of input i) at the status-quo is ambiguous. Hence, the sign of RIP is ambiguous.

Remark 7. *(5.17) implies*

(1) if RIP_Ψ is negative, then at least one of inputs coal, gas, and oil has a negative marginal profit.[38]

(2) RIP_Ψ is higher, the higher are the profitabilities of fossil fuels and the higher is the increase in cost due to a one-percent increase in afforestation at the status-quo.

Empirically, RIP_Ψ is computed by the formula below that follows from (5.14) and (5.17)

$$RIP_\Psi = \frac{\left[[F_e - p_c]\alpha_c c_0^2 + [F_e - p_g]\alpha_g g_0^2 + [F_o - p_o]\alpha_o o_0^2 + [G_f + G_a] s a_0^2\right]}{\sqrt{\alpha_c^2 c_0^2 + \alpha_g^2 g_0^2 + \alpha_o^2 o_0^2 + s^2 a_0^2}} \tag{5.18}$$

6. Reforms leading to environmentally permissible and non-differential increases in profit at the status-quo.

In the intuitive exposition provided in Section 3, we focussed on the non-differential (actual) impacts of marginal (local) reforms on profit and emission levels, starting from the status-quo. But in Sections 4 and 5, we switched to studying *differential* (*i.e.,* linear approximations of) changes in profit and emission levels induced by riaps (reforms in active policies) with a view to construct two general measures of MAC, which allow for inefficiencies at the status-quo, and that can be computed readily using status-quo data.

But, in this paper, exploiting the allocative inefficiency of the status-quo, we are also interested in constructing and studying reforms that lead to actual (non-differential)

[38] See, for example, Figure 4, where the marginal profit of oil is negative.

increases in profit and decreases in emission relative to the status-quo. In this section, we show that the differentially profit increasing and emission-non increasing riaps that were derived from lpcaps by using (5.2) in Section 5 can be employed to construct such riaps.

Another point to note is that, although the proportional changes in active policies implied by lpcaps are marginal (lie within the unit circle), the changes in the *levels* of active policies (*i.e.,* the riaps) they induce (see (5.2)) need not be so, *i.e.,* these riaps need not be marginal (local) reforms. Moreover, the actual (non-differential) changes in profit and emission levels induced by such such nonlocal (non-marginal) riaps may be quite large. These points are reflected in our empirical results.

Throughout this section, in addition to Assumptions 1 to 3, we will maintain the following assumption.

Assumption 4: At the tight status-quo S_0, $\nu_0 \gg 0_6$, $z_0 \neq 0$, profit $\pi_0 := \Pi(\nu_0, \Theta_0) \neq 0$, and ε and Ψ are not proportional.[39]

The following observation is useful for the analysis. (2.3) implies that, starting from the tight status-quo S_0, the total amount of emission due a riap $\delta = \langle \delta_l,\ \delta_c,\ \delta_g,\ \delta_r,\ \delta_o,\ \delta_a \rangle$ is

$$\begin{aligned} z_\delta &:= Z\left(c_0 + \delta_c,\ g_0 + \delta_g,\ o_0 + \delta_o,\ a_0 - d_0 + \delta_a\right) \\ &\equiv \alpha_c(c_0 + \delta_c) + \alpha_g(g_0 + \delta_g) + \alpha_o(o_0 + \delta_o) - s(a_0 - d_0 + \delta_a). \end{aligned}$$

Since Z and, hence, $\mathcal{Z}$ are linear functions, the differential change in emission induced by riap δ at the status-quo S_0 is the same as the actual change in emission induced by it, *i.e.,*

$$z_\delta - z_0 = \nabla_\nu \mathcal{Z} \cdot \delta = \alpha_c\delta_c + \alpha_g\delta_g + \alpha_o\delta_o - s\delta_a. \tag{6.1}$$

To construct reforms that lead to non-differential increases in profit and decreases in the emission level, we will distinguish between two cases based on the sign of $\Psi \cdot \varepsilon$, which (as seen in Remark 2) determines whether or not the emission constraint is binding at the optimum of problem (5.5).

6.1. *Existence of reforms leading to non-differential increases in profit and decreases in emission when $\Psi \cdot \varepsilon < 0$.*

Under Assumption 4, we are in case (ii) of Remark 2. Let $\bar{\delta} = \langle \bar{\delta}_l, \bar{\delta}_c, \bar{\delta}_g, \bar{\delta}_r, \bar{\delta}_o, \bar{\delta}_a \rangle \in \mathbf{R}^6$ be the riap derived from lpcap $\bar{q}$ using (5.2).[40] Note, while $\bar{q}$ being a lpcap has length one, $\bar{\delta}$ need not be a marginal reform. Define a riap $\mathbf{a} := \bar{\kappa}\bar{\delta}$, where $\bar{\kappa} \in \mathbf{R}_+$ solves

$$\max_{\kappa} \left\{ \Pi\left(\nu_0 + \kappa\bar{\delta}, \Theta_0\right) \;\middle|\; \kappa \in [0,1] \right\}. \tag{6.2}$$

[39] As noted in Remark 2, when ε and Ψ are proportional at the status-quo, no lpcaps that increase profit without decreasing the emission level exist.

[40] Recall that $\bar{q}$ is the lpcap that solves (5.5) with $\dot{z} = 0$ (see Section 6.2.)

Since Π is a continuous function and we are optimising over a compact set, by Bolzano-Weirstrass theorem, a solution exists. It is unique as Π is strictly concave in ν. Denote the profit and emission levels associated with the active policy vector $\nu_0 + \mathbf{a}$ as $\pi_{\mathbf{a}} := \Pi\left(\nu_0 + \bar{\kappa}\bar{\delta}, \Theta_0\right)$ and $z_{\mathbf{a}} := Z\left(c_0 + \bar{\kappa}\bar{\delta}_c,\ g_0 + \bar{\kappa}\bar{\delta}_g,\ o_0 + \bar{\kappa}\bar{\delta}_o,\ a_0 - d_0 + \bar{\kappa}\bar{\delta}_a\right)$.[41]

Theorem 4 states that, starting from the status-quo S_0, riap $\mathbf{a}$ leads to a non-differential increase in profit. Further, depending on the sign of the dot product $\Psi \cdot \varepsilon$, it results in either a decrease or no change in the emission level. In particular, $\mathbf{a}$ is a profit increasing and emission decreasing riap if $\Psi \cdot \varepsilon < 0$.

Theorem 4: *Starting from status-quo S_0,*

(i) $\mathbf{a}$ *is a riap that leads to a non-differential increase in profit ($\pi_{\mathbf{a}} > \pi_0$) with no change in the emission level ($z_{\mathbf{a}} = z_0$) when $\Psi \cdot \varepsilon \geq 0$.*

(ii) If $\Psi \cdot \varepsilon < 0$, then $\mathbf{a}$ *is a riap that leads to a non-differential increase in profit ($\pi_{\mathbf{a}} > \pi_0$) and decrease in emission ($z_{\mathbf{a}} < z_0$).*

Another point to note from Theorem 4 is that we have used riap $\bar{\delta}$ (which is derived from lpcap $\bar{q}$ that solves problem (5.5)) and results in a *differential* increase in profit, to construct riap $\mathbf{a}$ that results in a *non-differential* increase in profit with no increase in emission. In particular, reform $\mathbf{a}$ was obtained by scaling down riap $\bar{\delta}$ by the factor $\bar{\kappa}$ defined in (6.2).

6.2. Existence of reforms that lead to non-differential increases in profit and decreases in emission when $\Psi \cdot \varepsilon \geq 0$.

In this section, using a fixed point argument, we show that riaps that lead to simultaneous (non-differential) increase in profit and decrease in emission levels in the more ubiquitous case $\Psi \cdot \varepsilon \geq 0$ exist.

The first step towards this end is to consider the *dual* of the optimisation problem (5.5), where we seek, starting from the status-quo S_0, an lpcap that maximises proportional reductions in emission subject to no proportional reductions in profit, *i.e.,* we seek an lpcap $\underline{q}$ that solves[42]

$$\max_{q \in \mathbf{R}^6} \left\{ -\Psi \cdot q \;\middle|\; \varepsilon \cdot q \geq 0 \ \wedge\ q \cdot q \leq 1 \right\}. \tag{6.3}$$

Let $\mathbf{b} = \langle \mathbf{b}_l, \mathbf{b}_c, \mathbf{b}_g, \mathbf{b}_r, \mathbf{b}_o, \mathbf{b}_a \rangle$ be the riap corresponding to $\underline{q}$ as defined in (5.2). Then (5.3) and (5.4) imply

$$|\pi_0|\ \varepsilon \cdot \underline{q} = \nabla_\nu \Pi \cdot \mathbf{b} \quad \text{and} \quad |z_0|\ \Psi \cdot \underline{q} = \nabla_\nu \mathcal{Z} \cdot \mathbf{b}. \tag{6.4}$$

[41] See (6.1) to find the emission level associated with $\nu_0 + \mathbf{a}$.

[42] The optimum of this problem can be characterised analogously to the optimum of problem (5.5). See Section A2 of the appendix.

Define the profit and emission levels corresponding to the active policy vector $\nu_0 + \mathbf{b}$ as $\pi_{\mathbf{b}} := \Pi(\nu_0 + \mathbf{b}, \Theta_0)$ and $z_{\mathbf{b}} := Z\left(c_0 + \mathbf{b}_c,\ g_0 + \mathbf{b}_g,\ o_0 + \mathbf{b}_o,\ a_0 - d_0 + \mathbf{b}_a\right)$, resp. The lemma below states that, if $\Psi \cdot \varepsilon \geq 0$ then, starting from the status-quo S_0, riap $\mathbf{b}$ reduces both the emission and profit below the status-quo levels. To clarify this lemma, note that (6.4) imply that reform $\mathbf{b}$ does not result in a *differential* decrease in profit (this follows from the constraint of problem (6.3)). However, Lemma 1 states that it does result in a *non-differential* decrease in profit.

Lemma 1: *If* $\Psi \cdot \varepsilon \geq 0$ *then* $\pi_{\mathbf{b}} < \pi_0$ *and* $\nabla_\nu \mathcal{Z} \cdot \mathbf{b} = z_{\mathbf{b}} - z_0 < 0$.

Now choose any $\overset{*}{\pi} \in (\pi_0, \pi_a)$. We show that there is a convex combination of riaps $\mathbf{a}$ and $\mathbf{b}$ that, when implemented at the tight status-quo S_0, yields profit $\overset{*}{\pi}$, which by its definition is bigger that the status-quo profit π_0, and also results in a decrease in the emission level. Define the sets

$$\Omega := \left\{ \delta \in \mathbf{R}^6 \mid \delta = \gamma \mathbf{a} + (1-\gamma)\mathbf{b} \text{ for some } \gamma \in [0,1] \right\},$$
$$\Omega_{\mathbf{a}} := \left\{ \delta \in \Omega \mid \Pi\left(\nu_0 + \delta, \Theta_0\right) \geq \overset{*}{\pi} \right\} \quad \text{and} \quad \Omega_{\mathbf{b}} := \left\{ \delta \in \Omega \mid \Pi\left(\nu_0 + \delta, \Theta_0\right) \leq \overset{*}{\pi} \right\}.$$

It is clear that Ω is a non-empty, convex, and compact set as it is the set of all convex combinations of vectors $\mathbf{a}$ and $\mathbf{b}$. Since Π is a continuous and strictly concave function and Ω is compact, the sets $\Omega_{\mathbf{a}}$ and $\Omega_{\mathbf{b}}$ are also convex and compact, so that the Cartesian product $\Omega_{\mathbf{a}} \times \Omega_{\mathbf{b}}$ is also a convex and compact set. When $\Psi \cdot \varepsilon \geq 0$, Theorem 4, Lemma 1, and the definition of $\overset{*}{\pi}$ imply that $\pi_{\mathbf{b}} < \pi_0 < \overset{*}{\pi} < \pi_{\mathbf{a}}$. Hence, $\mathbf{a} \in \Omega_{\mathbf{a}}$ and $\mathbf{b} \in \Omega_{\mathbf{b}}$. Hence, $\Omega_{\mathbf{a}} \times \Omega_{\mathbf{b}}$ is a non-empty set. $\Omega_{\mathbf{a}} \times \Omega_{\mathbf{b}}$ is a subset of the product space $\mathbf{R}^6 \times \mathbf{R}^6$. Define the product metric $\rho : \mathbf{R}^6 \times \mathbf{R}^6 \longrightarrow \mathbf{R}_+$ with image

$$\rho(\langle \hat{\delta}_1, \hat{\delta}_2 \rangle,\ \langle \bar{\delta}_{\mathbf{a}}, \bar{\delta}_{\mathbf{b}} \rangle) = \|\hat{\delta}_1 - \bar{\delta}_{\mathbf{a}}\| + \|\hat{\delta}_2 - \bar{\delta}_{\mathbf{b}}\|.$$

Then the product space $\mathbf{R}^6 \times \mathbf{R}^6$ combined with the product metric ρ is a metric space.[43]

Choose $\bar{\gamma} \in (0,1)$ and define a function $\mathcal{U} : \Omega_{\mathbf{a}} \times \Omega_{\mathbf{b}} \longrightarrow \mathbf{R}^6$ with image

$$\mathcal{U}(\delta_{\mathbf{a}}, \delta_{\mathbf{b}}) = \bar{\gamma}\delta_{\mathbf{a}} + (1-\bar{\gamma})\delta_{\mathbf{b}}.$$

Thus, given any two riaps $\delta_{\mathbf{a}} \in \Omega_{\mathbf{a}}$ and $\delta_{\mathbf{b}} \in \Omega_{\mathbf{b}}$, function $\mathcal{U}$ defines a convex combination of these riaps with weight $\bar{\gamma}$.

Lemma 2: *Suppose* $\Psi \cdot \varepsilon \geq 0$. *Then* $\mathcal{U}$ *is a continuous function and* $\mathcal{U} : \Omega_{\mathbf{a}} \times \Omega_{\mathbf{b}} \longrightarrow \Omega$.[44]

[43] See, for instance, Ok [2007], pp. 192–3. Product metric ρ will be employed in the proofs of theorems and lemmas to follow.

[44] *I.e.,* $\mathcal{U}\left(\delta_{\mathbf{a}}, \delta_{\mathbf{b}}\right) \in \Omega$ for all $\langle \delta_{\mathbf{a}}, \delta_{\mathbf{b}} \rangle \in \Omega_{\mathbf{a}} \times \Omega_{\mathbf{b}}$.

Define a function $\mathcal{T} : \Omega_{\mathbf{a}} \times \Omega_{\mathbf{b}} \longrightarrow \Omega_{\mathbf{a}} \times \Omega_{\mathbf{b}}$ with image

$$\begin{aligned}\mathcal{T}(\delta_{\mathbf{a}}, \delta_{\mathbf{b}}) &= \left\langle \mathcal{U}(\delta_{\mathbf{a}}, \delta_{\mathbf{b}}), \ \delta_{\mathbf{b}} \right\rangle && \text{if} \quad \Pi(\nu_0 + \mathcal{U}(\delta_{\mathbf{a}}, \delta_{\mathbf{b}}), \Theta_0) > \overset{*}{\pi} \\ &= \left\langle \delta_{\mathbf{a}}, \ \mathcal{U}(\delta_{\mathbf{a}}, \delta_{\mathbf{b}}) \right\rangle && \text{if} \quad \Pi(\nu_0 + \mathcal{U}(\delta_{\mathbf{a}}, \delta_{\mathbf{b}}), \Theta_0) < \overset{*}{\pi} \\ &= \left\langle \mathcal{U}(\delta_{\mathbf{a}}, \delta_{\mathbf{b}}), \ \mathcal{U}(\delta_{\mathbf{a}}, \delta_{\mathbf{b}}) \right\rangle && \text{if} \quad \Pi(\nu_0 + \mathcal{U}(\delta_{\mathbf{a}}, \delta_{\mathbf{b}}), \Theta_0) = \overset{*}{\pi}.\end{aligned} \tag{6.5}$$

Lemma 3: *Suppose $\Psi \cdot \varepsilon \geq 0$. Then $\mathcal{T}$ is a continuous function.*

The next lemma states that, if there is a riap in Ω that results in profit $\overset{*}{\pi}$ starting from the status-quo S_0, then it is unique.

Lemma 4: *Suppose $\Psi \cdot \varepsilon \geq 0$. Suppose $\tilde{\delta} \in \Omega$ and $\hat{\delta} \in \Omega$ are such that $\Pi\left(\nu_0 + \tilde{\delta}, \Theta_0\right) = \Pi\left(\nu_0 + \hat{\delta}, \Theta_0\right) = \overset{*}{\pi}$. Then $\tilde{\delta} = \hat{\delta}$.*

Let the unique riap in Ω that results in profit $\overset{*}{\pi}$ be denoted by $\overset{*}{\delta} := \langle \overset{*}{\delta}_l, \ \overset{*}{\delta}_c, \ \overset{*}{\delta}_g, \ \overset{*}{\delta}_r, \ \overset{*}{\delta}_o, \ \overset{*}{\delta}_a \rangle$. Hence, $\Pi\left(\nu_0 + \overset{*}{\delta}, \Theta_0\right) = \overset{*}{\pi}$. As it is in Ω, there must exist a unique $\overset{*}{\gamma} \in (0, 1)$ such that $\overset{*}{\gamma}\mathbf{a} + (\mathbf{1} - \overset{*}{\gamma})\mathbf{b} = \overset{*}{\delta}$.

Lemma 5, which is obvious, gives the positions of various riaps in Ω relative to $\overset{*}{\delta}$. Intuitively, it says that riaps in $\Omega_{\mathbf{a}}$ (resp., $\Omega_{\mathbf{b}}$) are those convex combinations of riaps $\mathbf{a}$ and $\mathbf{b}$ that give weights higher than $\overset{*}{\gamma}$ to riap $\mathbf{a}$ (resp., $\mathbf{b}$).

Lemma 5: *Suppose $\Psi \cdot \varepsilon \geq 0$. Let $\gamma \in [0, 1]$ and $\delta := \gamma\mathbf{a} + (\mathbf{1} - \gamma)\mathbf{b}$. Then*

$$\gamma \geq \overset{*}{\gamma} \iff \delta \in \Omega_{\mathbf{a}} \quad \textit{and} \quad \gamma \leq \overset{*}{\gamma} \iff \delta \in \Omega_{\mathbf{b}}.$$

Theorem 5 states that riap $\overset{*}{\delta}$, which results in profit $\overset{*}{\pi}$, exists at the status-quo. This is because the function $\mathcal{T}$ has a fixed point in $\Omega_{\mathbf{a}} \times \Omega_{\mathbf{b}}$, which is precisely equal to $\langle \overset{*}{\delta}, \overset{*}{\delta} \rangle$. Moreover, starting from the status-quo, not only does reform $\overset{*}{\delta}$ increase the profit but it also reduces the emission level.

Theorem 5: *Suppose $\Psi \cdot \varepsilon \geq 0$. Then the following are true:*

(i) $\langle \overset{*}{\delta}, \ \overset{*}{\delta} \rangle$ *is the unique fixed point of* $\mathcal{T}$,

(ii) $\overset{*}{\pi} := \Pi\left(\nu_0 + \overset{*}{\delta}, \Theta_0\right) > \pi_0$, *and*

(iii) $\overset{*}{z} := Z\left(c_0 + \overset{*}{\delta}_c, \ g_0 + \overset{*}{\delta}_g, \ o_0 + \overset{*}{\delta}_o, \ a_0 - d_0 + \overset{*}{\delta}_a\right) < z_0$.

Proof: (i) Lemma 3 shows that $\mathcal{T}$ is a continuous function from the non-empty, convex, and compact set $\Omega_{\mathbf{a}} \times \Omega_{\mathbf{b}}$ into itself. Hence, from Brower's fixed point theorem, $\mathcal{T}$ has a fixed point, *i.e.,* there exists $\langle \overset{*}{\delta}_{\mathbf{a}},\ \overset{*}{\delta}_{\mathbf{b}} \rangle \in \Omega_{\mathbf{a}} \times \Omega_{\mathbf{b}}$ such that

$$\mathcal{T}\left(\overset{*}{\delta}_{\mathbf{a}}, \overset{*}{\delta}_{\mathbf{b}}\right) = \langle \overset{*}{\delta}_{\mathbf{a}}, \overset{*}{\delta}_{\mathbf{b}} \rangle, \tag{6.6}$$

Define $\overset{*}{u} := \mathcal{U}\left(\overset{*}{\delta}_{\mathbf{a}}, \overset{*}{\delta}_{\mathbf{b}}\right) = \bar{\gamma}\overset{*}{\delta}_{\mathbf{a}} + (1-\bar{\gamma})\overset{*}{\delta}_{\mathbf{b}}$. Suppose $\overset{*}{\delta}_{\mathbf{a}} \neq \overset{*}{\delta}_{\mathbf{b}}$. Then $\overset{*}{u} \neq \overset{*}{\delta}_{\mathbf{a}}$ and $\overset{*}{u} \neq \overset{*}{\delta}_{\mathbf{b}}$. There are three possibilities, all of which contradict (6.6):

(a) $\Pi\left(\nu_0 + \overset{*}{u}, \Theta_0\right) = \overset{*}{\pi}$: In this case, $\mathcal{T}\left(\overset{*}{\delta}_{\mathbf{a}}, \overset{*}{\delta}_{\mathbf{b}}\right) = \langle \overset{*}{u}, \overset{*}{u} \rangle \neq \langle \overset{*}{\delta}_{\mathbf{a}}, \overset{*}{\delta}_{\mathbf{b}} \rangle$.

(b) $\Pi\left(\nu_0 + \overset{*}{u}, \Theta_0\right) > \overset{*}{\pi}$: In this case, $\mathcal{T}\left(\overset{*}{\delta}_{\mathbf{a}}, \overset{*}{\delta}_{\mathbf{b}}\right) = \langle \overset{*}{u}, \overset{*}{\delta}_{\mathbf{b}} \rangle \neq \langle \overset{*}{\delta}_{\mathbf{a}}, \overset{*}{\delta}_{\mathbf{b}} \rangle$.

(c) $\Pi\left(\nu_0 + \overset{*}{u}, \Theta_0\right) < \overset{*}{\pi}$: In this case, $\mathcal{T}\left(\overset{*}{\delta}_{\mathbf{a}}, \overset{*}{\delta}_{\mathbf{b}}\right) = \langle \overset{*}{\delta}_{\mathbf{a}}, \overset{*}{u} \rangle \neq \langle \overset{*}{\delta}_{\mathbf{a}}, \overset{*}{\delta}_{\mathbf{b}} \rangle$.

Hence, $\overset{*}{\delta}_{\mathbf{a}} = \overset{*}{\delta}_{\mathbf{b}} = \overset{*}{u}$. Hence, $\overset{*}{u} \in \Omega_{\mathbf{a}} \cap \Omega_{\mathbf{b}}$. Hence, $\Pi\left(\nu_0 + \overset{*}{u}, \Theta_0\right) = \overset{*}{\pi}$. Hence, Lemma 4 implies $\overset{*}{u} = \overset{*}{\delta}$. Hence, (6.6) implies $\mathcal{T}\left(\overset{*}{\delta}, \overset{*}{\delta}\right) = \langle \overset{*}{\delta}, \overset{*}{\delta} \rangle$.

(ii) Since we chose $\overset{*}{\pi} \in (\pi_0, \pi_{\mathbf{a}})$, it follows that $\overset{*}{\pi} = \Pi\left(\nu_0 + \overset{*}{\delta}, \Theta_0\right) > \pi_0$.

(iii) From (6.1), Theorem 4, and Lemma 1, we have

$$\begin{aligned} \overset{*}{z} - z_0 &= \nabla_\nu \mathcal{Z} \cdot \overset{*}{\delta} = \nabla_\nu \mathcal{Z} \cdot [\overset{*}{\gamma}\mathbf{a} + (\mathbf{1} - \overset{*}{\gamma})\mathbf{b}] \\ &= \overset{*}{\gamma}\nabla_\nu \mathcal{Z} \cdot \mathbf{a} + (\mathbf{1} - \overset{*}{\gamma})\nabla_\nu \mathcal{Z} \cdot \mathbf{b} \ = \ (\mathbf{1} - \overset{*}{\gamma})\nabla_\nu \mathcal{Z} \cdot \mathbf{b} < \mathbf{0}. \ \blacksquare \end{aligned}$$

6.3. An algorithm to compute profit increasing and emission decreasing reforms.

We provide below an algorithm based on Theorems 4 and 5 that constructs reforms (usually non-marginal) at the status-quo, which result in non-differential increases in profit and decreases in the emission level. In, particular, in the case when $\Psi \cdot \varepsilon \geq 0$, the algorithm constructs a sequence using function $\mathcal{T}$ that converges to the fixed point of $\mathcal{T}$.

ALGORITHM

<u>*Step 1.*</u> Solve problems (4.6) and (6.2) to compute riap $\mathbf{a}$.

If $\Psi \cdot \varepsilon < 0$ then, starting from the tight status-quo S_0, $\mathbf{a}$ increases profit and decreases emission (see Theorem 4) and the algorithm stops.

If $\Psi \cdot \varepsilon \geq 0$ then proceed to Step 2.

<u>*Step 2.*</u> Solve problem (6.3) to compute riap $\mathbf{b}$.

<u>*Step 3.*</u> Choose $\overset{*}{\pi} \in (\pi_0, \pi_{\mathbf{a}})$. Choose $\bar{\gamma} \in (0, 1)$.

<u>*Step 4.*</u> Construct the sequence $\left\{\langle \delta^t_{\mathbf{a}}, \delta^t_{\mathbf{b}} \rangle\right\}$ in $\Omega_{\mathbf{a}} \times \Omega_{\mathbf{b}}$ such that

(i) $\langle \delta^0_{\mathbf{a}}, \delta^0_{\mathbf{b}} \rangle = \langle \mathbf{a}, \mathbf{b} \rangle$ and

(ii) $\langle \delta^t_{\mathbf{a}}, \delta^t_{\mathbf{b}} \rangle = \mathcal{T}\left(\delta^{t-1}_{\mathbf{a}}, \delta^{t-1}_{\mathbf{b}}\right)$ for all $t > 0$.

Theorem 6, below, states that this sequence converges to the fixed point $\langle \overset{*}{\delta}, \overset{*}{\delta} \rangle$ of function $\mathcal{T}$, where $\overset{*}{\delta}$ increases profit and decreases emission (see Theorem 5). The algorithm stops.

Theorem 6: *Suppose* $\Psi \cdot \varepsilon \geq 0$. *Then* $\left\{ \langle \delta^t_{\mathbf{a}}, \delta^t_{\mathbf{b}} \rangle \right\} \longrightarrow \langle \overset{*}{\delta}, \overset{*}{\delta} \rangle$.

7. Data and estimation procedures.

Our sample consists of 118 countries – 26 countries from Africa, 31 from Asia, 39 from Europe, 13 from North America, 7 from South America, and 2 from Oceania.

7.1. Data.[45]

The data on GDP, capital, and labour for these countries was derived from Version 4 of the Extended Penn Tables (EPWT) for four years, 1990, 2000, 2005, and 2010. Real GDP and capital are measured in 2005 purchasing power parity in USD, while labour is measured as number of workers.

Energy data was obtained from the World Development Indicators (WDI), International Energy Agency (IEA), and US Energy Information Agency (US-EIA) for four years 1990, 2000, 2005, and 2010. Energy is measured in kilotons of oil equivalents (ktoe).

Country-level data on gross CO_2 emissions from fossil-fuel combustion was obtained from IEA for 2010. For parity with carbon sequestration by forests, the emissions of CO_2 are converted into million tons of carbon (mtc) by multiplying them by a factor, 12/44.[46] The emission factors for coal, gas, and oil, resp., are derived from IPCC (2006) guidelines. Employing appropriate conversion factors, these amount to $\alpha_c = 0.001141854$, $\alpha_g = 0.000650857$, and $\alpha_o = 0.000856391$ mtc per ktoe of coal, gas, and oil, resp.

Country-level forest including afforestation data was obtained from the Global Forest Resources Assessment (FRA) 2010 of the Food and Agricultural Organisation. The carbon sequestration factor s in our theoretical model is defined as the amount of carbon sequestered per unit volume of forest and is measured in million tons of carbon per millimetre cube (mtc per mm3). Based on information provided by FRA we derive the carbon sequestration factor for each country.

International data on the price of labour, the wage rate, is obtained from EPWT [2014] in 2005 USD purchasing power parity. Prices (levelised costs of producing electricity (LCOE)) of electrical energy from coal, gas, and renewables are derived from the Projected Costs of Generating Electricity (PCGE) [2010] published by IEA and the Nuclear Energy Agency (NEA), which is based on cost data from 190 power plants in 21 countries. We

[45] For a more detailed description of the data, we refer the reader to the appendix.
[46] See *e.g.,* EPA brochure: http://www.epa.gov/cpd/pdf/brochure.pdf.

extrapolate the findings of the PCGE report to countries outside its sample. The price of oil energy is taken to be the crude oil price, which is obtained from the BP Statistical Review as 572591.7 USD per ktoe. Costs of afforestation and maintaining current stocks of forests are derived from Chp. 24 of the Second Assessment Report (SAR) of IPCC (1996) and Sathaye and Ravindranathan (SR) (1998), which compile estimates from different case-studies in a manner that facilitates attribution of these costs to countries in our sample.

7.2. Estimation procedures.

We estimated the production function under Cobb Douglas and nested and non-nested CES specifications. We found that only the Cobb Douglas form fits the data well:

$$y = f(k, l, e, o) = Ak^{\beta_k} l^{\beta_l} e^{\beta_e} o^{\beta_o}.$$

Moreover, this specification implies that all inputs (including stationary and non-stationary energy inputs) are essential, which, as discussed in Section 3, is a desirable property. For the estimation, an unbalanced panel of 118 countries across four years, 1990, 2000, 2005, and 2010 was employed, and a stochastic production frontier that allows for total factor productivity (TFP) differentials across countries was estimated. Thus, production frontiers of countries differ due to differences in their TFP. Estimates of coefficients of all inputs can be found in Table 1a, while the descriptive statistics of TFP can be found in Table 1b. These coefficients are all positive and significant. The sum of input coefficients is $\beta_k + \beta_l + \beta_e + \beta_o = 0.8306 < 1$, indicating that the technologies of countries exhibit decreasing returns to scale.

The ATA, RIP, and MAC for each country are computed employing (5.16), (5.18), and (5.13), resp.

To construct riaps that lead to non-differential increases in profit and decreases in emission, the algorithm in Section 6.3 is implemented for each country using MATLAB. For every country in our sample, Steps 1 and 2 of the alogorithm were coded to numerically compute riaps $\mathbf{a}$ and $\mathbf{b}$ with $\bar{\gamma} = 0.5$ and $\overset{*}{\pi} = .5\mathbf{a} + .5\mathbf{b}$. A code was created to construct sequence $\left\{\langle \delta^t_{\mathbf{a}}, \delta^t_{\mathbf{b}} \rangle\right\}$ and to solve for the fixed point $\langle \overset{*}{\delta}, \overset{*}{\delta} \rangle$, numerically.

8. Results.[47]

None of the 118 countries in our sample maximises its profit at the status-quo subject to its existing emission level. Hence, status-quo of each country in our sample is allocatively inefficient and profit increasing and emission non-increasing reforms exist for every country. We present results on ATA, RIP, and MAC before presenting the results on unilateral

[47] For a complete set of results that cover all countries, please see appendix.

Table 1a : Stochastic production frontier estimation with Cobb Douglas specification

Coefficients	Estimates	p-value
capital	0.3722823	0
stationary energy	0.0819302	0
oil	0.130042	0
labour	0.2463787	0

Table 1b : Descriptive statistics of country-specific total factor productivity

min	9.0617487
max	11.323674
mean	10.18564957
median	10.2365058
std dev	0.488024176

Table 2: Descriptive statistics of ATA

min	0.1202
max	1905.8898
mean	55.6128
median	8.6717
std dev	205.9294

Table 3 : ATA, RIP, MAC, and energy from fossil fuels in countries with highest ATA

Country	ATA	RIP	MAC	Energy from fossil fuels
China	1905.89	227456.39	119.34	2207807.83
United States	1087.51	1152288.08	1059.56	2053288.86
India	380.02	212303.93	558.66	532696.86
Russia	329.39	-10133.09	0.00	665571.85
Japan	247.74	407911.41	1646.50	450920.65
Germany	153.45	256809.02	1673.54	294470.66
South Korea	135.84	89897.08	661.78	236056.60
South Africa	122.94	52.08	0.42	137600.78
Brazil	117.95	179706.85	1523.61	171850.99
Iran	117.51	53232.02	453.01	227646.81
Saudi Arabia	113.81	20580.87	180.84	197942.59
Canada	112.01	116133.65	1036.79	214216.17
United Kingdom	98.73	219375.16	2221.96	204148.99
Mexico	95.98	142682.99	1486.61	170914.86
France	88.52	230312.77	2601.77	153712.96
Italy	86.54	188077.06	2173.38	170080.11
Australia	83.84	60323.19	719.46	141826.99
Indonesia	81.83	71630.18	875.33	151907.28
Poland	70.65	27085.06	383.38	99480.72
Spain	69.86	142722.18	2043.11	119088.76

efficiency-improving reforms and the extent of efficiency improvements they entail. Our empirical findings confirm our theoretical results in Sections 4 to 6.

8.1. International differences in ATA

ATA is measured in tons of carbon. Table 2 shows that the ATA ranges in our sample of countries from 0.12 to $1,905.89$. The average ATA is 55.61. Nearly 90% of the countries have low (less than 100) ATAs. Table 3 provides the list of 20 countries with the highest ATAs. The top 5 countries in this group are China, USA, India, Russia, and Japan.

We demonstrated (see Remark 6) that the ATA is related to the extents of fossil-fuel usage and afforestation efforts by countries. Indeed, the rank correlation between fossil-fuel usage and ATA is very high, nearly 0.986. Countries listed in Table 3 are also the largest users of fossil fuels.

8.2. International differences in RIP

RIP is measured in millions of USD. Table 4 shows that the value of RIP ranges in our sample from $-20,079.3$ to $1,152,288.08$. The average RIP is $40,700.79$. Lists of countries with the highest and lowest RIPs can be found in Tables 5a and 5b, resp. 12 countries have RIPs greater than $100,000$, with 5 countries (namely, Germany, France, China, UK, and India) having RIPs between $200,000$ to $400,000$ and two countries (USA and Japan) having RIPs greater than $400,000$. USA has the highest RIP in our sample, which is nearly three times that of Japan.

The magnitude of RIP depends on the profitabilities of fossil fuels and the cost of one-percent increase in afforestation (see part (2) of Remark 7) . Indeed, we find that the rank correlation coefficient between RIP and the average of profitabilities of fossil fuels and cost of afforestation is 0.897. The top 12 countries with respect to RIP are also the top 12 countries with respect to average profitability (see Tables 5a).

Table 5b shows that there are 9 countries in our sample, including Russia and several other countries from the Soviet block, with negative RIPs. From the definition of RIP (see (5.12)) it follows that, in these countries, the dot product $\Psi \cdot \varepsilon$ is negative. Remark 2 implies that these are the countries for which the emission constraint is not binding at the optimum of problem (5.5). In this set, marginal profits, profitabilities, and elasticities of profit with respect to coal and gas are negative for countries that utilise these inputs and the profitability of oil is negative for countries which do not utilise coal and gas (this corroborates part (1) of Remark 7).[48]

[48] For elasticities of the profit function in these countries, see Table 12b.

Table 4 : Descriptive statistics of RIP

min	-20079.29
max	1152288.08
mean	40700.79
median	6502.41
std dev	121844.39

Table 5a: RIPs and average profitability in countries with highest RIP

country	RIP	Avg. Profitability
United States	1152288.08	413835.85
Japan	407911.41	138380.46
Germany	256809.02	91339.51
France	230312.77	63226.66
China	227456.39	233457.32
United Kingdom	219375.16	80415.31
India	212303.93	119034.95
Italy	188077.06	63424.03
Brazil	179706.85	46674.43
Spain	142722.18	43355.10
Mexico	142682.99	44908.00
Canada	116133.65	34452.77
South Korea	89897.08	29105.44
Indonesia	71630.18	23833.59
Turkey	67408.51	30592.88

Table 5b: RIPs and average profitability in countries with lowest RIP

country	RIP	Avg. Profitability
Uzbekistan	-20079.29	-3635.947976
Trinidad &Tobago	-15747.08	-2558.300852
Russia	-10133.09	24000.75279
Turkmenistan	-6848.12	-1308.92777
Ukraine	-5577.35	5949.802007
Kazakhstan	-3380.18	2709.368595
Zimbabwe	-723.03	83.90781941
Bosnia & Herzegovina	-444.08	517.3444709
Malta	-7.02	-1.754960167
Bhutan	15.38	72.78585291
South Africa	52.08	8410.658842
Gambia	119.44	45.14281788
Cape Verde	131.22	34.27352093
Moldova	315.01	214.8807537
Mongolia	329.69	225.1790206

8.3. International differences in MAC and relationship to differences in ATAs and RIPs.

MAC is measured in USD per ton of carbon. The MACs of 118 countries in our sample vary from zero to $6,700.24$. Table 6 shows that the mean and median MACs are $1,345.7$ and $1,082.5$, respectively. Recall that MAC is the ratio of the RIP and ATA whenever $RIP > 0$ and is zero whenever $RIP \leq 0$ (see (5.13)).

Countries with lowest MACs (between zero and 200) are listed in Table 7a. This set includes countries with negative RIP, which, as discussed in the previous sub-section, implies that $\Psi \cdot \varepsilon < 0$ for these countries and, hence, from Remark 2, the emission constraint of problem (5.5) is not binding for these countries. Hence, starting from the optimum of problem (5.5), a small proportionate increase in the emission cap results in no proportionate reduction in profit. This explains why MACs are zero for these countries.[49]

Although, majority of the countries (more than 94%) have MACs less that $3,000$, Table 7b shows 7 outliers whose MACs are exceptionally high (greater than $3,000$). Primarily, this is because of their very low ATAs. In this set, Haiti has the lowest ATA and MAC. In countries with higher ATAs, Table 7b shows that the RIP is also disproportionately higher, resulting in even higher MACs.

Table 3 shows considerable variations in the MACs of countries with the highest ATAs. China and USA are the countries with the highest ATAs (more than $1,000$). But the MAC of USA is nearly nine times that of China. This is because, not only does China have a higher ATA than USA, but the RIP for USA is also more than five times higher. China and India have similar RIPs, nevertheless India's MAC is nearly five times that of China. This is because India has a lower ATA than China (China's ATA is more than five times that of India). UK, Germany, and India have similar RIPs, but because Germany's ATA is bigger than UK's and much smaller than India's, we find that the MAC of Germany is higher than India's but lower than UK's. RIP for France is bigger than UK's, while its ATA is lower than UK's. Hence it has a higher MAC than UK. Table 7c shows that the MAC for most major European powers lies at the higher end of the world spectrum (between 2,000 and 3,000).

We now study the differences in the MACs across OECD and non-OECD countries, excluding the seven outliers listed in Table 7b. Table 8a shows that the MAC ranges from 383 to $2,942$ in the OECD countries and from zero to $2,930$ in the non-OECD countries. The mean MAC is $1,543$ in the set of OECD countries, which is higher than 937 in the set of non-OECD countries. Table 8b reveals that a big proportion (80%) of OECD countries (*i.e.,* 27 countries) have high MACs (more than 1000), while a big proportion (61%) of Non-OECD countries (*i.e.,* 47 countries) lie at the lower end of the MAC spectrum (MAC less than 1000).

[49] Refer to Figure 4 for intuition. See also Theorem 3.

Table 6 : Descriptive statistics of MAC

min	0.00
max	6700.24
mean	1345.73
median	1082.47
std dev	1213.30

Table 7b : Countries with highest MAC

Country	MAC	ATA	RIP
Haiti	3104.71	0.63	1961.98
Nigeria	4361.38	10.92	47631.47
Mozambique	4526.80	0.72	3252.10
Zambia	5078.40	0.65	3306.85
Tanzania	5090.42	1.46	7456.10
Ethiopia	5290.28	1.92	10165.35
Nepal	6700.24	0.95	6393.21

Table 7c : MACs in major European countries

Country	MAC	Country	MAC
Switzerland	2942.34	Spain	2043.11
France	2601.77	Norway	1885.90
Ireland	2501.37	Portugal	1870.96
Austria	2302.21	Iceland	1745.63
Denmark	2254.34	Germany	1673.54
United Kingdom	2221.96	Finland	1645.33
Italy	2173.38	Netherlands	1194.72
Sweden	2150.96	Poland	383.38

See also Table 3.

Table 7a : Countries with lowest MAC

Country	MAC	ATA	RIP
Bosnia & Herzegovina	0.00	5.72	-444.08
Kazakhstan	0.00	42.75	-3380.18
Malta	0.00	2.25	-7.02
Russia	0.00	329.39	-10133.09
Trinidad &Tobago	0.00	14.22	-15747.08
Turkmenistan	0.00	12.74	-6848.12
Ukraine	0.00	53.98	-5577.35
Uzbekistan	0.00	27.34	-20079.29
Zimbabwe	0.00	2.25	-723.03
South Africa	0.42	122.94	52.08
Bhutan	5.12	3.00	15.38
Kyrgyz Republic	118.51	4.04	479.31
China	119.34	1905.89	227456.39
Belarus	165.82	18.57	3079.08
Mongolia	172.73	1.91	329.69
Saudi Arabia	180.84	113.81	20580.87
Latvia	183.04	5.74	1050.94

Table 8a : Descriptive statistics of MAC in OECD and Non OECD countries

	OECD	Non-OECD
min	383.38	0.00
max	2942.34	2930.00
mean	1543.31	937.28
median	1552.81	772.43
std dev	649.00	808.41

Table 8b : Distribution of MAC in OECD and Non OECD countries

	OECD		Non-OECD	
MAC	Percentage	Number	Percentage	Number
0 - 1000	20%	7	61%	47
1000 - 2000	50%	17	29%	22
> 2000	30%	10	10%	8

8.4. Efficiency increasing reforms.

Starting from an inefficient status-quo, efficiency increasing riaps increase profit without increasing the emission level. In Section 6, we provided the theory and methodology to construct two such reforms: **a** and $\overset{*}{\delta}$. Running the codes for the algorithm in Section 6.3, we compute reforms **a** and riap $\overset{*}{\delta}$ and the potential efficiency improvements they entail.

8.4.1. Extent of efficiency improvements.

While Theorem 4 stated that **a** is a reform that results in a non-differential increase in profit and a reduction in emission at status-quos characterised by $\Psi \cdot \varepsilon < 0$, Theorem 5 stated that reform $\overset{*}{\delta}$ has these properties in the cases where $\Psi \cdot \varepsilon \geq 0$. Table 9 provides summary statistics of estimated efficiency gains from implementing these riaps.

The levels of profit increases range from 57.7 million USD to 777,113.5 million USD with average and median increases being 23,960.7 and 3,982.6 million USD, resp. The global profit increases by 8.3%.

On the other hand, emission reductions range from .006 million tons to 1,016.4 million tons. The mean and median reductions are 22.6 and 2.3 million tons, resp. Global net emissions reduce by 30.5%.

Table 10 lists the 15 biggest gainers from these reforms. It lists, both, countries which have the biggest increases in profit and countries which can reduce emission the most. It should be noted that, Ethiopia and Democratic Republic of Congo, who are among the biggest gainers of profit, have negative profits at the status-quo. The rank correlation between profit increase and emission reduction is 0.737, hence countries that can increase profit the most tend also to be the countries that can reduce emission the most under these reforms.

8.4.2. Structure of reform **a** *and its recommendations.*

Recall that, by construction, riap $\mathbf{a} = \langle \mathbf{a_l}, \mathbf{a_c}, \mathbf{a_g}, \mathbf{a_o}, \mathbf{a_r}, \mathbf{a_a} \rangle = \bar{\kappa}\bar{\bar{\delta}}$. For several reasons, **a** is a theoretically appealing riap. Firstly, it is proportional to riap $\bar{\delta}$ that is derived, using (5.2), from the optimal lpcap $\bar{q}$, which solves problem (5.5). Secondly, as stated in Theorem 4, for all countries, it ensures non-differential increases in profit with no increase in emission level. Thirdly, as also stated in the Theorem 4, for countries where the emission constraint is not binding at the optimum of problem (5.5), **a** not only leads to increases in profit but also to positive reductions in emission level. Fourthly, as seen in Section 6.2, it is employed along with reform **b** to construct reform $\overset{*}{\delta}$, which results in non-differential increases in profit and reductions in emission in countries for which the emission constraint is binding at the optimum. Thus, it is worthwhile studying the structure of reform **a**.

Table 9 : Descriptive statistics of efficiency gains

Profit increase in millions of USD	
min	57.72
max	777113.46
mean	23960.67
median	3982.55
std dev	80465.68
global increase in profit	2827359.31
global profit at status-quo	34201437.55
global % increase	8.27
Emission reduction in millions of tons	
min	0.0062
max	1016.4302
mean	22.6095
median	2.3399
std dev	107.5762
global decrease in emission	2667.9230
global net emission at status-quo	8753.7431
global % reduction	30.4775

Table 10 : Biggest gainers from efficiency-increasing reforms

Countries with biggest increases in profit		**Countries with biggest reductions in emission**	
Country	**Profit Increase**	**Country**	**Emission reduction**
United States	777113.46	China	1016.43
China	296292.01	United States	568.60
Japan	210055.53	India	140.81
Russia	145126.01	Japan	116.99
Germany	121249.43	Germany	71.81
United Kingdom	112828.78	South Africa	68.50
India	106374.78	United Kingdom	47.17
France	83009.19	Canada	44.64
Spain	49060.16	Australia	40.11
Ethiopia	46532.04	South Korea	35.44
Canada	43817.40	Poland	33.19
Ukraine	42072.91	France	33.01
Democratic Republic of Congo	36742.74	Spain	24.94
Italy	35935.80	Netherlands	22.88
Nigeria	33655.10	Indonesia	22.16
(in millions of USD)		(in millions of tons)	

Table 11 shows that riap **a** is definitely a non-marginal reform – its size is considerably bigger than one. Table 11 also shows that, for a majority of countries in our sample, reform **a** recommends decreases in coal and gas by 73% and 69%, resp. In more than 90% of countries it recommends *increase* in afforestation, while it recommends reduction in renewables in more than 66% of countries. However, the reasons for these prescriptions by reform **a** vary across countries depending upon the sign of the dot product $\Psi \cdot \varepsilon$. The structure of **a** for 15 countries with biggest profit increases under reform **a** and for which $\Psi \cdot \varepsilon \geq 0$ is given in Table 12a, while the structure of **a** for countries where $\Psi \cdot \varepsilon < 0$ is given in Table 12b.

From Remark 3 and (5.10) it follows that riap $\bar{\delta}$ and, hence, riap **a** (as it is proportional to $\bar{\delta}$) recommend increase in an input if and only if its marginal proportional benefit is bigger than its marginal proportional cost.

Since, the emission constraint is non-binding in countries where $\Psi \cdot \varepsilon < 0$, the structure of **a** is based entirely on marginal proportional benefits of inputs, *i.e.,* on the profitability (or marginal profits) of inputs (see part (iii) of see Remark 3). Table 12b, shows that the marginal proportional benefits (given in rows marked ε) of fossil fuels coal and gas are negative in all countries which consume positive amounts of these fossil fuels. In this sense, these countries are over-consuming these fossil fuels. From part (iii) of Remark 3 and (5.10), it follows that lpcap $\bar{q}$ and, hence, riaps $\bar{\delta}$ and **a**, recommend reductions in the consumption of coal and gas in these countries. Part (iii) of Remark 3 also implies that, in these countries, reform **a** must recommend decrease in afforestation, which, as seen in Table 12b, is confirmed by data.

Since $\Psi \cdot \varepsilon \geq 0$ for countries in Table 12a, the emission constraint of problem (5.5) is binding and, hence, the Lagrange multiplier of the emission constraint $\bar{\bar{\mu}}$ is positive in these countries. Hence, recommendations of lpcap $\bar{q}$ and, hence, riaps $\bar{\delta}$ and **a**, are based on both the marginal proportional benefits and the marginal proportional costs. In contrast to countries in Table 12b, for almost all countries in Table 12a, the marginal proportional benefits of fossil fuels are positive. The fact that riap **a** still recommends decreasing coal and gas consumption in these countries must be because the marginal proportional costs of coal and gas (in the face of the emission cap) must be high enough in these countries to offset their positive marginal proportional benefits. Further, riap **a** recommends increasing afforestation in these countries. Given that the marginal proportional benefit and the marginal proportional cost of afforestation are always negative (see part (ii) of Remark 3) this can be true if and only if the negative marginal proportional cost of afforestation can offset its negative marginal proportional benefit.

Table 11 : Recommendations of reform a and its size

countries (% and number)	Percentage	Number
$a_l < 0$	77.12	91
$a_c < 0$	72.88	86
$a_g < 0$	69.49	82
$a_o < 0$	9.32	11
$a_r < 0$	66.95	79
$a_a < 0$	8.47	10
Descriptive statistics of size of reform a		
min	27039.25	
max	392592852.90	
mean	11198258.93	
median	2832720.57	
std dev	37974447.49	

Table 12a : Structure of reform a and marginal proportional benefits for biggest gainers with $\Psi \bullet \varepsilon \geq 0$

Country		labour	coal	gas	oil	renewables	afforestation
United States	a	-97222757.62	-40400.07	-40062.107	95151.90	-6484.80	24.08
	ε	-0.63	0.03	0.004	0.24	-0.02	-0.000271185
China	a	-392592807.18	-112704.70	-475.366	151026.81	-19833.35	0.80
	ε	-0.33	0.01	-0.001	0.21	-0.04	-0.000266585
Japan	a	-38969727.15	-11704.67	-4754.005	19219.26	-890.21	3.22E-12
	ε	-0.55	0.03	0.013	0.24	-0.01	-1.45278E-10
Germany	a	-23741274.32	-8233.93	-7156.662	16430.78	-2969.73	0.03
	ε	-0.46	0.03	0.002	0.24	-0.03	-1.41108E-05
United Kingdom	a	-18396181.71	-1995.42	-9005.309	9526.03	-298.43	0.05
	ε	-0.53	0.02	0.033	0.25	-0.01	-1.85668E-05
India	a	-43833896.70	-110907.84	-2243.364	150868.10	-42248.49	2.97
	ε	-0.01	0.03	0.003	0.17	-0.03	-3.27329E-05
France	a	-16185391.03	-468.27	-4334.983	4087.26	-11243.42	0.31
	ε	-0.46	0.00	0.002	0.25	-0.07	-4.76925E-05

Table 13 : Descriptive statistics of efficiency gains in the absence of reforms in renewable energy

Profit increase in millions of USD	
min	35.70
max	775617.43
mean	23133.34
median	3749.62
std dev	80102.84
global increase in profit	2729734.10
global profit at status-quo	34201437.55
global % increase	7.98
Emission reduction in millions of tons	
min	0.01
max	1024.63
mean	22.58
median	2.14
std dev	108.30
global decrease in emission	2664.98
global net emission at status-quo	8753.74
global % reduction	30.44

Table 12b : Structure of reform a and marginal proportional benefits for countries with $\Psi \bullet \varepsilon < 0$

Country		labour	coal	gas	oil	renewables	afforestation
Bosnia &	a	-2011090.65	-218.16	-0.79	183.11	-10.50	-1.00E-12
Herzegovina	ε	-6.14	-0.33	-0.03	0.91	-0.14	-7.34901E-08
Kazakhstan	a	4660527.99	-10258.97	-807.43	8138.31	-19.65	-2.13E-12
	ε	0.11	-0.06	-0.02	0.15	-0.0046	-4.1969E-09
Malta	a	-72974.17	0	0	-7.28	0.76	-1.67E-11
	ε	-0.18	0	0	0.00	0.15	-6.93776E-08
Russia	a	-34357929.11	-6249.70	-106422.77	80533.43	-6144.74	-0.64
	ε	-0.17	-0.02	-0.10	0.20	-0.04	-0.001151865
Trinidad & Tobago	a	183365.03	0	-17688.62	593.78	-0.02	-1.18E-12
	ε	0.22	0	-0.62	0.24	-0.0008	-9.0048E-09
Turkmenistan	a	320191.26	0	-13143.69	1392.44	0	-1.78E-11
	ε	0.07	0	-0.38	0.15	0	-9.5907E-08
Ukraine	a	-12966518.04	-3127.21	-8569.83	7225.70	-8385.00	-0.01
	ε	-0.32	-0.05	-0.10	0.28	-0.19	-0.000306787
Uzbekistan	a	-9699126.57	-20.53	-22276.05	954.06	-25.80	-0.000120115
	ε	-7.45	-0.14	-4.96	1.75	-0.26	-0.002259051
Zimbabwe	a	-3859369.57	-153.50	0	71.85	-3272.36	-4.03E-12
	ε	-1.10	-0.10	0	0.13	-0.64	-5.1478E-08

8.4.3. Trade-offs between short-run efficiency-improving reforms and long-run climate policies in the renewable energy sector.

The fact that optimal lpcap $\bar{q}$ and, hence, riaps $\bar{\bar{\delta}}$ and **a**, recommend reduction in renewables in a majority of countries may seem a counter-intuitive policy at a first glance (see Table 11). But, Tables 12a and 12b also show that, for almost all these countries, the marginal proportional benefit from renewables is negative.[50] This is consistent with our intuition that, currently, the cost of producing renewable energy is high, which reduces its profitability. Thus, the recommendation of riap **a** to reduce renewable usage, based purely on its current profitability, is not surprising.

However, current climate policies encourage a switch to renewables all over the world, even if, in the short run, this switch is costly on our resources. This is because such a policy is helpful for securing long-run environmental sustainability.

Suppose, in order not to hinder this long-run objective, our short-run reform framework excludes renewable energy as a short-run active policy, *i.e.,* suppose, we do not interfere with the renewable energy sector, while designing short-run reforms.[51] The entire exercise developed in Sections 2 to 6 can now be repeated in this new environment with a smaller set of active policy variables. We can ask the question whether efficiency gains can still be possible.

Table 13 shows that it is still possible to design reforms that increase profit and reduce emission levels in all countries, albeit, the efficiency gains are slightly smaller than in the case where we could also have short-run reforms in the renewable energy sector.[52]

9. Conclusions.

This paper allows for inefficiencies in the way countries manage their resources. Exploiting these inefficiencies, it develops (i) a methodology for constructing unilateral efficiency-improving reforms that allow inefficiently operating economies to increase their producer surpluses (profits), while also decreasing their emission levels and (ii) a measure of MAC, based on local input-policy reforms, that becomes relevant when an economy can not implement, due to existing institutional and political constraints, a radical instantaneous jump from its inefficient status-quo to the optimum under the self-imposed

[50] Recall that, according to Remark 3, marginal proportional costs of renewables are zero, so that the prescribed sign of renewable reform depends only on its marginal proportional benefit and, hence, on the marginal profit of renewables.

[51] This means, we include inherited renewable energy generation as a part of the environment (Θ_0) that is fixed in the short-run at the status-quo.

[52] In fact, we conjecture (based on a continuity argument) that, short-run reforms, which *increase* renewable energy and, at the same time, increase profit and reduce emission levels, could exist. But the immediate efficiency gains they would entail will be much smaller.

emission-constraint. International differences in this measure of MAC allow multilateral efficiency improvements, whereby countries with high MACs can achieve reductions in their emission targets with lower losses in their producer surpluses by purchasing abatement more cheaply from countries with lower MACs.

The data, which we could gather for 118 countries, combined with the theory we develop, shows that allocative inefficiencies are pervasive. The unilateral efficiency improving reform we construct using a fixed point argument indicates that efficiency gains may not be insignificant. In particular, in the context of the model we present, this reform results in more than 8% increase in global profit and more than 30% reduction in global emission.

In this paper, we find that OECD countries, especially many western European countries that have set for themselves high emission-reduction targets, have significantly higher MACs than most non-OECD countries. Differences in these MACs *should* trigger considerable purchases of abatement by the OECD block from the non-OECD countries with lower MACs such as Russia, China, and India at mutually beneficial prices.

However, we believe that such potentials, currently, seem not to be fully, estimated, appreciated, and exploited. Though there are many instruments in place such as the Clean Development Mechanism (CDM); direct government-to-government transactions through International Emission Trading (IET); and several transnational climate governance (TCG) initiatives by subnational entities, private sector associations, individual firms, and NGOs for promoting multilateral efficiency improvements in meeting emission-reduction targets, according to Michaelowa (2015), barriers to import of emission credits have increased significantly in recent times, and lack of demand has led to the crash of price of international emission credits.[53] For example, large surpluses of emission allowances were granted to Eastern Europe, Russia, and Ukraine in the first commitment period of the Kyoto protocol, which were later deemed as "hot air." So, in the second commitment period, further purchases of emission credits from these countries were banned by many Western countries, some of which have high voluntary emission-reduction targets. But, under our analysis, many countries in the former Soviet block have the lowest MACs. As another (anecdotal) example, we refer to the case of TATA steel, which recently announced that it was "considering shutting down its UK Port Talbot plant owing to cheap imports of Chinese steel, *high energy costs*, and a weak demand." This has caused serious concerns in UK as "it puts thousands of jobs at risk."

Ambitious emission reduction targets of western European countries, which it seems they are planning to attain mainly by unilateral efforts, put tremendous pressures on their domestic energy sectors raising considerably their domestic energy prices. This can be eased by multilateral efficiency improvements that tap international differences in MACs.

[53] For a review of these instruments and their performance, see Michaelowa (2015) and Michaelowa and Michaelowa (2016).

A necessary condition for this is to have a better understanding and more credible and accurate estimates of current international differences in MACs.

APPENDIX

A1. Proofs of lemmas and theorems.

Proof of Theorem 1: Since 0_6 is an emission non-increasing reform with magnitude less than one, it satisfies constraints of problem (4.6). Hence, $\dot{\mathbf{\Pi}}(\nu_0, z_0, \Theta_0, 0) = \nabla_\nu \Pi(\nu_0, z_0, \Theta_0) \cdot \overset{*}{\nu} \geq 0 = \nabla_\nu \Pi(\nu_0, z_0, \Theta_0) \cdot 0_6$.

Since $\overset{*}{\nu}$ solves problem (4.6) and $\overset{*}{\mu}$ and $\overset{*}{\lambda}$ are the values of the Lagrange multipliers at the optimum of problem (4.6), the vector $\langle \overset{*}{\nu}, \overset{*}{\mu}, \overset{*}{\lambda} \rangle$ solves the Kuhn-Tucker first-order conditions (4.12).

Suppose $\nabla_\nu \mathcal{Z} \cdot \overset{*}{\nu} = 0$. Premultiplying both sides of the first equation of (4.12) by $\nabla_\nu \mathcal{Z}$, we obtain

$$\nabla_\nu \mathcal{Z} \cdot \nabla_\nu \Pi - \overset{*}{\mu} \nabla_\nu \mathcal{Z} \cdot \nabla_\nu \mathcal{Z} - 2\overset{*}{\lambda} \nabla_\nu \mathcal{Z} \cdot \overset{*}{\nu} = 0$$

Since $\nabla_\nu \mathcal{Z} \cdot \overset{*}{\nu} = 0$ and $(\nabla_\nu \mathcal{Z} \cdot \nabla_\nu \mathcal{Z})^{-1}$ exists, we have

$$\nabla_\nu \mathcal{Z} \cdot \nabla_\nu \Pi - \overset{*}{\mu} \nabla_\nu \mathcal{Z} \cdot \nabla_\nu \mathcal{Z} = 0 \quad \Longrightarrow \quad \overset{*}{\mu} = (\nabla_\nu \mathcal{Z} \cdot \nabla_\nu \mathcal{Z})^{-1} (\nabla_\nu \mathcal{Z} \cdot \nabla_\nu \Pi).$$

Suppose $\nabla_\nu \mathcal{Z} \cdot \overset{*}{\nu} < 0$. Then the second condition in (4.12) immediately implies $\overset{*}{\mu} = 0$.

(1) (i) Suppose $\nabla_\nu \mathcal{Z} \cdot \overset{*}{\nu} = 0$ and $\nabla_\nu \Pi - \overset{*}{\mu} \nabla_\nu \mathcal{Z} \neq 0_6$.

The set of first conditions in (4.12) imply that $\overset{*}{\lambda}\overset{*}{\nu} \neq 0$. Hence, $\overset{*}{\nu} \neq 0$ and $\overset{*}{\lambda} > 0$. The third set of conditions in (4.12), hence, imply that

$$\overset{*}{\nu} \cdot \overset{*}{\nu} = 1. \tag{A.1}$$

The first equation in (4.12) implies

$$\overset{*}{\nu} = \frac{1}{2\overset{*}{\lambda}} \left[\nabla_\nu \Pi - \overset{*}{\mu} \nabla_\nu \mathcal{Z} \right].$$

This, combined with (A.1), gives us

$$\overset{*}{\lambda} = \frac{1}{2} \sqrt{\left[\nabla_\nu \Pi - \overset{*}{\mu} \nabla_\nu \mathcal{Z} \right] \cdot \left[\nabla_\nu \Pi - \overset{*}{\mu} \nabla_\nu \mathcal{Z} \right]} \quad \Longrightarrow \quad \overset{*}{\nu} = \frac{\nabla_\nu \Pi - \overset{*}{\mu} \nabla_\nu \mathcal{Z}}{\sqrt{\left[\nabla_\nu \Pi - \overset{*}{\mu} \nabla_\nu \mathcal{Z} \right] \cdot \left[\nabla_\nu \Pi - \overset{*}{\mu} \nabla_\nu \mathcal{Z} \right]}}.$$

Post multiplying both sides of the first equation in (4.12) by $\overset{*}{\nu}$ we obtain

$$\nabla_\nu \Pi \cdot \overset{*}{\nu} - \overset{*}{\mu} \nabla_\nu \mathcal{Z} \cdot \overset{*}{\nu} = \overset{*}{\lambda}\overset{*}{\nu} \cdot \overset{*}{\nu}.$$

Since $\nabla_\nu \mathcal{Z} \cdot \overset{*}{\nu} = 0$ and (A.1) holds we have

$$\dot{\mathbf{\Pi}}(\nu_0, z_0, \Theta_0, 0) = \nabla_\nu \Pi \cdot \overset{*}{\nu} = \overset{*}{\lambda} > 0.$$

Hence, $\dot{\mathbf{\Pi}}(\nu_0, z_0, \Theta_0, 0) > 0$.

(ii) Suppose $\nabla_\nu \mathcal{Z} \cdot \overset{*}{\nu} = 0$ and $\nabla_\nu \Pi - \overset{*}{\mu} \nabla_\nu \mathcal{Z} = 0_6$.

First set of conditions of (4.12) imply that $\overset{*}{\lambda}\overset{*}{\nu} = 0$. Suppose $\overset{*}{\lambda} > 0$. Then the third condition of (4.12) implies that (A.1) is true. But this implies that $\overset{*}{\nu} \neq 0_6$. This contradicts the fact that $\overset{*}{\lambda}\overset{*}{\nu} = 0$. Hence, $\overset{*}{\lambda} = 0$. Suppose $\overset{*}{\mu} = 0$. Then $\nabla_\nu \Pi = 0_6$, which is a contradiction to maintained Assumption 1. Hence, $\overset{*}{\mu} > 0$. $\nabla_\nu \Pi - \overset{*}{\mu} \nabla_\nu \mathcal{Z} = 0_6$ implies that $\nabla_\nu \Pi = \overset{*}{\mu} \nabla_\nu \mathcal{Z}$. Hence, under the maintained assumptions of this case, $\dot{\mathbf{\Pi}}(\nu_0, z_0, \Theta_0, 0) = \nabla_\nu \Pi \cdot \overset{*}{\nu} = \overset{*}{\mu} \nabla_\nu \mathcal{Z} \cdot \overset{*}{\nu} = 0$. This implies that any $\dot{\nu} \in \omega(\nu_0, z_0, \Theta_0, 0)$ as long as $\|\dot{\nu}\| \leq 1$ and $\nabla_\nu \Pi \cdot \dot{\nu} = 0$. In particular, $0_6 \in \omega(\nu_0, z_0, \Theta_0, 0)$.

(2) Suppose $\nabla_\nu \mathcal{Z} \cdot \overset{*}{\nu} < 0$.

From our earlier conclusions of this theorem, we have $\overset{*}{\mu} = 0$. If $\nabla_\nu \Pi - \overset{*}{\mu} \nabla_\nu \mathcal{Z} = 0_6$, then this implies $\nabla_\nu \Pi = 0_6$, which is a contradiction to maintained Assumption 1. Hence, $\nabla_\nu \Pi - \overset{*}{\mu} \nabla_\nu \mathcal{Z} \neq 0_6$. Hence, from the first condition of (4.12), we have $\nabla_\nu \Pi - \overset{*}{\mu} \nabla_\nu \mathcal{Z} = \nabla_\nu \Pi = 2\overset{*}{\lambda}\overset{*}{\nu} \neq 0_6$. This implies

$$\overset{*}{\nu} = \frac{1}{2\overset{*}{\lambda}} \nabla_\nu \Pi.$$

Combined with (A.1) (which also holds for this case), this implies

$$\overset{*}{\lambda} = \frac{1}{2}\sqrt{\nabla_\nu \Pi \cdot \nabla_\nu \Pi} \quad \Longrightarrow \quad \overset{*}{\nu} = \frac{\nabla_\nu \Pi}{\sqrt{\nabla_\nu \Pi \cdot \nabla_\nu \Pi}}.$$

Combined with Assumption 1, this also implies that $\dot{\mathbf{\Pi}}(\nu_0, z_0, \Theta_0, 0) = \nabla_\nu \Pi \cdot \overset{*}{\nu} = \nabla_\nu \Pi \cdot \frac{\nabla_\nu \Pi}{\sqrt{\nabla_\nu \Pi \cdot \nabla_\nu \Pi}} = \|\nabla_\nu \Pi\| > 0$. ■

Proof of Theorem 2:

(**1**) (i) Proof of $\nabla_\nu \mathcal{Z} \cdot \nabla_\nu \Pi < 0 \implies \nabla_\nu \mathcal{Z} \cdot \overset{*}{\nu} < 0$. Define $\overset{*}{\delta} := \frac{\nabla_\nu \Pi}{\|\nabla_\nu \Pi\|}$. Then $\overset{*}{\delta} \cdot \overset{*}{\delta} = 1$. Hence, $\overset{*}{\delta}$ solves the problem

$$\max_\delta \left\{ \nabla_\nu \Pi \cdot \delta \;\middle|\; \delta \cdot \delta = 1 \right\}. \tag{A.2}$$

The definition of $\overset{*}{\delta}$ implies $\nabla_\nu \mathcal{Z} \cdot \overset{*}{\delta} \equiv \nabla_\nu \mathcal{Z} \cdot \frac{\nabla_\nu \Pi}{\|\nabla_\nu \Pi\|} < 0$. Hence, (A.2) implies that $\overset{*}{\delta}$ also solves

$$\max_\delta \left\{ \nabla_\nu \Pi \cdot \delta \;\middle|\; \nabla_\nu \mathcal{Z} \cdot \delta \leq 0 \;\wedge\; \delta \cdot \delta = 1 \right\}.$$

Hence, $\overset{*}{\nu} := \overset{*}{\delta} = \omega(\nu_0, z_0, \Theta_0, 0)$ (*i.e.*, $\omega(\nu_0, z_0, \Theta_0, 0)$ is a singleton). But this implies $\nabla_\nu \mathcal{Z} \cdot \overset{*}{\nu} \equiv \nabla_\nu \mathcal{Z} \cdot \frac{\nabla_\nu \Pi}{\|\nabla_\nu \Pi\|} < 0$.

(ii) Proof of $\nabla_\nu \mathcal{Z} \cdot \overset{*}{\nu} < 0 \implies \nabla_\nu \mathcal{Z} \cdot \nabla_\nu \Pi < 0$.
Suppose $\overset{*}{\nu} \in \omega(\nu_0, z_0, \Theta_0, 0)$ and $\nabla_\nu \mathcal{Z} \cdot \overset{*}{\nu} < 0$. Suppose $\nabla_\nu \mathcal{Z} \cdot \nabla_\nu \Pi > 0$. Theorem 1 implies that $\nabla_\nu \Pi \cdot \overset{*}{\nu} > 0$. Remark 1 implies that $\overset{*}{\nu} \cdot \overset{*}{\nu} = 1$. Define a function $\delta : [0,1] \longrightarrow \mathbf{R}^6$ with image $\delta(\lambda) = \lambda \overset{*}{\nu} + (1-\lambda)\frac{\nabla_\nu \Pi}{\|\nabla_\nu \Pi\|}$, where $\lambda \in [0,1]$. Then $\nabla_\nu \mathcal{Z} \cdot \delta(\lambda) = \lambda \nabla_\nu \mathcal{Z} \cdot \overset{*}{\nu} + (1-\lambda)\nabla_\nu \mathcal{Z} \cdot \frac{\nabla_\nu \Pi}{\|\nabla_\nu \Pi\|}$. Since, $\nabla_\nu \mathcal{Z} \cdot \overset{*}{\nu} < 0$ and $\nabla_\nu \mathcal{Z} \cdot \nabla_\nu \Pi > 0$, continuity of δ implies that there exists $\hat{\lambda} \in (0,1)$ such that $\nabla_\nu \mathcal{Z} \cdot \delta(\hat{\lambda}) = 0$. The definition of δ implies that $\nabla_\nu \Pi \cdot \delta(\hat{\lambda}) = \hat{\lambda} \nabla_\nu \Pi \cdot \overset{*}{\nu} + (1-\hat{\lambda})\nabla_\nu \Pi \cdot \frac{\nabla_\nu \Pi}{\|\nabla_\nu \Pi\|} = \hat{\lambda}\|\nabla_\nu \Pi\| \cos\theta + (1-\hat{\lambda})\|\nabla_\nu \Pi\|$, where θ is the angle between $\nabla_\nu \Pi$ and $\overset{*}{\nu}$. Since, $\nabla_\nu \Pi \cdot \overset{*}{\nu} > 0$, we have $0 < \cos\theta < 1$. Hence, $\|\nabla_\nu \Pi\| \cos\theta < \|\nabla_\nu \Pi\|$. Hence, $\hat{\lambda}\|\nabla_\nu \Pi\| \cos\theta + (1-\hat{\lambda})\|\nabla_\nu \Pi\| > \|\nabla_\nu \Pi\| \cos\theta = \nabla_\nu \Pi \cdot \overset{*}{\nu}$. Hence, $\nabla_\nu \Pi \cdot \delta(\hat{\lambda}) > \nabla_\nu \Pi \cdot \overset{*}{\nu}$. Further, $\|\delta(\hat{\lambda})\| = \sqrt{\left[\hat{\lambda}\overset{*}{\nu} + (1-\hat{\lambda})\frac{\nabla_\nu \Pi}{\|\nabla_\nu \Pi\|}\right] \cdot \left[\hat{\lambda}\overset{*}{\nu} + (1-\hat{\lambda})\frac{\nabla_\nu \Pi}{\|\nabla_\nu \Pi\|}\right]} = \sqrt{\hat{\lambda}^2 + (1-\hat{\lambda})^2 + 2\hat{\lambda}(1-\hat{\lambda})\cos\theta} < \sqrt{\hat{\lambda}^2 + (1-\hat{\lambda})^2 + 2\hat{\lambda}(1-\hat{\lambda})} = 1$. Hence, $\hat{\delta} := \frac{\delta(\hat{\lambda})}{\|\delta(\hat{\lambda})\|} > \delta(\hat{\lambda})$. Hence, $\nabla_\nu \Pi \cdot \hat{\delta} > \nabla_\nu \Pi \cdot \delta(\hat{\lambda}) > \nabla_\nu \Pi \cdot \overset{*}{\nu}$, $\hat{\delta} \cdot \hat{\delta} = 1$, and $\nabla_\nu \mathcal{Z} \cdot \hat{\delta} = 0$. This contradicts the optimality of $\overset{*}{\nu}$ in solving problem (4.6). Hence, $\nabla_\nu \mathcal{Z} \cdot \nabla_\nu \Pi > 0$ implies $\nabla_\nu \mathcal{Z} \cdot \overset{*}{\nu} = 0$.
(**2**) From Theorem 1, it follows that $\overset{*}{\mu} = 0$ if and only if $\nabla_\nu \mathcal{Z} \cdot \overset{*}{\nu} < 0$ or $\nabla_\nu \mathcal{Z} \cdot \nabla_\nu \Pi = 0$. From part (1) of this theorem, $\nabla_\nu \mathcal{Z} \cdot \overset{*}{\nu} < 0$ is true if and only if $\nabla_\nu \mathcal{Z} \cdot \nabla_\nu \Pi < 0$. Conclusions follow. ■

Proof of Theorem 4: Since ε and Ψ are not proportional, the definition of $\mathbf{a}$, (5.4), Remark 2, and (6.1) imply

$$z_{\mathbf{a}} - z_0 = \nabla_\nu \mathcal{Z} \cdot \mathbf{a} = \bar{\kappa}\ \nabla_\nu \mathcal{Z} \cdot \bar{\delta} = \bar{\kappa}\ [z_0\ \Psi \cdot \bar{q}] \begin{array}{lll} = 0 & \text{if} & \Psi \cdot \varepsilon \geq 0 \\ < 0 & \text{if} & \Psi \cdot \varepsilon < 0. \end{array} \tag{A.3}$$

We prove next that $\bar{\kappa} > 0$. Given the uniqueness of solution to (6.2), this will imply that $\pi_{\mathbf{a}} > \pi_0$ (for otherwise both $\kappa = 0$ and $\bar{\kappa}$ would be solutions to (6.2)).

Since ε and Ψ are not proportional, Remark 2 implies $\varepsilon \cdot \bar{q} > 0$. Hence, (5.3) implies that $|\pi_0|\ \varepsilon \cdot \bar{q} = \nabla_\nu \Pi \cdot \bar{\delta} > 0$. Hence, riap $\bar{\delta}$ leads to a differential increase in profit at S_0. The definition of a derivative implies that for any sequence $\{\kappa^t\} \longrightarrow 0$ with $\kappa^t \in (0,1)$ for all $t \geq 0$, we have

$$\left\{ \frac{\Pi\left(\nu_0 + \kappa^t \bar{\delta}, \Theta_0\right) - \Pi\left(\nu_0, \Theta_0\right)}{\kappa^t} \right\} \longrightarrow \nabla_\nu \Pi \cdot \bar{\delta} =: \alpha > 0. \tag{A.4}$$

Since $\alpha > 0$, for any given scalar ϵ such that $0 < \epsilon < \alpha$, we have $b > 0$ for all $b \in N_\epsilon(\alpha)$. Since the sequence in (A.4) converges, there exists $\bar{t} > 0$ such that

$$\begin{aligned}
&\frac{\Pi\left(\nu_0 + \kappa^t \bar{\delta}, \Theta_0\right) - \Pi\left(\nu_0, \Theta_0\right)}{\kappa^t} \in N_\epsilon(\alpha) \quad \forall\, t \geq \bar{t} \\
\Longrightarrow\ &\frac{\Pi\left(\nu_0 + \kappa^t \bar{\delta}, \Theta_0\right) - \Pi\left(\nu_0, \Theta_0\right)}{\kappa^t} > 0 \ \forall\, t \geq \bar{t} \\
\Longrightarrow\ &\Pi\left(\nu_0 + \kappa^t \bar{\delta}, \Theta_0\right) > \Pi\left(\nu_0, \Theta_0\right) \ \forall\, t \geq \bar{t}.
\end{aligned}$$

Thus, there exists $\kappa \in \mathbf{R}$ with $0 < \kappa < 1$ such that $\Pi\left(\nu_0 + \kappa\bar{\delta}, \Theta_0\right) > \pi_0$. Hence, (6.2) implies that $\bar{\kappa} > 0$. ■

Proof of Lemma 1. Analogous to problem (5.5), it can be stated and shown that if $\Psi \cdot \varepsilon \geq 0$, then the constraint of problem (6.3) is binding at the optimum.[54] Hence, $\varepsilon \cdot \underline{q} = 0$. (6.4) then implies that $\nabla_\nu \Pi \cdot \mathbf{b} = 0$. Hence, $\nu_0 + \mathbf{b}$ lies on the tangent hyperplane of graph of function $\Pi(\cdot\ ,\ \Theta_0)$ passing through ν_0 and with normal $\nabla_\nu \Pi$. Strict concavity of function Π implies that the set $\geq_\Pi (\pi_0) := \{\delta \in \mathbf{R}^6 \setminus \{0_6\} \mid \Pi(\nu_0 + \delta, \Theta_0) \geq \pi_0\}$ is contained in the upper half-space $H_{++} := \{\delta \in \mathbf{R}^6 \mid \nabla_\nu \Pi \cdot \delta > 0\}$. Hence, $\mathbf{b} \notin \geq_\Pi (\pi_0)$. Hence, $\pi_{\mathbf{b}} := \Pi(\nu_0 + \mathbf{b}, \Theta_0) < \pi_0$. Since Ψ and ε are not proportional, analogous to Remark 2, it can be stated and shown that $-\Psi \cdot \underline{q} > 0$ (equivalently, $\Psi \cdot \underline{q} < 0$). (6.4) then implies that $\nabla_\nu \mathcal{Z} \cdot \mathbf{b} < 0$. From (6.1) it follows that $z_{\mathbf{b}} - z_0 < 0$. ■

Proof of Lemma 2. Let $\langle \delta_{\mathbf{a}}, \delta_{\mathbf{b}} \rangle \in \Omega_{\mathbf{a}} \times \Omega_{\mathbf{b}}$. Hence, there exist $\gamma_{\mathbf{a}}$ and $\gamma_{\mathbf{b}}$, both in $[0,1]$, such that $\delta_{\mathbf{a}} = \gamma_{\mathbf{a}}\mathbf{a} + (1-\gamma_{\mathbf{a}})\mathbf{b}$ and $\delta_{\mathbf{b}} = \gamma_{\mathbf{b}}\mathbf{a} + (1-\gamma_{\mathbf{b}})\mathbf{b}$. Hence, $u = \mathcal{U}(\delta_{\mathbf{a}}, \delta_{\mathbf{b}}) \equiv \bar{\gamma}\delta_{\mathbf{a}} + (1-\bar{\gamma})\delta_{\mathbf{b}}$ implies $u = \gamma'\mathbf{a} + (1-\gamma')\mathbf{b}$, where $\gamma' = \bar{\gamma}\gamma_{\mathbf{a}} + (1-\bar{\gamma})\gamma_{\mathbf{b}} > 0$ and $1 - \gamma' = (1-\gamma_{\mathbf{a}})\bar{\gamma} + (1-\bar{\gamma})(1-\gamma_{\mathbf{b}}) > 0$. Thus, $\gamma' \in [0,1]$. Hence, $\mathcal{U}(\delta_{\mathbf{a}}, \delta_{\mathbf{b}}) \in \Omega$.

We now show that $\mathcal{U}$ is a continuous function. Suppose $\bar{u}$ and $\langle \bar{\delta}_{\mathbf{a}}, \bar{\delta}_{\mathbf{b}} \rangle$ are such that $\bar{u} = \mathcal{U}\left(\bar{\delta}_{\mathbf{a}}, \bar{\delta}_{\mathbf{b}}\right) \equiv \bar{\gamma}\bar{\delta}_{\mathbf{a}} + (1-\bar{\gamma})\bar{\delta}_{\mathbf{b}}$. Given $\epsilon > 0$, we have to find $\eta > 0$ such that $\mathcal{U}(\delta_{\mathbf{a}}, \delta_{\mathbf{b}}) \in N_\epsilon(\bar{u})$ whenever $\langle \delta_{\mathbf{a}}, \delta_{\mathbf{b}} \rangle \in N_\eta\left(\bar{\delta}_{\mathbf{a}}, \bar{\delta}_{\mathbf{b}}\right)$. The latter is true when, employing the product metric ρ, we have

$$\rho\left(\langle \bar{\delta}_{\mathbf{a}}, \bar{\delta}_{\mathbf{b}} \rangle, \langle \delta_{\mathbf{a}}, \delta_{\mathbf{b}} \rangle\right) \equiv \|\bar{\delta}_{\mathbf{a}} - \delta_{\mathbf{a}}\| + \|\bar{\delta}_{\mathbf{b}} - \delta_{\mathbf{b}}\| < \eta. \tag{A.5}$$

Let $u := \mathcal{U}(\delta_{\mathbf{a}}, \delta_{\mathbf{b}}) \equiv \bar{\gamma}\delta_{\mathbf{a}} + (1-\bar{\gamma})\delta_{\mathbf{b}}$. Then

$$\begin{aligned}
\|\bar{u} - u\| &= \|\bar{\gamma}[\bar{\delta}_{\mathbf{a}} - \delta_{\mathbf{a}}] + (1-\bar{\gamma})[\bar{\delta}_{\mathbf{b}} - \delta_{\mathbf{b}}]\| \\
&\leq \bar{\gamma}\|\bar{\delta}_{\mathbf{a}} - \delta_{\mathbf{a}}\| + (1-\bar{\gamma})\|\bar{\delta}_{\mathbf{b}} - \delta_{\mathbf{b}}\| \text{ (employing triangular inequality)} \\
&< \bar{\gamma}[\eta - \|\bar{\delta}_{\mathbf{b}} - \delta_{\mathbf{b}}\|] + (1-\bar{\gamma})\|\bar{\delta}_{\mathbf{b}} - \delta_{\mathbf{b}}\| \ = \ \bar{\gamma}\eta + \|\bar{\delta}_{\mathbf{b}} - \delta_{\mathbf{b}}\|(1-2\bar{\gamma}) \\
&< \bar{\gamma}\eta + \eta(1-2\bar{\gamma}) \quad = \quad \eta(1-\bar{\gamma}),
\end{aligned} \tag{A.6}$$

[54] See Section 2A of the appendix.

where the last two inequalities follow from (A.5). Clearly, $\|\bar{u}-u\| < \epsilon$ whenever $\eta(1-\bar{\gamma}) < \epsilon$. Hence, given $\epsilon > 0$, choosing η such that $0 < \eta < \frac{\epsilon}{1-\bar{\gamma}}$ ensures that $\mathcal{U}(\delta_{\mathbf{a}}, \delta_{\mathbf{b}}) \in N_\epsilon(\bar{u})$, whenever $\langle \delta_{\mathbf{a}}, \delta_{\mathbf{b}} \rangle \in N_\eta\left(\bar{\delta}_{\mathbf{a}}, \bar{\delta}_{\mathbf{b}}\right)$. Thus, $\mathcal{U}$ satisfies the definition of a continuous function. ■

Proof of Lemma 3. Let $\langle \bar{\delta}_{\mathbf{a}}, \bar{\delta}_{\mathbf{b}} \rangle \in \Omega_{\mathbf{a}} \times \Omega_{\mathbf{b}}$ and $\bar{u} := \mathcal{U}\left(\bar{\delta}_{\mathbf{a}}, \bar{\delta}_{\mathbf{b}}\right) \equiv \bar{\gamma}\bar{\delta}_{\mathbf{a}} + (1-\bar{\gamma})\bar{\delta}_{\mathbf{b}}$. Consider a sequence $\left\{\langle \delta^t_{\mathbf{a}}, \delta^t_{\mathbf{b}} \rangle\right\} \longrightarrow \langle \bar{\delta}_{\mathbf{a}}, \bar{\delta}_{\mathbf{b}} \rangle$. We need to show that $\left\{\mathcal{T}\left(\delta^t_{\mathbf{a}}, \delta^t_{\mathbf{b}}\right)\right\} \longrightarrow \mathcal{T}\left(\bar{\delta}_{\mathbf{a}}, \bar{\delta}_{\mathbf{b}}\right)$. Three cases are possible:

(i) $\Pi\left(\nu_0 + \bar{u}, \Theta_0\right) > \overset{*}{\pi} \implies \mathcal{T}\left(\bar{\delta}_{\mathbf{a}}, \bar{\delta}_{\mathbf{b}}\right) = \langle \bar{u}, \bar{\delta}_{\mathbf{b}} \rangle$: Continuity of function Π implies there exists $\epsilon > 0$ such that $\Pi\left(\nu_0 + u,\ \Theta_0\right) > \overset{*}{\pi}$ whenever $u \in N_\epsilon(\bar{u})$. Continuity of function $\mathcal{U}$ (as proved in Lemma 2) implies that there exists $\eta > 0$ such that $\mathcal{U}(\delta_{\mathbf{a}}, \delta_{\mathbf{b}}) \in N_\epsilon(\bar{u})$ whenever $\langle \delta_{\mathbf{a}}, \delta_{\mathbf{b}} \rangle \in N_\eta\left(\bar{\delta}_{\mathbf{a}}, \bar{\delta}_{\mathbf{b}}\right)$. The convergence property of a sequence implies that there exists $T > 0$ such that $\langle \delta^t_{\mathbf{a}}, \delta^t_{\mathbf{b}} \rangle \in N_\eta\left(\bar{\delta}_{\mathbf{a}}, \bar{\delta}_{\mathbf{b}}\right)$ for all $t \geq T$. Hence, $\Pi\left(\nu_0 + \mathcal{U}\left(\delta^t_{\mathbf{a}}, \delta^t_{\mathbf{b}}\right), \Theta_0\right) > \overset{*}{\pi}$ for all $t \geq T$. Hence, $\mathcal{T}\left(\delta^t_{\mathbf{a}}, \delta^t_{\mathbf{b}}\right) = \left\langle \mathcal{U}\left(\delta^t_1, \delta^t_2\right), \delta^t_2 \right\rangle$ for all $t \geq T$. Continuity of $\mathcal{U}$, hence, implies that $\left\{\mathcal{T}\left(\delta^t_{\mathbf{a}}, \delta^t_{\mathbf{b}}\right)\right\} \longrightarrow \left\langle \mathcal{U}\left(\bar{\delta}_{\mathbf{a}}, \bar{\delta}_{\mathbf{b}}\right), \bar{\delta}_{\mathbf{b}} \right\rangle = \langle \bar{u}, \bar{\delta}_{\mathbf{b}} \rangle = \mathcal{T}\left(\bar{\delta}_{\mathbf{a}}, \bar{\delta}_{\mathbf{b}}\right)$.

(ii) $\Pi\left(\nu_0 + \bar{u}, \Theta_0\right) < \overset{*}{\pi} \implies \mathcal{T}\left(\bar{\delta}_{\mathbf{a}}, \bar{\delta}_{\mathbf{b}}\right) = \left\langle \bar{\delta}_{\mathbf{a}}, \bar{u} \right\rangle$: Proof is similar to case (i).

(iii) $\Pi\left(\nu_0 + \bar{u}, \Theta_0\right) = \overset{*}{\pi} \implies \mathcal{T}\left(\bar{\delta}_{\mathbf{a}}, \bar{\delta}_{\mathbf{b}}\right) = \langle \bar{u}, \bar{u} \rangle$: Choose ϵ such that $0 < \epsilon \leq \bar{\gamma}\|\bar{\delta}_{\mathbf{a}} - \bar{\delta}_{\mathbf{b}}\|$.

Step 1. We show that, if there exists $\eta > 0$ such that $\mathcal{T}(\delta_{\mathbf{a}}, \delta_{\mathbf{b}}) \in N_\epsilon\left(\mathcal{T}\left(\bar{\delta}_{\mathbf{a}}, \bar{\delta}_{\mathbf{b}}\right)\right)$ whenever $\langle \delta_{\mathbf{a}}, \delta_{\mathbf{b}} \rangle \in N_\eta\left(\bar{\delta}_{\mathbf{a}}, \bar{\delta}_{\mathbf{b}}\right)$, then $\mathcal{T}(\delta_{\mathbf{a}}, \delta_{\mathbf{b}}) = \langle u, u \rangle$ for all $\langle \delta_{\mathbf{a}}, \delta_{\mathbf{b}} \rangle \in N_\eta\left(\bar{\delta}_{\mathbf{a}}, \bar{\delta}_{\mathbf{b}}\right)$, where $u := \mathcal{U}(\delta_{\mathbf{a}}, \delta_{\mathbf{b}}) \equiv \bar{\gamma}\delta_{\mathbf{a}} + (1-\bar{\gamma})\delta_{\mathbf{b}}$. Suppose not. Then, there exists $\langle \delta_{\mathbf{a}}, \delta_{\mathbf{b}} \rangle \in N_\eta\left(\bar{\delta}_{\mathbf{a}}, \bar{\delta}_{\mathbf{b}}\right)$ such that either $\mathcal{T}(\delta_{\mathbf{a}}, \delta_{\mathbf{b}}) = \langle u, \delta_{\mathbf{b}} \rangle$ or $\mathcal{T}(\delta_{\mathbf{a}}, \delta_{\mathbf{b}}) = \langle \delta_{\mathbf{a}}, u \rangle$. WOLOG, assume $\mathcal{T}(\delta_{\mathbf{a}}, \delta_{\mathbf{b}}) = \langle u, \delta_{\mathbf{b}} \rangle$. Then,[55]

$$\begin{aligned}
\rho\left(\mathcal{T}\left(\bar{\delta}_{\mathbf{a}}, \bar{\delta}_{\mathbf{b}}\right), \mathcal{T}(\delta_{\mathbf{a}}, \delta_{\mathbf{b}})\right) &= \rho\left(\langle \bar{u}, \bar{u} \rangle, \langle u, \delta_{\mathbf{b}} \rangle\right) = \|\bar{u} - u\| + \|\bar{u} - \delta_{\mathbf{b}}\| \\
&< \eta(1-\bar{\gamma}) + \|\bar{u} - \delta_{\mathbf{b}}\| = \eta(1-\bar{\gamma}) + \|\bar{\gamma}\bar{\delta}_{\mathbf{a}} + (1-\bar{\gamma})\bar{\delta}_{\mathbf{b}} - \delta_{\mathbf{b}}\| \\
&\leq \eta(1-\bar{\gamma}) + \bar{\gamma}\|\bar{\delta}_{\mathbf{a}} - \bar{\delta}_{\mathbf{b}}\| + \|\bar{\delta}_{\mathbf{b}} - \delta_{\mathbf{b}}\| \\
&< \eta(1-\bar{\gamma}) + \bar{\gamma}\|\bar{\delta}_{\mathbf{a}} - \bar{\delta}_{\mathbf{b}}\| + \eta \quad = \quad \eta(2-\bar{\gamma}) + \bar{\gamma}\|\bar{\delta}_{\mathbf{a}} - \bar{\delta}_{\mathbf{b}}\|
\end{aligned}$$

Thus, $\rho\left(\mathcal{T}\left(\bar{\delta}_{\mathbf{a}}, \bar{\delta}_{\mathbf{b}}\right), \mathcal{T}(\delta_{\mathbf{a}}, \delta_{\mathbf{b}})\right) < \epsilon$ if $\eta(2-\bar{\gamma}) + \bar{\gamma}\|\bar{\delta}_{\mathbf{a}} - \bar{\delta}_{\mathbf{b}}\| < \epsilon \leq \bar{\gamma}\|\bar{\delta}_{\mathbf{a}} - \bar{\delta}_{\mathbf{b}}\|$. But this implies $\eta < 0$, which is a contradiction. The same contradiction can be demonstrated if $\mathcal{T}(\delta_{\mathbf{a}}, \delta_{\mathbf{b}}) = \langle \delta_{\mathbf{a}}, u \rangle$. Hence, $\mathcal{T}(\delta_{\mathbf{a}}, \delta_{\mathbf{b}}) = \langle u, u \rangle$ for all $\langle \delta_{\mathbf{a}}, \delta_{\mathbf{b}} \rangle \in N_\eta\left(\bar{\delta}_{\mathbf{a}}, \bar{\delta}_{\mathbf{b}}\right)$.

Step 2. Given $0 < \epsilon \leq \bar{\gamma}\|\bar{\delta}_{\mathbf{a}} - \bar{\delta}_{\mathbf{b}}\|$, we now show the existence of $\eta > 0$ such that $\mathcal{T}(\delta_{\mathbf{a}}, \delta_{\mathbf{b}}) \in N_\epsilon\left(\mathcal{T}\left(\bar{\delta}_{\mathbf{a}}, \bar{\delta}_{\mathbf{b}}\right)\right)$ whenever $\langle \delta_{\mathbf{a}}, \delta_{\mathbf{b}} \rangle \in N_\eta\left(\bar{\delta}_{\mathbf{a}}, \bar{\delta}_{\mathbf{b}}\right)$. Step 1 showed that, for the η with the required property, we have $\mathcal{T}(\delta_{\mathbf{a}}, \delta_{\mathbf{b}}) = \langle u, u \rangle$ whenever $\langle \delta_{\mathbf{a}}, \delta_{\mathbf{b}} \rangle \in N_\eta\left(\bar{\delta}_{\mathbf{a}}, \bar{\delta}_{\mathbf{b}}\right)$, where $u := \mathcal{U}(\delta_{\mathbf{a}}, \delta_{\mathbf{b}}) \equiv \bar{\gamma}\delta_{\mathbf{a}} + (1-\bar{\gamma})\delta_{\mathbf{b}}$. Thus,

$$\rho\left(\langle \bar{u}, \bar{u} \rangle, \langle u, u \rangle\right) = 2\|\bar{u} - u\| < 2\eta(1-\bar{\gamma}).$$

[55] We employ (A.6) to compute $\|\bar{u} - u\|$. Also, (A.5) implies $\|\bar{\delta}_{\mathbf{b}} - \delta_{\mathbf{b}}\| < \eta$.

Therefore, $\rho\left(\langle \bar{u}, \bar{u}\rangle, \langle u, u\rangle\right) < \epsilon$ whenever $2\eta(1-\bar{\gamma}) < \epsilon$, *i.e.,* whenever $0 < \eta < \frac{\epsilon}{2(1-\bar{\gamma})}$.
Step 3. Choose η such that $0 < \eta < \frac{\epsilon}{2(1-\bar{\gamma})}$. Then, from the convergence property of a sequence, there exists $T > 0$ such that $\left\langle \delta^t_{\mathbf{a}}, \delta^t_{\mathbf{b}}\right\rangle \in N_\eta\left(\bar{\delta}_{\mathbf{a}}, \bar{\delta}_{\mathbf{b}}\right)$ for all $t \geq T$. Hence, steps 1 and 2 imply that $\mathcal{T}\left(\delta^t_{\mathbf{a}}, \delta^t_{\mathbf{b}}\right) = \left\langle \mathcal{U}\left(\delta^t_{\mathbf{a}}, \delta^t_{\mathbf{b}}\right), \mathcal{U}\left(\delta^t_{\mathbf{a}}, \delta^t_{\mathbf{b}}\right)\right\rangle$ for all $t \geq T$. Continuity of $\mathcal{U}$, hence, implies that $\left\{\mathcal{T}\left(\delta^t_{\mathbf{a}}, \delta^t_{\mathbf{b}}\right)\right\} \longrightarrow \left\langle \mathcal{U}\left(\bar{\delta}_{\mathbf{a}}, \bar{\delta}_{\mathbf{b}}\right), \mathcal{U}\left(\bar{\delta}_{\mathbf{a}}, \bar{\delta}_{\mathbf{b}}\right)\right\rangle = \langle \bar{u}, \bar{u}\rangle = \mathcal{T}\left(\bar{\delta}_{\mathbf{a}}, \bar{\delta}_{\mathbf{b}}\right)$. ■

Proof of Lemma 4: Suppose $\tilde{\delta} \neq \hat{\delta}$. Since both lie in Ω, there exist $\tilde{\gamma}$ and $\hat{\gamma}$ in $(0,1)$ and $\tilde{\gamma} \neq \hat{\gamma}$ and $\tilde{\delta} = \tilde{\gamma}\mathbf{a} + (\mathbf{1}-\tilde{\gamma})\mathbf{b}$ and $\hat{\delta} = \hat{\gamma}\mathbf{a} + (\mathbf{1}-\hat{\gamma})\mathbf{b}$. WOLOG assume $\tilde{\gamma} > \hat{\gamma}$. Then there exists $\gamma \in (0,1)$ such that $\tilde{\gamma} = \gamma + \hat{\gamma}(1-\gamma)$, namely, $\gamma = \frac{\tilde{\gamma}-\hat{\gamma}}{1-\hat{\gamma}}$. Then $\tilde{\delta} = \gamma\mathbf{a} + (\mathbf{1}-\gamma)\hat{\delta}$. Strict concavity of the profit function then implies $\Pi\left(\nu_0 + \hat{\delta}, \Theta_0\right) > \overset{*}{\pi}$, which is a contradiction. ■

Proof of Lemma 5. Note $\gamma \geq \overset{*}{\gamma}$ if and only if there exists $\gamma' \in (0,1)$ such that $\delta = \gamma'\mathbf{a} + (\mathbf{1}-\gamma')\overset{*}{\delta}$. (Simply choose γ' that solves $\gamma = \gamma' + (1-\gamma')\overset{*}{\gamma}$, namely, $\gamma' = \frac{\gamma-\overset{*}{\gamma}}{1-\overset{*}{\gamma}}$.) There exists $\gamma' \in (0,1)$ such that $\delta = \gamma'\mathbf{a} + (\mathbf{1}-\gamma')\overset{*}{\delta}$ if and only if (i) $\gamma = \overset{*}{\gamma}$ or (ii) $\gamma > \overset{*}{\gamma}$ are true. If (i) then $\gamma = \overset{*}{\gamma} \iff \gamma' = 0 \iff \delta = \overset{*}{\delta}$. If (ii) then $\gamma > \overset{*}{\gamma} \iff \gamma' > 0$. Given strict concavity of the profit function, this is true if and only if $\Pi\left(\nu_0 + \delta, \Theta_0\right) > \overset{*}{\pi}$. Hence, $\gamma \geq \overset{*}{\gamma} \iff \delta \in \Omega_{\mathbf{a}}$.

From this it follows that $\delta \notin \Omega_{\mathbf{a}} \iff \Pi\left(\nu_0 + \delta, \Theta_0\right) < \overset{*}{\pi} \iff \gamma < \overset{*}{\gamma}$. Combined with the fact that $\Pi\left(\nu_0 + \overset{*}{\delta}, \Theta_0\right) = \overset{*}{\pi}$, it follows that $\gamma \leq \overset{*}{\gamma} \iff \delta \in \Omega_{\mathbf{b}}$. ■

Proof of Theorem 6: For all $t \geq 0$, define $u^t := \mathcal{U}\left(\delta^t_{\mathbf{a}}, \delta^t_{\mathbf{b}}\right)$ and define $\gamma^t_{\mathbf{a}}$ and $\gamma^t_{\mathbf{b}}$ so that they solve $\delta^t_{\mathbf{a}} = \gamma^t_{\mathbf{a}}\mathbf{a} + (\mathbf{1}-\gamma^{\mathbf{t}}_{\mathbf{a}})\mathbf{b}$ and $\delta^t_{\mathbf{b}} = \gamma^t_{\mathbf{b}}\mathbf{a} + (\mathbf{1}-\gamma^{\mathbf{t}}_{\mathbf{b}})\mathbf{b}$, resp. Two cases arise:
<u>*Case 1.*</u> There exists $t' > 0$ such that $\Pi\left(\nu_0 + u^{t'}, \Theta_0\right) = \overset{*}{\pi}$. This implies $\langle \delta^{t'+1}_{\mathbf{a}}, \delta^{t'+1}_{\mathbf{b}}\rangle \equiv \mathcal{T}\left(\delta^{t'}_{\mathbf{a}}, \delta^{t'}_{\mathbf{b}}\right) = \langle u^{t'}, u^{t'}\rangle$. Hence, from the definition of $\mathcal{T}$ it follows that for all $t \geq t'$, we have $\mathcal{T}\left(\delta^t_{\mathbf{a}}, \delta^t_{\mathbf{b}}\right) = \langle u^{t'}, u^{t'}\rangle$. Moreover, Lemma 4 implies that $u^{t'} = \overset{*}{\delta}$. Thus, we conclude that $\left\{\langle \delta^t_{\mathbf{a}}, \delta^t_{\mathbf{b}}\rangle\right\} \longrightarrow \langle \overset{*}{\delta}, \overset{*}{\delta}\rangle$.
<u>*Case 2.*</u> For all $t \geq 0$, $\Pi\left(\nu_0 + u^t, \Theta_0\right) \neq \overset{*}{\pi}$. This implies that for any $t \geq 0$ one and only one of the following is true.

(i) $\langle \delta^{t+1}_{\mathbf{a}}, \delta^{t+1}_{\mathbf{b}}\rangle \equiv \mathcal{T}\left(\delta^t_{\mathbf{a}}, \delta^t_{\mathbf{b}}\right) = \langle u^t, \delta^t_{\mathbf{b}}\rangle$ if $\Pi\left(\nu_0 + u^t, \Theta_0\right) > \overset{*}{\pi}$
(ii) $\langle \delta^{t+1}_{\mathbf{a}}, \delta^{t+1}_{\mathbf{b}}\rangle \equiv \mathcal{T}\left(\delta^t_{\mathbf{a}}, \delta^t_{\mathbf{b}}\right) = \langle \delta^t_{\mathbf{a}}, u^t\rangle$ if $\Pi\left(\nu_0 + u^t, \Theta_0\right) < \overset{*}{\pi}$.

Suppose (i) is true at time point t. Since $u^t = \bar{\gamma}\delta^t_{\mathbf{a}} + (1-\bar{\gamma})\delta^t_{\mathbf{b}}$ with $\bar{\gamma} \in (0,1)$, we have $u^t = \bar{\gamma}[\gamma^t_{\mathbf{a}}\mathbf{a} + (\mathbf{1}-\gamma^{\mathbf{t}}_{\mathbf{a}})\mathbf{b}] + (\mathbf{1}-\bar{\gamma})[\gamma^{\mathbf{t}}_{\mathbf{b}}\mathbf{a} + (\mathbf{1}-\gamma^{\mathbf{t}}_{\mathbf{b}})\mathbf{b}]$. This implies that

$$u^t = [\bar{\gamma}\gamma^t_{\mathbf{a}} + (1-\bar{\gamma})\gamma^t_{\mathbf{b}}]\mathbf{a} + [\bar{\gamma}(\mathbf{1}-\gamma^{\mathbf{t}}_{\mathbf{a}}) + (\mathbf{1}-\bar{\gamma})(\mathbf{1}-\gamma^{\mathbf{t}}_{\mathbf{b}})]\mathbf{b}. \tag{A.7}$$

Since $u^t \in \Omega_{\mathbf{a}}$ and $\Pi\left(\nu_0 + u^t, \Theta_0\right) > \overset{*}{\pi}$, we have $u^t \neq \overset{*}{\delta}$. Combined with Lemma 5 and (A.7), this implies

$$\bar{\gamma}\gamma_{\mathbf{a}}^t + (1-\bar{\gamma})\gamma_{\mathbf{b}}^t > \overset{*}{\gamma}. \tag{A.8}$$

Since $\delta_{\mathbf{a}}^t \in \Omega_{\mathbf{a}}$ and $\delta_{\mathbf{b}}^t \in \Omega_{\mathbf{b}}$, Lemma 5 implies $\gamma_{\mathbf{a}}^t \geq \overset{*}{\gamma} \geq \gamma_{\mathbf{b}}^t$. Moreover, $\gamma_{\mathbf{a}}^t \neq \gamma_{\mathbf{b}}^t$.[56] Hence, (A.8) is true if and only if[57]

$$\gamma_{\mathbf{a}}^t > \bar{\gamma}\gamma_{\mathbf{a}}^t + (1-\bar{\gamma})\gamma_{\mathbf{b}}^t > \overset{*}{\gamma} \implies \gamma_{\mathbf{a}}^t > \overset{*}{\gamma} \geq \gamma_{\mathbf{b}}^t. \tag{A.9}$$

Since, $\delta_{\mathbf{a}}^{t+1} = u^t \in \Omega_{\mathbf{a}}$ and $\Pi\left(\nu_0 + u^t, \Theta_0\right) > \overset{*}{\pi}$, Lemma 5 implies that $\gamma_{\mathbf{a}}^{t+1} > \overset{*}{\gamma}$. Since $\delta_{\mathbf{b}}^{t+1} = \delta_{\mathbf{b}}^t \in \Omega_{\mathbf{b}}$, Lemma 5 also implies that $\gamma_{\mathbf{b}}^{t+1} \leq \overset{*}{\gamma}$. Moreover, $\delta_{\mathbf{b}}^{t+1} = \delta_{\mathbf{b}}^t$ implies that $\gamma_{\mathbf{b}}^t = \gamma_{\mathbf{b}}^{t+1}$. Hence,

$$\gamma_{\mathbf{a}}^{t+1} > \overset{*}{\gamma} \geq \gamma_{\mathbf{b}}^{t+1} = \gamma_{\mathbf{b}}^t. \tag{A.10}$$

Since $u^t = \delta_{\mathbf{a}}^{t+1}$, we have $u^t = \gamma_{\mathbf{a}}^{t+1}\mathbf{a} + (\mathbf{1} - \gamma_{\mathbf{a}}^{\mathbf{t+1}})\mathbf{b}$. Hence, from (A.7), we have

$$\gamma_{\mathbf{a}}^{t+1}\mathbf{a} + (\mathbf{1} - \gamma_{\mathbf{a}}^{\mathbf{t+1}})\mathbf{b} = \left[\bar{\gamma}\gamma_{\mathbf{a}}^t + (1-\bar{\gamma})\gamma_{\mathbf{b}}^t\right]\mathbf{a} \;+\; \left[\bar{\gamma}(\mathbf{1} - \gamma_{\mathbf{a}}^{\mathbf{t}}) + (\mathbf{1} - \bar{\gamma})(\mathbf{1} - \gamma_{\mathbf{b}}^{\mathbf{t}})\right]\mathbf{b}. \tag{A.11}$$

From (A.11) it follows that

$$\gamma_{\mathbf{a}}^{t+1} = \bar{\gamma}\gamma_{\mathbf{a}}^t + (1-\bar{\gamma})\gamma_{\mathbf{b}}^t. \tag{A.12}$$

Hence, (A.9), (A.10), and (A.12) imply that

$$\gamma_{\mathbf{b}}^t = \gamma_{\mathbf{b}}^{t+1} \leq \overset{*}{\gamma} < \gamma_{\mathbf{a}}^{t+1} < \gamma_{\mathbf{a}}^t. \tag{A.13}$$

Proceeding in an exactly similar manner, we can show that, if (ii) is true at time point t, then

$$\gamma_{\mathbf{b}}^t < \gamma_{\mathbf{b}}^{t+1} < \overset{*}{\gamma} \leq \gamma_{\mathbf{a}}^{t+1} = \gamma_{\mathbf{a}}^t. \tag{A.14}$$

Hence, we conclude that, in Case 2, $\{\gamma_{\mathbf{a}}^t\}$ is a non-increasing sequence and $\{\gamma_{\mathbf{b}}^t\}$ is a non-decreasing sequence in the compact set $[0,1]$. Hence, the two sequences converge. Given $\delta_{\mathbf{a}}$ and $\delta_{\mathbf{b}}$ are continuous functions of $\gamma_{\mathbf{a}}$ and $\gamma_{\mathbf{b}}$, respectively, we have:

$$\{\langle\gamma_{\mathbf{a}}^t,\ \gamma_{\mathbf{b}}^t\rangle\} \longrightarrow \langle\overset{*}{\gamma}_{\mathbf{a}}, \overset{*}{\gamma}_{\mathbf{b}}\rangle \in [0,1]\times[0,1] \implies \{\langle\delta_{\mathbf{a}}^t,\ \delta_{\mathbf{b}}^t\rangle\} \longrightarrow \langle\overset{*}{\delta}_{\mathbf{a}}, \overset{*}{\delta}_{\mathbf{b}}\rangle \in \Omega_{\mathbf{a}}\times\Omega_{\mathbf{b}}. \tag{A.15}$$

We show that $\overset{*}{\gamma}_{\mathbf{a}} = \overset{*}{\gamma}_{\mathbf{b}} = \overset{*}{\gamma}$ or, equivalently, $\overset{*}{\delta}_{\mathbf{a}} = \overset{*}{\delta}_{\mathbf{b}} = \overset{*}{\delta}$. Since $\mathcal{T}$ is a continuous function (see Lemma 3), (A.15) implies

$$\left\{\mathcal{T}\left(\delta_{\mathbf{a}}^t,\ \delta_{\mathbf{b}}^t\right)\right\} \longrightarrow \mathcal{T}\left(\overset{*}{\delta}_{\mathbf{a}}, \overset{*}{\delta}_{\mathbf{b}}\right). \tag{A.16}$$

[56] If not then, $\delta_{\mathbf{a}}^t = \delta_{\mathbf{b}}^t$, which implies $u^t = \delta_{\mathbf{a}}^t = \delta_{\mathbf{b}}^t$ and $u^t \in \Omega_{\mathbf{a}} \cap \Omega_{\mathbf{b}}$. This is true if and only if $\Pi\left(\nu_0 + u^t, \Theta_0\right) = \overset{*}{\pi}$, which is a contradiction to maintained assumption in (i).

[57] Suppose $\gamma_{\mathbf{a}}^t \leq \bar{\gamma}\gamma_{\mathbf{a}}^t + (1-\bar{\gamma})\gamma_{\mathbf{b}}^t$. This implies $\gamma_{\mathbf{a}}^t \leq \gamma_{\mathbf{b}}^t$, which contradicts $\gamma_{\mathbf{a}}^t \geq \gamma_{\mathbf{b}}^t$ and $\gamma_{\mathbf{a}}^t \neq \gamma_{\mathbf{b}}^t$.

(A.15) also implies that for every $\eta > 0$ there exists $T \geq 0$ such that for all $t \geq T$, we have $\langle \delta^t_{\mathbf{a}}, \delta^t_{\mathbf{b}} \rangle \in N_\eta \left(\overset{*}{\delta}_{\mathbf{a}}, \overset{*}{\delta}_{\mathbf{b}} \right)$. But since $\langle \delta^t_{\mathbf{a}}, \delta^t_{\mathbf{b}} \rangle = \mathcal{T}\left(\delta^{t-1}_{\mathbf{a}}, \delta^{t-1}_{\mathbf{b}} \right)$ for all $t \geq 0$, we have $\mathcal{T}\left(\delta^{t-1}_{\mathbf{a}}, \delta^{t-1}_{\mathbf{b}} \right) \in N_\eta \left(\overset{*}{\delta}_{\mathbf{a}}, \overset{*}{\delta}_{\mathbf{b}} \right)$ for all $t \geq T$. Equivalently, $\mathcal{T}\left(\delta^{t}_{\mathbf{a}}, \delta^{t}_{\mathbf{b}} \right) \in N_\eta \left(\overset{*}{\delta}_{\mathbf{a}}, \overset{*}{\delta}_{\mathbf{b}} \right)$ for all $t \geq T' := T - 1$. Hence, for every $\eta > 0$ there exists $T' > 0$ such that for all $t \geq T'$, we have $\mathcal{T}\left(\delta^{t}_{\mathbf{a}}, \delta^{t}_{\mathbf{b}} \right) \in N_\eta \left(\overset{*}{\delta}_{\mathbf{a}}, \overset{*}{\delta}_{\mathbf{b}} \right)$. Hence,

$$\left\{ \mathcal{T}\left(\delta^t_{\mathbf{a}},\ \delta^t_{\mathbf{b}} \right) \right\} \longrightarrow \langle \overset{*}{\delta}_{\mathbf{a}}, \overset{*}{\delta}_{\mathbf{b}} \rangle. \tag{A.17}$$

From (A.16) and (A.17) it follows that

$$\mathcal{T}\left(\overset{*}{\delta}_{\mathbf{a}}, \overset{*}{\delta}_{\mathbf{b}} \right) = \langle \overset{*}{\delta}_{\mathbf{a}}, \overset{*}{\delta}_{\mathbf{b}} \rangle, \tag{A.18}$$

i.e., $\langle \overset{*}{\delta}_{\mathbf{a}}, \overset{*}{\delta}_{\mathbf{b}} \rangle$ is a fixed point of $\mathcal{T}$. Hence, Theorem 5 implies that $\overset{*}{\delta}_{\mathbf{a}} = \overset{*}{\delta}_{\mathbf{b}} = \overset{*}{\delta}$. Since, by construction, $\left\langle \delta^{t+1}_{\mathbf{a}}, \delta^{t+1}_{\mathbf{b}} \right\rangle = \mathcal{T}\left(\delta^t_{\mathbf{a}}, \delta^t_{\mathbf{b}} \right)$ for all $t \geq 0$, (A.16) and (A.18) imply $\left\{ \langle \delta^t_{\mathbf{a}}, \delta^t_{\mathbf{b}} \rangle \right\} \longrightarrow \langle \overset{*}{\delta}_{\mathbf{a}}, \overset{*}{\delta}_{\mathbf{b}} \rangle = \langle \overset{*}{\delta}, \overset{*}{\delta} \rangle$. ■

A2. Characterising the solutions of problems (5.5) and (6.3).

Let $s = \langle \nu_0, z_0, \Theta_0 \rangle$ be a tight status-quo. Suppose $\bar{q} \in \mathbf{R}^6$ solves

$$\max_{q \in \mathbf{R}^6} \left\{ \varepsilon \cdot q \;\middle|\; \Psi \cdot q \leq 0 \quad \text{and} \quad q \cdot q \leq 1 \right\} \tag{A.19}$$

and $\underline{q} \in \mathbf{R}^6$ solves

$$\max_{q \in \mathbf{R}^6} \left\{ -\Psi \cdot q \;\middle|\; \varepsilon \cdot q \geq 0 \quad \text{and} \quad q \cdot q \leq 1 \right\}. \tag{A.20}$$

The Lagrangian for (A.19) and (A.20) are, resp.,

$$\tilde{L} = \varepsilon \cdot q - \tilde{\mu} \Psi \cdot q - \tilde{\lambda} q \cdot q \quad \text{and}$$
$$\hat{L} = -\Psi \cdot q + \hat{\mu} \varepsilon \cdot q - \hat{\lambda} q \cdot q.$$

At the optima of (A.19) and (A.20) suppose the Lagrange multipliers take the values $\bar{\tilde{\mu}}$, $\underline{\hat{\mu}}$, $\bar{\tilde{\lambda}}$, and $\underline{\hat{\lambda}}$. Then,

$$\begin{aligned}
\bar{\tilde{\mu}} &= \frac{\varepsilon \cdot \Psi}{\Psi \cdot \Psi} \qquad \text{if} \qquad \varepsilon \cdot \Psi \geq 0 \\
&= 0 \qquad\qquad \text{if} \qquad \varepsilon \cdot \Psi < 0 \\
\underline{\hat{\mu}} &= \frac{\varepsilon \cdot \Psi}{\varepsilon \cdot \varepsilon} \qquad \text{if} \qquad \varepsilon \cdot \Psi \geq 0 \\
&= 0 \qquad\qquad \text{if} \qquad \varepsilon \cdot \Psi < 0.
\end{aligned}$$

Case 1: Suppose $\varepsilon \cdot \Psi < 0$. Then

$$\Psi \cdot \bar{q} < 0, \qquad \varepsilon \cdot \underline{q} > 0$$

$$\bar{q} = \frac{\varepsilon}{\sqrt{\varepsilon \cdot \varepsilon}}, \qquad \bar{\bar{\lambda}} = \frac{1}{2}\sqrt{\varepsilon \cdot \varepsilon}, \qquad \underline{q} = \frac{-\Psi}{\sqrt{\Psi \cdot \Psi}}, \qquad \underline{\hat{\lambda}} = \frac{1}{2}\sqrt{\Psi \cdot \Psi}. \tag{A.21}$$

Case 2: Suppose $\varepsilon \cdot \Psi \geq 0$. Then two cases become possible:

(i) ε and Ψ are not proportional.

In this case

$$\bar{q} = \frac{\varepsilon - \bar{\bar{\mu}}\Psi}{\sqrt{[\varepsilon - \bar{\bar{\mu}}\Psi] \cdot [\varepsilon - \bar{\bar{\mu}}\Psi]}}, \qquad \bar{\bar{\lambda}} = \frac{1}{2}\sqrt{[\varepsilon - \bar{\bar{\mu}}\Psi] \cdot [\varepsilon - \bar{\bar{\mu}}\Psi]},$$
$$\underline{q} = \frac{-\Psi + \underline{\hat{\mu}}\varepsilon}{\sqrt{[-\Psi + \underline{\hat{\mu}}\varepsilon] \cdot [-\Psi + \underline{\hat{\mu}}\varepsilon]}}, \qquad \underline{\hat{\lambda}} = \frac{1}{2}\sqrt{[-\Psi + \underline{\hat{\mu}}\varepsilon] \cdot [-\Psi + \underline{\hat{\mu}}\varepsilon]}.$$

(ii) ε and Ψ are proportional.

In this case any $q \in \mathbf{R}^6$ such that $\varepsilon \cdot q = 0$ and $q \cdot q \leq 1$ is a solution to (A.19). In particular, 0_6 is a solution to (A.19). Similarly, any q such that $-\Psi \cdot q = 0$ and $q \cdot q \leq 1$ is a solution to (A.20). In particular, 0_6 is a solution to (A.20). Thus, we have

$$\bar{q} \in \{q \in \mathbf{R}^n \mid \varepsilon \cdot q = 0 \text{ and } q \cdot q \leq 1\},$$
$$\underline{q} \in \{q \in \mathbf{R}^n \mid -\Psi \cdot q = 0 \text{ and } q \cdot q \leq 1\}, \qquad \text{and}$$
$$\bar{\bar{\lambda}} = \underline{\hat{\lambda}} = 0.$$

The signs of the objective functions of (A.19) and (A.20) at the optima (*i.e.,* the signs of $\varepsilon \cdot \bar{q}$ and $-\Psi \cdot \underline{q}$, resp.) are:

$$\begin{aligned} &\varepsilon \cdot \bar{q} > 0 \quad \text{and} \quad -\Psi \cdot \underline{q} > 0 \quad \text{if} \quad \varepsilon \neq \kappa\Psi \quad \text{for all } \kappa > 0 \\ &\varepsilon \cdot \bar{q} = 0 \quad \text{and} \quad -\Psi \cdot \underline{q} = 0 \quad \text{if} \quad \varepsilon = \kappa\Psi \quad \text{for some } \kappa > 0. \end{aligned} \tag{A.22}$$

A3. Data description and country tables with full set of results.

Our sample consists of 118 countries – 26 countries from Africa, 31 from Asia, 39 from Europe, 13 from North America, 7 from South America, and 2 from Oceania.

Data on GDP, capital, labour, and energy.

The data on GDP, capital, and labour for these countries was derived from Version 4 of the Extended Penn Tables (EPWT) for four years, 1990, 2000, 2005, and 2010. Real GDP and capital are measured in 2005 purchasing power parity in USD, while labour is measured as number of workers.

Data on total energy use and its split into fossil-fuel and alternative (including nuclear and renewable) energy sources was obtained from the World Development Indicators (WDI), published by the World Bank and International Energy Agency (IEA), for four years 1990, 2000, 2005, and 2010. Energy is measured in kilotonnes of oil equivalents (ktoe). The split of fossil fuel energy into coal, gas, and oil was obtained from the country tables of the US Energy Information Agency (US-EIA). A similar country-level detailed split of energy from alternative sources into independent sources such as nuclear, hydro, wind, solar, *etc.*, was not available for an extensive coverage of countries.

Emission data and emission factors.

Country-level data on CO_2 emissions from fossil-fuel combustion was obtained from IEA for 2010. For parity with carbon sequestration by forests, the emissions of CO_2 are converted into million tonnes of carbon (mtc) by multiplying them by a factor, 12/44.[58] The emission factors α_c, α_g, and α_o, for coal, gas, and oil, respectively, are derived from IPCC (2006) guidelines. Table 2.3 of these guidelines was employed to derive an emission factors representative of these three fossil fuels. The emission factors for coal, gas, and oil, were taken to be 100,000, 57,000, and 75,000 kilograms of CO_2 per tera joules of energy, respectively. Employing appropriate conversion factors, these amount to $\alpha_c = 0.001141854$, $\alpha_g = 0.000650857$, and $\alpha_o = 0.000856391$ mtc per ktoe of coal, gas, and oil, respectively.

Forest data and computation of carbon sequestration factors.

Country-level forest data was obtained from the Global Forest Resources Assessment (FRA) 2010 of the Food and Agricultural Organisation. In its guidelines for country reporting (2008), FRA asks countries to estimate and report the carbon content of their forests in 2010 in accordance with the guidelines given in IPCC (2006). Measures of carbon sequestered in forests are based on the estimates of forest biomass, which in turn are derived from data on the growing stock (volume) of forests measured in million meter-cube (mm3). The growing stock of forests is converted into forest biomass, based on conversion factors given in IPCC (2006). Forest biomass, measured in millions of tonnes (mt), is a sum of the above ground biomass (AGB), the below ground biomass (BGB), and dead wood.[59] A biomass expansion and contraction factor (BECF) converts the forest stock into AGB. The BGB is obtained by multiplying the AGB by the root-shoot ratio (RSR). The forest biomass is converted into carbon stock in forest, measured in mtc, by multiplying the former by a carbon fraction (CF). In general, the BCEF, the RSR, and

[58] See *e.g.,* EPA brochure: http://www.epa.gov/cpd/pdf/brochure.pdf.
[59] In our analysis we omit the dead wood component as it is usually small for countries.

the CF depend on the type of trees and other vegetation in forests, and hence will be forest-type and more generally country-specific. IPCC (2006) provides a global default value of CF equal to 0.47, though many countries prefer to measure this independently.

Since carbon sequestration depends ultimately on the volume (stock) of forests, we measure the extent of forests of any country, and its afforestation and deforestation in mm3. Thus, the carbon sequestration factor s in our theoretical model is defined as the amount of carbon sequestered per unit volume of forest and is measured in mtc per mm3. From the discussion above, it is derived as $s = BECF \times (1 + RSR) \times CF$. Individual country reports of FRA were employed to compute, for each country, its BECF as the ratio of the reported volume of forests and the amount of AGB. RSR was computed as the ratio of reported amounts of BGB and AGB. Finally, country statements on CF, in combination with the derived BCEF and RSR, were employed to derive the value of s for each country.

Given time series of data on forest stock from FRA, the change in the forest stock over time is computed. The change in the forest stock could be attributed to afforestation, deforestation, and natural regeneration. FRA does not provide independent information on these three components.[60] As a result, we make the following assumptions: We assume that there is some positive afforestation in every country.[61] Change in the forest stock during the period 2009-2010 is computed. If the change in the forest stock is positive, it is attributed completely to afforestation. If it is found to be negative then a country is assigned a small amount of afforestation, which is assumed to be 0.0001 mm3 by default.

Price data and imputations of energy costs and expenditures on afforestation.

Cost of energy.

International data on the price of labour, the wage rate, is obtained from EPWT [2014] in 2005 USD purchasing power parity. Prices of various sources of energy (excluding oil) are derived from the Projected Costs of Generating Electricity (PCGE) [2010] published by IEA and the Nuclear Energy Agency (NEA). This report includes 21 countries (16 OECD; 3 non-member countries including Brazil, Russia, and South Africa; industry participants from Australia, France, and European Union; and some plants from China). It is based on cost data for 190 power plants from these countries. The study computes levelised costs of producing electricity (LCOE) from various sources – coal, gas, nuclear, and renewables. LCOE includes investment costs, fuel costs, operations and maintenance costs, etc. For fossil fuels, an additional carbon (externality) cost is added for some (mainly OECD)

[60] FRA provides information on the increase in area of planted forests, but the conversion of forest area into forest volume is very much dependent on the forest type and the tree species involved. Factors that convert a country's forest area into its growing stock are not readily available. As discussed above, carbon sequestration depends on the extent of the growing stock of forest.

[61] We do not study reforms in the deforestation policy.

countries. This is like a fixed Pigouvian tax levied per unit of emission. However, in our model, the emission-policy instrument is a carbon cap, which makes the Pigouvian tax redundant. Hence, in our analysis, we consider the LCOE net of the carbon cost. In this report, LCOE are computed in USD per mega-watt-hours of electricity generated. We convert this into USD per ktoe of electricity generated employing appropriate conversion factors. In our analysis we employ the LCOE computed at a 5% discount rate.[62] The study demonstrates great variations in the LCOE across regions, depending upon regional differences in factors such as availability of low cost fuels (some regions may be richly endowed with fuels, while other have to import them at higher prices), natural endowments of renewable resources, capital and other input costs, etc.

In the absence of other data sources that estimate LCOE for an extensive coverage of countries, we extrapolate the findings of the PCGE report to countries outside its sample of study. *E.g.,* with respect to coal-fired power plants, the extrapolation scheme is as follows: We impute the LCOE in China to all countries in Asia, except the Middle Eastern countries, Japan, South Korea, and countries in the former Soviet Block. Estimates for Japan and South Korea are provided by PCGE. To the countries in the former Soviet Block in both Asia and Europe we impute Russia's LCOE. The average LCOE for plants in Europe in the sample considered by PCGE is imputed to all countries in Europe except for those for which we have data from PCGE. Coal does not seem to be an important source of electrical energy in most African and Gulf countries due to their poor endowment of coal. These countries rely on gas-powered or other renewable sources of energy (such as hydroelectricity). At the same time, we assume that other input costs such as labour and capital costs in these countries are closer to those in Asia. Hence, we impute Japan's LCOE (which is the highest in Asia) to all countries in Africa that produce some coal-fired electricity, except South Africa, for which PCGE provides estimates. South Africa seems to be an exception in Africa in this regards, as it produces and imports coal-fired electricity. To all countries in North America, we impute the LCOE of either USA or Mexico, which are provided by PCGE. To all countries in South America, we impute the LCOE of Brazil, and to countries in Oceania in our sample, namely, Australia and New Zealand, we impute the LCOE in Australia. Similar extrapolations are done also for imputing LCOE to countries in the case of gas-powered and renewable electricity.

Several internet sources and repositories of the International Energy Agency (IEA) and US-IEIA were employed to obtain country-level energy profiles for nuclear and renewable sources of energy. The profiles noted, for each country, the different types of alternative sources of energy in place as well as its plans and intentions for extensions to

[62] In the PCGE report these are computed for 5% and 10% discount rates.

newer renewable sources in the future.[63] Note, the geography and the natural endowments of any country define or place constraints on the types of renewable energy sources available to it currently and in the future, *e.g.,* hydropower may not be a viable source of renewable energy in a country comprising chiefly deserts. It can only be a small-scale provider of power for countries with no big rivers. For each country, the LCOE for renewable energy is computed as the average levelised cost computed over renewables that were noted in the profile of the country. A more disaggregated approach could not be followed due to lack of country-level data on the actual amounts of energy produced by each renewable source.[64]

Oil is a significant source of non-electrical energy. Oil-powered electricity generation is relatively more costly in most countries. Moreover, the split of data on oil energy between electrical and non-electrical usage is not available at the level of all countries. At the same time, consumption of oil is highly positive for all countries. Thus, we assume that it is an essential input in production for all countries.[65] Hence, we assume that all the data on oil energy pertains to non-electrical energy. Hence, the price of oil energy is taken to be the crude oil price, which is obtained from the BP Statistical Review as 83.7 USD per barrel in 2010. This is equivalent to 572591.7 USD per ktoe.

Cost of afforestation.

There are several case studies that have been conducted for selected countries, which study costs of afforestation and maintaining current stocks of forests. These estimates vary widely. Even within a single country, estimates vary widely depending on the location of forests and the methodologies used. Given the wide variation in estimates in recent studies and due to incomplete coverage of countries by these studies, we rely on two earlier sources: Chp. 24 of the Second Assessment Report (SAR) of IPCC (1996) and Sathaye and Ravindranathan (SR) (1998), which compile estimates from different case-studies in a manner that greatly facilitates attribution of comprehensive costs of forests (including afforestation costs) to all countries in our sample. We consider seven climate zones of the world – polar, boreal, cold desert, temperate, Mediterranean (sub-tropical), warm deserts, and tropical. For each country, we identify the major zones that it lies in. SAR and SR tabulate ranges of costs of forests across different latitudinal belts of the world in USD per tonn of carbon (tc) sequestered. In our analysis, the comprehensive cost of afforestation is captured by the term $G_f + G_a$, and is hence measured in USD per meter-cube (m3) of forest. Hence, we multiply the cost estimates in the SAR and SR by the sequestration factor s, which is measured in tc per m3. Parts of different continents

[63] Note, the information on the actual amount of energy produced from each source was not available consistently for all countries in the sample.

[64] Note, this information is needed to compute profit elasticity of renewables in our model.

[65] This is unlike coal and gas. There are countries in our sample which consume no coal or gas. *E.g.,* some African countries consume no coal.

that lie in each latitudinal belt are identified by the SAR and SR. We relate the latitudinal belts to the climate zones. Employing their tables, for each continent, we attribute costs of forest to the different types of climate zones within which it lies. For every country, depending on the continent that it lies in, an estimate of the cost of forest is obtained as the average of costs across different climate zones that it lies in. There seem no studies which explicitly report costs of forests in the desert regions of the world. It should be noted, however, that the sequestration factors are very high for deserts, and many desert countries derive huge profits from export of fossil-fuel resources, which could be used in part to finance afforestation reforms. Hence, it is not clear at the outset that afforestation cannot form a part of profit increasing and emission-non increasing set of reforms for desert countries. Since forest cost estimates are not available for these countries, we impute the highest forest costs in the SAR and SR to these countries, namely, 29 and 27 USD per tc sequestered for warm and cold desert regions, respectively.

Afforestation costs are the highest in Africa and Oceania (Australia and New Zealand). Significant parts of Africa and Australia are deserts, and this explains the high afforestation costs in these regions. Asia has a wide variation in its climatic zones. Although, significant parts of Asia lie in tropical, subtropical, and temperate zones with lower afforestation costs, regions of several countries in Asia are also deserts. Hence, the average afforestation cost in Asia is higher than in Europe and the Americas. The countries with the highest afforestation costs in our sample lie in major warm or cold deserts of the world.

ATA, RIP, MAC, fossil fuel energy consumption, and average profitability in all countries.

Country	ATA	RIP	MAC	Fossil-fuel energy	Avg. profitability
Albania	1.14	2019.64	1775.14	1391.84	1137.539099
Algeria	23.41	18275.95	780.61	45511.00	21270.12101
Angola	3.88	4392.08	1130.94	5173.18	3189.714773
Argentina	37.76	33421.44	885.21	73067.83	35509.00925
Armenia	2.34	1252.52	534.76	4041.10	1765.318616
Australia	83.84	60323.19	719.46	141826.99	67411.34025
Austria	14.45	33275.53	2302.21	27252.43	20180.80614
Azerbaijan	7.06	3954.17	560.00	13490.92	5817.381842
Bangladesh	12.96	13509.65	1042.40	24884.87	12802.49289
Belarus	18.57	3079.08	165.82	28232.95	10443.53209
Belgium	32.72	35429.18	1082.67	57261.30	30907.73368
Benin	1.52	524.29	344.68	1776.18	767.3295396
Bhutan	3.00	15.38	5.12	120.80	46.39338183
Bolivia	2.88	1867.00	647.47	5294.73	2388.205125
Bosnia & Herzegovina	5.72	-444.08	0.00	6570.82	2044.151848
Botswana	1.11	2208.33	1987.03	1617.55	1275.662749
Brazil	117.95	179706.85	1523.61	171850.99	117225.262
Brunei	1.99	1502.13	756.37	3684.75	1729.623383
Bulgaria	10.25	3853.52	376.01	14699.92	6187.89305
Cambodia	4.04	1358.67	336.28	4772.95	2045.222439
Cameroon	1.64	4319.07	2638.00	2096.96	2139.223878
Canada	112.01	116133.65	1036.79	214216.17	110153.9458
Cape Verde	0.08	131.22	1582.04	137.44	89.58271723
Chile	16.09	19873.37	1235.34	27327.33	15738.93109
China	1905.89	227456.39	119.34	2207807.83	812390.0359
Colombia	13.29	35529.25	2673.60	24599.89	20047.47757
Congo DemRep	0.71	2060.50	2902.07	1015.32	1025.508389
Congo Republicof	0.82	728.90	893.19	1548.95	759.5550576
Costa Rica	2.98	3896.79	1307.05	2535.77	2145.181725
Croatia	5.13	5704.81	1112.27	7553.35	4421.095065
Cuba	8.17	10480.53	1282.88	9943.86	6810.85485
Cyprus	2.58	1330.88	515.15	3033.34	1455.602891
Czech Republic	19.15	13783.85	719.66	32672.76	15491.91877
Denmark	9.40	21192.80	2254.34	17760.24	12987.4827
Dominican Republic	4.89	9231.69	1886.38	6935.74	5390.771409
Egypt	42.03	35471.91	844.03	80234.82	38582.92056
Estonia	1.23	2653.29	2160.81	2012.48	1555.66398
Ethiopia	1.92	10165.35	5290.28	2243.74	4137.004218
Finland	11.42	18792.91	1645.33	20307.37	13037.23321
France	88.52	230312.77	2601.77	153712.96	128038.0864
Gabon	0.75	1840.99	2453.14	946.72	929.4887988
Gambia	0.14	119.44	857.23	170.73	96.76975097
Georgia	1.64	1463.76	892.89	2678.51	1381.304097

Germany	153.45	256809.02	1673.54	294470.66	183811.0444
Ghana	2.81	3045.21	1082.28	3285.53	2111.187398
Greece	19.15	31000.12	1619.02	30943.24	20654.16851
Guatemala	3.10	8995.42	2898.76	3916.26	4304.927978
Haiti	0.63	1961.98	3104.71	737.91	900.1741853
Honduras	2.11	2583.15	1226.72	2571.84	1719.030203
Hungary	10.36	13398.57	1293.54	21611.19	11673.37176
Iceland	0.95	1662.16	1745.63	1199.42	954.1774929
India	380.02	212303.93	558.66	532696.86	248460.2675
Indonesia	81.83	71630.18	875.33	151907.28	74539.76572
Iran	117.51	53232.02	453.01	227646.81	93665.44688
Ireland	8.12	20309.17	2501.37	15068.87	11795.38604
Israel	14.60	17645.01	1208.45	24363.34	14007.65161
Italy	86.54	188077.06	2173.38	170080.11	119414.5681
Jamaica	2.66	1671.74	629.39	3135.99	1603.460234
Japan	247.74	407911.41	1646.50	450920.65	286359.9329
Jordan	5.01	1210.43	241.82	8067.82	3094.416376
Kazakhstan	42.75	-3380.18	0.00	55446.79	17369.78985
Kenya	3.61	6611.62	1833.00	4378.78	3664.669496
Korea Republicof	135.84	89897.08	661.78	236056.60	108696.5088
Kyrgyz Republic	4.04	479.31	118.51	2882.03	1121.793146
Latvia	5.74	1050.94	183.04	3094.88	1383.852058
Lebanon	5.38	2844.28	529.05	6430.60	3093.417086
Libya	13.48	9594.05	711.53	21387.50	10331.67619
Lithuania	3.10	5892.51	1903.30	5870.23	3921.947166
Luxembourg	2.84	4183.75	1473.74	4560.61	2915.732178
Malaysia	35.59	25418.27	714.10	73299.84	32917.90413
Malta	2.25	-7.02	0.00	2631.86	875.6971947
Mexico	95.98	142682.99	1486.61	170914.86	104564.6103
Moldova	1.47	315.01	214.36	2848.24	1054.904982
Mongolia	1.91	329.69	172.73	2426.07	919.2226403
Morocco	10.96	9181.25	837.85	15740.45	8310.886895
Mozambique	0.72	3252.10	4526.80	915.95	1389.590103
Namibia	0.92	869.99	945.97	1093.50	654.8028791
Nepal	0.95	6393.21	6700.24	1289.95	2561.370974
Netherlands	53.51	63926.97	1194.72	103362.68	55781.05423
New Zealand	11.91	7716.00	648.04	13707.04	7144.983391
Nicaragua	1.28	958.30	750.64	1536.09	831.8904324
Nigeria	10.92	47631.47	4361.38	16864.46	21502.28454
Norway	12.32	23239.40	1885.90	17030.41	13427.37565
Pakistan	27.43	33636.46	1226.44	56148.89	29937.59185
Panama	5.02	1072.48	213.47	5939.08	2338.860256
Peru	9.17	22022.87	2400.59	16444.32	12825.45306
Philippines	16.33	25789.63	1579.29	26300.16	17368.70765
Poland	70.65	27085.06	383.38	99480.72	42212.14244
Portugal	12.78	23917.59	1870.96	20975.67	14968.68188

Romania	14.17	16171.10	1141.39	27807.25	14664.1717
Russian Federation	329.39	-10133.09	0.00	665571.85	218589.3813
Saudi Arabia	113.81	20580.87	180.84	197942.59	72879.08809
Senegal	1.66	1746.95	1052.78	2110.58	1286.395999
Slovak Republic	6.99	5442.43	778.46	13771.86	6407.09279
Slovenia	4.77	3577.01	750.30	5469.00	3016.926296
South Africa	122.94	52.08	0.42	137600.78	45925.26603
Spain	69.86	142722.18	2043.11	119088.76	87293.59897
Sri Lanka	3.67	7334.40	1997.04	4357.34	3898.473034
Sudan	5.74	13239.79	2306.09	6703.97	6649.832988
Sweden	17.56	37779.85	2150.96	20826.12	19541.17711
Switzerland	11.96	35178.89	2942.34	17198.87	17463.23852
Tajikistan	0.56	1647.93	2930.00	928.56	859.0185821
Tanzania	1.46	7456.10	5090.42	2361.76	3273.108721
Thailand	53.60	37161.72	693.28	106444.19	47886.50523
Trinidad &Tobago	14.22	-15747.08	0.00	23606.27	2624.467455
Tunisia	4.36	5995.93	1375.67	7747.79	4582.692021
Turkey	51.81	67408.51	1300.97	99650.69	55703.67341
Turkmenistan	12.74	-6848.12	0.00	23486.73	5550.449129
Ukraine	53.98	-5577.35	0.00	100427.44	31634.69016
United Arab Emirates	46.34	9864.54	212.89	90156.49	33355.7875
United Kingdom	98.73	219375.16	2221.96	204148.99	141207.6252
United States	1087.51	1152288.08	1059.56	2053288.86	1068888.15
Uruguay	2.65	3092.10	1168.80	3017.46	2037.399548
Uzbekistan	27.34	-20079.29	0.00	47792.57	9246.86954
Vietnam	23.22	17935.20	772.43	39239.28	19065.90061
Yemen	5.76	3998.68	694.18	7611.86	3872.099105
Zambia	0.65	3306.85	5078.40	760.35	1355.950758
Zimbabwe	2.25	-723.03	0.00	2593.52	624.2493432

Profit increasing and net emission reducing reforms of Theorems 4 and 5 and extent of efficiency improvements in all countries.

Country	Profit increase	Emission reduc.	Labour	Coal	Gas	Oil	Renewables	Afforestation	Size of reform
Albania	8347.23	0.64	-570016.5	-2.050449	-0.028198	-742.6879	-0.446336545	0.004957748	570016.939
Algeria	3952.60	3.47	8344614.4	-5.28E+00	-12047.49	5110.308	-0.126495503	2.11E-10	8344624.642
Angola	2402.09	1.69	-2482572	0.00E+00	-52.06644	-1928.819	-3688.135423	3.65E-09	2482575.33
Argentina	7438.20	9.85	-6699504	-2.19E+01	-20356.94	3997.1532	231.280838	1.76E-10	6699536.387
Armenia	126.24	0.28	-486481.1	-5.71E-05	-342.7269	-68.37174	-177.7900404	1.10E-09	486481.306
Australia	30661.27	40.11	-4233693	-2.77E+04	-4432.185	-6479.063	-39.61614419	6.29E-11	4233790.925
Austria	13279.23	5.87	-2060884	-5.45E+02	-2217.937	-4438.761	59.7726371	1.51E-10	2060889.739
Azerbaijan	3850.68	2.45	2351356.7	0.00E+00	-4916.155	876.16598	-9.619897311	3.58E-10	2351362
Bangladesh	28902.87	7.76	-27431554	-3.45E+01	-10709.09	-872.14	-86.38211451	5.76E-10	27431555.69
Belarus	4501.59	10.64	-1344883	-0.353423	-9648.843	-225.1689	-172.1146065	10.78729127	1344917.598
Belgium	14667.88	13.05	-2191826	-210.1999	-4100.197	-11843.85	-1394.763145	0.00411564	2191862.727
Benin	3027.54	0.86	-1396828	0.00E+00	0	-1004.218	38.43956461	6.29E-09	1396828.538
Bhutan	101.23	1.71	12413.738	0.4967937	0	26.204066	0.519916022	3.356348596	12413.76628
Bolivia	618.25	1.02	-755036.3	0.00E+00	-1960.665	303.14609	-468.1692934	3.26E-09	755039.0037
Bosnia & Herzegovina	14490.77	0.09	-2011091	-2.18E+02	-0.786241	183.10663	-10.50328086	-1.00E-12	2011090.671
Botswana	350.05	0.18	460748.45	-3.91E+02	0	311.25285	-130.1811665	9.40E-09	460748.7353
Brazil	30602.82	21.96	-45779899	-7.16E+02	-1563.767	-23504.08	20.95438099	2.13E-11	45779904.84
Brunei	2088.93	0.80	119541.17	0.00E+00	-1343.068	81.517432	0	2.98E-09	119548.7415
Bulgaria	3208.12	5.06	-316306.7	-4575.345	-369.2287	1816.0031	-816.6547359	3.721919104	316346.3186
Cambodia	3297.27	2.48	-2372081	-2.08E+03	0	-121.3225	-312.5773472	7.78E-10	2372081.865
Cameroon	1031.54	0.28	5855657.2	0.00E+00	-14.57538	-310.588	2267.952195	3.11E-09	5855657.609
Canada	43817.40	44.64	-8231709	-4.91E+03	-19532.69	-30737.54	-5228.350912	2.61E-11	8231792.2
Cape Verde	57.72	0.03	-87736.97	0	0	-33.00591	1.162388738	0.010746536	87736.97736
Chile	2610.44	2.51	-3107469	-1091.925	-471.0154	-951.5886	21.89388859	0.308637466	3107469.573
China	296292.01	1016.43	-2.29E+08	-910961.5	-2087.227	32366.422	-11547.99005	6.166913257	228589455.1
Colombia	2075.97	0.88	-5960937	-5.25E+02	-1327.83	686.72781	312.6236661	4.10E-10	5960937.645
Congo DemRep	36742.74	0.38	-11621852	-5.36E+01	-0.032959	-374.8874	-528.1693779	4.29E-09	11621852.01
Congo Republicof	2596.87	0.48	-1209149	0.00E+00	-349.9036	-289.9163	8.291683089	5.75E-09	1209149.436
Costa Rica	2461.87	1.53	-878330.3	-0.865135	0	-659.1526	259.1121745	1.10357824	878330.6312
Croatia	1200.81	1.46	-832168.8	-52.70558	-392.8251	6.4182826	99.28956712	1.855043863	832168.8974

Cuba	2065.54	1.79	-2272593	0.010511	62.043946	-1531.402	151.5040167	0.594889045	2272593.681
Cyprus	594.02	0.94	-225825.8	0.4116461	0	-1094.803	18.88643843	0.000539454	225828.4594
Czech Republic	9589.84	10.60	-1748970	-7631.911	-1778.416	-441.2149	-1069.129703	0.766807478	1748988.39
Denmark	10999.12	4.41	-1373747	-1210.445	-1002.848	-2774.502	-13.84532961	0.004171853	1373751.016
Dominican Republic	684.06	0.29	2246529.3	2.78E+01	99.426898	-454.9065	233.7148503	2.24E-09	2246529.337
Egypt	1275.06	1.34	6589333.1	-2.12E+01	-8475.624	4903.7835	-71.313492	8.20E-05	6589340.361
Estonia	933.58	0.44	-347821.8	-5.86E-01	-91.14938	-443.2565	55.29472775	1.85E-09	347822.0588
Ethiopia	46532.04	1.07	-17623448	0.00E+00	0	-1249.886	-3592.639223	2.58E-09	17623448.91
Finland	6914.87	4.55	-1255128	-1.51E+03	-747.379	-2734.745	-2837.344408	2.15E-10	1255135.716
France	83009.19	33.01	-14604061	-1106.324	-9764.193	-29274.8	-10144.9292	0.686572892	14604096.87
Gabon	418.45	0.12	471577.05	0.00E+00	-3.441595	-138.8911	792.5489527	9.44E-09	471577.7334
Gambia	558.69	0.08	-239733	0	0	-38.21878	0.047936858	0.027753611	239733.0494
Georgia	791.07	0.65	361037.41	-1.567293	-803.232	367.85419	-388.1189314	1.022036086	361038.6957
Germany	121249.43	71.81	-19070971	-28660.35	-20527.96	-29996.08	-2385.5363	0.079211841	19071027.17
Ghana	9799.34	1.58	-4717063	0.00E+00	0	-1850.231	-452.4585688	2.83E-09	4717063.622
Greece	2433.25	1.63	-1778167	-905.2963	-66.04785	-630.8789	-23.47679718	0.030977999	1778167.381
Guatemala	305.77	0.10	-1265217	-2.49E+01	0	-87.70849	1395.40184	1.15E-09	1265218.183
Haiti	11300.27	0.35	-1929441	0.00E+00	0	-410.2601	-12.39361106	7.72E-09	1929441.113
Honduras	586.17	0.50	-1380808	-3.17E+00	0	-580.8262	202.4789949	1.28E-09	1380808.404
Hungary	6733.03	5.11	-1674451	-668.1046	-5205.32	-987.0476	-607.8927967	0.309333884	1674460.077
Iceland	869.07	0.40	-63479.35	-9.251065	0	-455.0984	-2536.779436	0.000121291	63531.65284
India	106374.78	140.81	-34564783	-191289.6	-4033.307	95516.398	-33314.62658	4.216686905	34565460.5
Indonesia	15613.75	22.16	-52439082	-9.73E+03	-5789.778	-8497.173	-4204.109768	7.76E-11	52439083.7
Iran	9616.58	15.05	14671703	-17.00614	-46198.51	17566.503	-4.365374707	0.008026506	14671786.23
Ireland	3652.45	1.68	-1131556	-126.4694	-784.3063	-1193.841	-1.709567672	0.011249214	1131557.309
Israel	7622.05	6.35	-1471634	-2.81E+03	-195.98	-3516.882	-1.499264964	1.25E-05	1471641.316
Italy	35935.80	20.11	-11639521	-1299.882	-21026.12	-5136.723	-497.835558	1.40024464	11639541.09
Jamaica	993.84	1.12	-561040.7	8.25E-02	0	-1307.904	66.57049005	1.49E-05	561042.1916
Japan	210055.53	116.99	-30567150	-4.14E+04	-16848.42	-68553.58	-698.2667741	9.36E-12	30567259.37
Jordan	279.74	1.13	-702821.4	0.00E+00	106.09127	-1400.313	-1.686703234	5.80E-10	702822.775
Kazakhstan	18294.35	5.27	4660528	-1.03E+04	-807.4306	8138.3056	-19.65035722	-2.13E-12	4660546.461
Kenya	2632.97	1.19	1157937.3	-7.31E+00	0	-1385.337	-9797.607246	1.47E-09	1157979.599

Korea Republicof	18176.73	35.44	-9748360	-22797.37	-6110.501	-5986.24	-1410.943554	0.690053324	9748390.666
Kyrgyz Republic	440.46	2.23	1012615.5	-88.93703	-20.31754	314.88257	-112.6011474	1.917518409	1012615.584
Latvia	1691.85	3.64	-331344.4	-0.718165	-120.0867	-49.61954	47.06236243	8.193780143	331344.3853
Lebanon	751.50	1.38	-567491.5	1.27E+01	0	-1634.091	63.12380435	6.85E-05	567493.8839
Libya	7928.82	4.43	1759505.8	0.00E+00	281.4995	-5390.069	0.014436604	3.58E-10	1759514.119
Lithuania	1591.15	0.99	-823446	-8.129096	-827.516	-490.3794	75.7231791	0.068578231	823446.5829
Luxembourg	1325.43	0.94	-180058.2	-9.12E-01	-136.0487	-987.6783	-7.577196641	7.35E-10	180060.9849
Malaysia	17046.99	12.47	8943498.2	-3.26E+03	-9545.228	-2963.717	-100.8033044	1.52E-10	8943504.337
Malta	459.28	0.01	-72974.17	0.00E+00	0	-7.282276	0.761058826	-1.67E-11	72974.17398
Mexico	3666.04	2.67	-3420549	-1.00E+03	-24670.14	16962.154	1581.569418	1.42E-10	3420680.575
Moldova	3172.28	0.87	-648119.1	-3.817395	-997.9468	-218.5086	-3.068219284	0.052449159	648119.895
Mongolia	246.70	0.54	428425.06	-8.56E+02	0	510.22893	-13.5785918	1.13E-09	428426.2172
Morocco	403.25	0.48	-3325846	9.55E+01	18.703269	-699.8434	58.52235969	4.89E-10	3325846.557
Mozambique	2995.12	0.33	2283808.1	-4.37E-02	-3.266851	-383.2129	-5108.360579	1.11E-08	2283813.89
Namibia	116.00	0.13	-294596.9	-7.86E-02	0	-151.0445	50.92987436	5.41E-09	294596.9746
Nepal	3936.19	0.45	-6579818	-3.70E+01	0	-474.55	-1986.603131	5.09E-09	6579818.489
Netherlands	27792.87	22.88	-4198939	-698.6628	-13327.09	-15653.67	-79.16002455	0.004534931	4198989.696
New Zealand	3136.30	5.69	-830532.1	-171.9941	-557.7147	-452.6829	-445.6572624	13.99744723	830532.5747
Nicaragua	4764.71	0.72	-927880	-1.14E+00	0	-837.8688	2.240473096	3.51E-09	927880.3936
Nigeria	33655.10	5.21	-18588411	-5.17E-02	-1005.166	-5324.33	-53348.00552	6.84E-10	18588488.28
Norway	5809.30	3.90	-1135532	-33.32904	-817.645	-862.0269	-584.3511255	6.471507473	1135532.793
Pakistan	8045.02	8.28	-20770863	-6.08E+02	-17462.73	4413.1668	-8973.490223	4.22E-10	20770872.7
Panama	220.33	0.95	-465778.3	1.56E+00	0	-1113.954	174.7110193	3.84E-10	465779.7064
Peru	3476.31	1.26	10389458	-8.51E+01	-1953.46	122.93009	322.5938149	1.32E-09	10389458.66
Philippines	1815.95	1.47	-11865961	-3153.546	-170.3529	2649.3185	-1664.836351	0.052956814	11865961.75
Poland	16751.81	33.19	-5017761	-32040.64	-1484.382	6625.368	-205.834771	3.018337986	5017867.993
Portugal	9119.47	4.79	-2861924	-137.1899	-614.2825	-4941.538	-161.7116266	0.001041095	2861928.119
Romania	6178.25	6.37	-3480897	-3273.232	-4481.49	1092.5314	-1740.838683	1.470088215	3480901.826
Russian Federation	145126.01	7.18	-34357929	-6249.697	-106422.8	80533.431	-6144.740687	-0.635555071	34358189.44
Saudi Arabia	1342.98	5.04	2282087.2	0.00E+00	6899.9499	-11131.84	-0.000186116	1.68E-11	2282124.827
Senegal	298.39	0.22	3384140.9	-1.28E+01	-0.022373	-238.7449	-327.0427415	5.03E-09	3384140.923
Slovak Republic	4020.85	4.05	-811449.3	-1947.346	-2396.922	9.7403996	-986.1949988	0.660379972	811455.8115

Slovenia	2401.83	2.66	-367656.7	-429.2926	-82.78624	-361.3357	-166.4972697	4.204069989	367657.1523
South Africa	13181.92	68.50	-4140360	-6.09E+04	-45.30375	1282.8157	-781.6069087	5.39E-11	4140808.125
Spain	49060.16	24.94	-11666446	-560.3418	-5461.587	-24012.85	-429.6149861	0.397887293	11666472.03
Sri Lanka	5319.56	1.75	-4076351	-8.69E-01	0	-2036.906	34.08873429	2.64E-09	4076351.069
Sudan	1596.16	0.75	-5752285	0.00E+00	0	-870.1001	-1253.411141	1.40E-09	5752285.625
Sweden	19348.87	8.19	-2225300	-199.254	-63.00231	-5281.456	-8036.554479	9.097110381	2225320.729
Switzerland	21834.92	5.51	-2277542	-0.940056	-354.3529	-6156.165	-178.8643683	0.025422588	2277550.364
Tajikistan	670.27	0.15	2496939.7	-17.1731	-41.79045	-121.2601	-166.3861879	0.000333278	2496939.66
Tanzania	19001.45	0.83	-8942436	-4.51E-01	-140.5487	-859.7846	-2053.815936	6.99E-09	8942436.663
Thailand	3953.21	6.20	6314100.4	-6006.323	-17076.43	13758.823	-7305.948498	0.00941622	6314145.565
Trinidad &Tobago	12364.73	11.00	183365.03	0.00E+00	-17688.62	593.78278	-0.015438631	-1.18E-12	184217.1953
Tunisia	634.73	0.51	-1571348	0	-370.5027	-314.0665	3.12217789	0.006454972	1571348.551
Turkey	23458.24	14.68	13763815	-16645.89	-9365.024	13242.902	-547.641186	1.912506953	13763834.36
Turkmenistan	4812.35	7.36	320191.26	0.00E+00	-13143.69	1392.4421	0	-1.78E-11	320463.9444
Ukraine	42072.91	2.96	-12966518	-3127.209	-8569.826	7225.6987	-8384.996844	-0.012274823	12966525.98
United Arab Emirates	10284.50	15.97	1951987.3	-1.14E+01	-25716.27	911.37956	0	3.53E-05	1952156.928
United Kingdom	112828.78	47.17	-14377901	-7050.467	-32473.79	-20937.48	-233.2458829	0.145634799	14377954.91
United States	777113.46	568.60	-68094479	-179844.8	-148247.5	-272906.9	-4541.929114	85.72890592	68095425.24
Uruguay	406.63	0.39	-458077.4	-0.002022	-0.675678	-112.7715	1226.342373	0.287213011	458079.0342
Uzbekistan	27829.87	13.70	-9699127	-20.53083	-22276.05	954.05912	-25.79991183	-0.000120115	9699152.197
Vietnam	14624.04	12.93	-18279263	-7436.181	-1022.361	-4022.534	-918.040834	0.292450206	18279265.06
Yemen	19658.60	3.27	-2448090	0.00E+00	-31.24183	-3794.473	0.259911513	1.07E-09	2448093.104
Zambia	4011.89	0.35	-2201177	0.00E+00	0	-404.5836	-2582.075311	9.37E-09	2201178.301
Zimbabwe	11872.87	0.11	-3859370	-1.54E+02	0	71.85205	-3272.363479	-4.03E-12	3859370.959

Structure of reform a in all countries.

Country	Labour	Coal	Gas	Oil	Renewables	Afforestation	Size of reform a
Albania	-1168663.8	-0.0934567	-0.000223	0.60620798	-0.9150918	0.000632997	1168663.824
Algeria	8661068.53	-4.5776143	-8877.7141	6753.16453	-0.1312926	1.78E-10	8661075.711
Angola	-3251061.5	0	-25.828041	19.6293109	-4829.8119	1.95E-09	3251065.073
Argentina	-6728468.8	-12.551989	-13112.024	9981.87171	232.280755	1.06E-10	6728489.012
Armenia	-421042.18	-3.65E-05	-223.95468	170.205564	-153.87463	7.43E-10	421042.2982
Australia	-5418468.3	-5611.127	-1365.5781	8519.33737	-50.702503	1.91E-11	5418478.041
Austria	-2365786.6	-192.55528	-795.08511	861.004755	68.6158582	6.16E-11	2365786.888
Azerbaijan	3140126.92	0	-2731.82	2076.18266	-12.846923	1.97E-10	3140128.795
Bangladesh	-55544261	-3.9822641	-1106.6025	846.327399	-174.90919	8.72E-11	55544260.74
Belarus	-2507682.3	-0.0617826	-2778.8083	2723.30558	-320.92663	1.354556725	2507685.352
Belgium	-2532666.9	-67.484643	-1499.9477	1230.6586	-1611.6561	0.001575778	2532668.142
Benin	-2966680.2	0	0	9.45E-07	81.6406047	4.96E-10	2966680.248
Bhutan	27039.1831	1.24676462	0	58.3011766	1.13246343	0.099188598	27039.24596
Bolivia	-886259.88	0	-1702.8839	1294.19144	-549.53608	2.33E-09	886262.6316
snia and Herzegov	-2011090.7	-218.1645	-0.7862415	183.106634	-10.503281	-1.00E-12	2011090.671
Botswana	468906.528	-328.12808	0	437.503863	-132.48617	8.31E-09	468906.8656
Brazil	-42126481	-394.94787	-905.84761	1215.04087	19.282138	1.26E-11	42126480.73
Brunei	172762.17	0	-390.07738	296.458736	0	1.38E-09	172762.8645
Bulgaria	-481532.63	-2704.1368	-311.10903	4507.6591	-1243.2422	1.850503515	481563.0295
Cambodia	-5078737.7	-209.97119	0	279.961425	-669.24292	9.33E-11	5078737.756
Cameroon	6273708.32	0	-13.011122	9.88845177	2429.86742	2.85E-09	6273708.789
Canada	-9323707.3	-1683.262	-7994.9553	8320.51252	-5921.9315	1.01E-11	9323716.478
Cape Verde	-103728.61	0	0	2.13054315	1.37425502	0.004479324	103728.6146
Chile	-2749048.9	-622.09211	-279.87402	1143.4863	19.3686135	0.192854888	2749049.195
China	-392592807	-112704.7	-475.36592	151026.811	-19833.347	0.796102743	392592852.9
Colombia	-5034969.7	-388.43015	-967.82157	1253.45078	264.060932	3.15E-10	5034969.998
Congo DemRep	-24462601	-2.8399602	-0.0017183	3.78791735	-1111.733	1.89E-10	24462601.11

Congo Republicof	-2538279.7	0	-37.473549	28.4798906	17.4061299	7.05E-10	2538279.671
Costa Rica	-1233038.4	-0.0731121	0	339.017181	363.752945	0.331962813	1233038.456
Croatia	-800806.59	-20.103133	-149.81246	840.368884	95.54761	0.97078049	800807.0508
Cuba	-2143387.5	0.01720962	74.9149198	284.493022	142.890431	0.334411702	2143387.524
Cyprus	-263507.29	0.53375221	0	-0.6371579	22.0378458	0.000188789	263507.2943
Czech Republic	-2659828.3	-2135.5591	-655.2253	3458.67328	-1625.9288	0.209896169	2659832.005
Denmark	-1748003.2	-345.37162	-278.25678	672.561871	-17.617273	0.001484736	1748003.415
Dominican Republi	2126529.31	30.0061405	98.5251272	-114.88724	221.230804	2.01E-09	2126529.325
Egypt	5773426.81	-17.819175	-6664.433	5088.79671	-62.483293	6.89E-05	5773432.903
Estonia	-380978.73	-0.1874928	-18.600999	14.3867462	60.5658346	8.40E-10	380978.736
Ethiopia	-35899662	0	0	3.36E-07	-7318.3483	3.47E-10	35899662.77
Finland	-1416462.2	-670.60179	-377.12074	1180.74689	-3202.0557	9.33E-11	1416466.512
France	-16185391	-468.26849	-4334.9831	4087.26423	-11243.424	0.308362111	16185396.04
Gabon	501021.832	0	-2.8824401	2.19065958	842.034893	8.77E-09	501022.5397
Gambia	-518438.39	0	0	5.71156879	0.10366659	0.002789776	518438.3917
Georgia	462800.545	-1.0313239	-540.1427	770.215735	-497.51535	0.718289251	462801.769
Germany	-23741274	-8233.9298	-7156.6617	16430.7751	-2969.732	0.02798292	23741282.7
Ghana	-9430967.9	0	0	6.27E-07	-904.61417	4.10E-10	9430967.919
Greece	-1522251.5	-518.81139	-24.548363	721.602738	-20.097993	0.022335458	1522251.732
Guatemala	-1075843.5	-20.140899	0	26.8545174	1186.5423	9.32E-10	1075844.134
Haiti	-3995452.8	0	0	7.98E-07	-25.664473	8.21E-10	3995452.847
Honduras	-1337209.9	-1.3284757	0	1.77130029	196.085812	6.66E-10	1337209.886
Hungary	-2181771.4	-213.80877	-1971.0039	1828.27288	-792.0702	0.104183057	2181773.17
Iceland	-81147.537	-6.0301222	0	8.09094043	-3242.8402	7.16E-05	81212.30715
India	-43833897	-110907.84	-2243.3637	150868.095	-42248.49	2.971378978	43834317.05
Indonesia	-50356735	-4275.3291	-2509.6243	7607.74947	-4037.1653	3.81E-11	50356735.5
Iran	14655457.6	-14.435846	-33047.173	25138.6453	-4.3605411	0.006319676	14655516.47
Ireland	-1052091.5	-59.902811	-307.68027	315.964397	-1.5895112	0.006733099	1052091.546
Israel	-1751752.9	-689.98658	19.8379019	904.90869	-1.7846428	4.61E-06	1751753.313

Italy	-10907221	-644.21014	-10315.784	9060.36115	-466.51426	0.801970264	10907229.77
Jamaica	-722563.4	0.38905246	0	-0.5127078	85.7360303	4.67E-06	722563.4005
Japan	-38969727	-11704.666	-4754.005	19219.255	-890.21272	3.22E-12	38969733.94
Jordan	-685904.43	0	290.569979	-220.83313	-1.6461042	2.19E-10	685904.5286
Kazakhstan	4660527.99	-10258.965	-807.43057	8138.30558	-19.650357	-2.13E-12	4660546.461
Kenya	1291829.92	-4.6492997	0	6.19906343	-10930.507	8.62E-10	1291876.165
Korea Republicof	-9192305.7	-9562.7085	-3592.3246	15654.3417	-1330.4622	0.335742117	9192324.773
Kyrgyz Republic	1712316.69	-10.047597	-6.4464751	1117.12687	-190.40674	0.756494543	1712317.067
Latvia	-686022.46	0.07637923	24.3116392	437.842985	97.4389246	0.911138193	686022.609
Lebanon	-573922.62	14.1529516	0	-18.857433	63.8391542	3.25E-05	573922.623
Libya	2255203.59	0	1198.32971	-910.73036	0.01850376	2.22E-10	2255204.09
Lithuania	-834930.74	-3.3995009	-317.19884	258.033435	76.7793015	0.033015453	834930.8453
Luxembourg	-190020.73	-0.1772984	-2.4870329	2.12654246	-7.9964381	3.35E-10	190020.732
Malaysia	11506472.8	-1729.0094	-4663.2144	5849.3866	-129.69092	9.93E-11	11506475.36
Malta	-72974.174	0	0	-7.2822757	0.76105883	-1.67E-11	72974.17398
Mexico	-2943730.3	-837.55751	-20646.378	16807.986	1361.10134	1.20E-10	2943851.16
Moldova	-1514388	-0.327159	-100.41351	78.3284435	-7.1691679	0.002244098	1514388.035
Mongolia	537370.727	-575.77296	0	767.696826	-17.031538	6.59E-10	537371.5844
Morocco	-2804055.9	113.589809	16.6408162	-164.10001	49.3408122	3.72E-10	2804055.887
Mozambique	3286310.66	-0.0211024	-1.5384838	1.19739096	-7350.7312	5.05E-09	3286318.878
Namibia	-262923.92	-0.0082561	0	0.01101242	45.4542494	3.30E-09	262923.9277
Nepal	-8848537.7	-15.678897	0	20.905185	-2671.5834	2.18E-09	8848538.143
Netherlands	-4983642.3	-178.62135	-4018.2906	3292.82163	-93.953548	0.001656371	4983644.99
New Zealand	-1006927.2	-25.061445	-137.51812	1961.07456	-540.30952	4.611916785	1006929.249
Nicaragua	-1933315.9	-0.1004521	0	0.13393641	4.66821374	3.60E-10	1933315.889
Nigeria	-26614575	-0.0312149	-588.59373	447.372742	-76382.778	3.95E-10	26614684.26
Norway	-1141279.1	-14.564765	-364.15201	1808.06404	-587.30862	3.239381652	1141280.772
Pakistan	-22090248	-400.96771	-11436.48	9226.34599	-9543.4949	2.93E-10	22090254.97
Panama	-447071	1.71732643	0	-2.2897671	167.693992	1.69E-10	447071.0275

Peru	10712444.7	-73.758016	-1663.9097	1362.91506	332.622574	1.20E-09	10712444.96
Philippines	-10270806	-2398.8159	-125.35818	3319.01373	-1441.0304	0.041778864	10270806.76
Poland	-7087504.2	-11080.945	-825.01342	15941.4515	-290.73819	1.064180334	7087530.803
Portugal	-3210856.2	-40.83454	-156.38878	173.587045	-181.42788	0.000444591	3210856.175
Romania	-4330162.2	-1704.4243	-2704.244	4736.74098	-2165.5666	0.787653581	4330166.463
Russian Federatior	-34357929	-6249.6972	-106422.77	80533.4308	-6144.7407	-0.635555071	34358189.44
Saudi Arabia	2072915.8	0	7727.03084	-5872.542	-0.0001691	1.27E-11	2072938.521
Senegal	3420817.07	-10.267917	-0.0094019	13.6976991	-330.58712	4.22E-09	3420817.086
Slovak Republic	-1329772	-913.22184	-1190.58	2234.62809	-1616.1385	0.233580489	1329775.756
Slovenia	-545279.39	-75.25274	-19.027199	649.013529	-246.93562	1.067109898	545279.8367
South Africa	-9337573.5	-7475.3203	-13.881402	9977.63774	-1762.7242	-1.59E-12	9337581.989
Spain	-12703320	-185.23125	-1589.9418	1551.78432	-467.79771	0.178585067	12703320.4
Sri Lanka	-5666145.8	-0.2054825	0	0.27397813	47.3834953	8.57E-10	5666145.774
Sudan	-5100523.3	0	0	1.59E-06	-1111.3935	9.52E-10	5100523.39
Sweden	-2861366	-80.03324	-27.412954	1806.84315	-10333.674	3.847250165	2861385.257
Switzerland	-3015780.7	-0.261278	-95.950426	76.8246855	-236.84117	0.009117231	3015780.68
Tajikistan	2893252.52	-13.67029	-33.269811	43.6779578	-192.79491	0.000269844	2893252.53
Tanzania	-17401446	-0.0928531	-27.939718	21.3579869	-3996.6029	1.34E-09	17401446.72
Thailand	5956875.79	-4979.9749	-14071.608	17344.9625	-6892.6094	0.008059509	5956923.727
Trinidad &Tobago	183365.032	0	-17688.624	593.782777	-0.0154386	-1.18E-12	184217.1953
Tunisia	-1366572.8	0	-152.03167	117.272146	2.71530046	0.004331686	1366572.767
Turkey	15905222.4	-11862.476	-5812.0568	21113.6406	-632.84453	1.57498395	15905241.87
Turkmenistan	320191.262	0	-13143.692	1392.44211	0	-1.78E-11	320463.9444
Ukraine	-12966518	-3127.2088	-8569.8255	7225.69866	-8384.9968	-0.012274823	12966525.98
nited Arab Emirate	2653612.79	-9.0282312	-13813.42	10510.2525	0	1.59E-05	2653669.552
United Kingdom	-18396182	-1995.4197	-9005.3088	9526.03071	-298.43254	0.051166931	18396186.49
United States	-97222758	-40400.072	-40062.107	95151.9008	-6484.797	24.08049361	97222821.05
Uruguay	-445527.3	-0.0014924	-0.4615908	289.495966	1192.74392	0.244373795	445528.992
Uzbekistan	-9699126.6	-20.530827	-22276.051	954.059116	-25.799912	-0.000120115	9699152.197

Vietnam	-28448798	-1470.6926	-196.59737	2205.88426	-1428.7862	0.071716861	28448798.05
Yemen	-5146145	0	49.0078129	-37.245929	0.54636155	1.07E-10	5146144.978
Zambia	-3660528.2	0	0	3.75E-06	-4293.9575	3.66E-09	3660530.697
Zimbabwe	-3859369.6	-153.50421	0	71.8520504	-3272.3635	-4.03E-12	3859370.959

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