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Too Small To Protect? The Role of Firm Size in Trade Agreements

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Too Small To Protect? The Role of Firm Size in Trade Agreements*

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Abstract

This paper develops a new model of a trade agreement that puts at center stage the competing interests between firms within a sector. Larger firms favor trade liberalization whereas smaller firms favor protection. Lobbying by firms for or against the agreement is modelled as an all-pay auction, thus incorporating the feature that binding contracts over contributions for policies cannot be written. A new motive for trade agreement formation is uncovered in this framework whereby governments' incentives to liberalize are driven by the lobbying process. If a proposed agreement is over non-tariff barriers then it always entails free trade. If a proposed agreement is over tariffs then it either entails free trade, which maximizes lobbying revenue, or the tariff revenue maximizing tariff. This outcome is supported by the surprising result that, off the equilibrium path, any tariff agreement that entails lobbying and positive tariffs yields lower expected revenue for the government than a free trade agreement involving no tariff revenue.

Keywords: All-pay auction, firm heterogeneity, non-tariff barriers, tariffs, trade agreement.
JEL Classification Numbers: F02, F12, F13, D44.

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1 Introduction

The outcome of a trade agreement is determined by a tension between competing interests. In real-world commentaries of trade agreements, this tension is usually identified between export sectors on the one hand who favor greater openness and import-competing sectors on the other who favor greater protection. For example, the UK government's Department for Trade and Industry report to Parliament on the outcome of the Uruguay Round focused first and foremost on the benefits to UK export sectors (DTI 1994, p.10).[1] The phasing out of the Multi-Fibre Agreement also received attention. The reason was that this lowered tariffs in industrialized countries on textiles and clothing, which the UK imports largely from developing countries but where it still maintains some competing interests. This basic approach of considering 'export sectors versus import competing sectors' is mirrored in the literature on the economics of trade agreements as well (Mayer 1981, Grossman and Helpman 1995, Hillman and Moser 1996, Maggi and Rodriguez-Clare 1998, Bagwell and Staiger 1999).

At the same time there appears to be an awareness that, even within a sector, firms' interests over a trade agreement may conflict. For example, DTI (1994, p.10, para. 5.13) in the assessment of the Uruguay round discussed above, also draws attention to the fact that UK firms both export industrial electronics and compete with foreign imports in the same sector. Perhaps surprisingly, given the present focus on the behavior of firms in economics, it has been political scientists rather than economists who have emphasized this division of interests between firms within a sector over trade policy and agreements. This division is expressed most clearly by Thacker (1998, 2000) in his study of Mexican business interests and their relationship with government and trade liberalization: "Where ... economies of scale give competitive advantages to large firms, size will also affect a firm's trade policy preferences. Under these conditions, larger firms will be more competitive internationally and therefore more likely to favor free trade. Smaller firms will be more protectionist" (Thacker 1998, 3). This view is also at the heart of work on the GATT and World Trade Organization (WTO) by Cowles (1998) and Aaronson (2001) for example.

The present paper develops a model of a trade agreement that puts at center stage the competing interests not of sectors but of firms. It determines the set of circumstances under which a trade

[1]The Uruguay Round was the last successfully completed round of trade talks under the General Agreement on Tariffs and Trade, or GATT.

agreement will be reached and the extent of trade liberalization in an environment where heterogeneous firms can lobby the government for the trade policy that they would prefer. Therefore, rather than an exporting sector and an import-competing sector as in traditional models of trade agreements, our model features exporting firms and import-competing firms within a single sector. In this setting, where larger more profitable firms apparently have greater resources to lobby the government in support of trade liberalization, we are asking whether smaller firms are necessarily 'too small to protect'?

The economic model is essentially that of Melitz (2003): in each of two countries, a continuum of monopolistically competitive firms produces a horizontally differentiated product at varying degrees of productivity. A prediction of the Melitz model is that, in a trading equilibrium, larger firms tend to export while smaller firms tend only to serve the domestic market. Each country is ex ante identical to the other. Firms produce horizontally differentiated products and exporters export different varieties in a trading equilibrium. Trade increases economic efficiency because traded goods are produced more efficiently and hence lower priced.

In our paper, starting from autarky, a trade agreement is reached according to the following procedure.[2] The two governments announce a proposed agreement to lower trade restrictions, to the same level, across the two countries. Trade restrictions are modelled as one of two possible instruments: iceberg transport costs or ad valorem tariffs. The proposed level of the restriction in each country can lie anywhere below autarky and is bounded from below at free trade. Each firm then has one of two options with regard to lobbying. It can contribute to a lobby group that supports the proposed trade agreement: the 'pro-trade lobby'. Or it can contribute to a lobby group that supports the status quo at autarky: the 'anti-trade lobby'. This captures the well known feature of real-world trade policy-making that firms form lobby groups to lobby for and against trade agreements. For example, the 'Alliance for GATT Now' group was formed to support the Uruguay Round vote whereas the 'Labor/Industry Coalition for International Trade' was (somewhat incongruously) formed to lobby against it (Dam 2001, 14). Each government then adopts the policy that raises the most revenue, taking into account both contributions by lobby groups and tariff

[2] Our model exhibits the standard feature that autarky is a Nash equilibrium in trade restrictions, wherein national welfares are below efficient levels. Therefore our approach follows the literature in considering a trade agreement as a way to escape from inefficient equilibrium trade policy outcomes. We will review below the similarities and differences of our approach to those of the prior literature.

revenues if any. Support for the agreement by the two governments must be unanimous, otherwise the agreement fails and autarky is maintained. It is through this set of policy interactions that the equilibrium trade agreement configuration will be determined. Our framework abstracts entirely from the role of consumers in order to focus on interactions between firms and their national government in lobbying over trade policy. This approach is tractable and approximates a standard utilitarian framework that incorporates consumer surplus, firm profits, tariff revenue and contributions by lobbies, but where there is a relatively large weight in the government's payoff function on contributions made by firms. This approach plays down the conventional terms-of-trade motive for trade agreement formation, which is also present in our model as we will explain below, and brings to the forefront the forces that we want to focus on.[3]

We follow Hillman (1989) by modelling lobbying over trade policy as an all-pay auction (with complete information).[4] This type of contest has three defining features. First, as in a regular first price auction, the highest contribution wins the contest with certainty. This assumption is generally regarded to capture rent-seeking, of which trade policy is an instrument (Krueger 1974), where contribution size plays a decisive role in swinging the outcome in a lobby group's favor. Second, as in an all-pay auction but unlike in a regular first price auction, there is complete information about the lobby groups' valuations of success. This seems reasonable in a trade setting where the gains from trade to the various different parties can be estimated in a straight forward way. Third, once contributions have been made they cannot be retrieved regardless of the outcome. This assumption is reasonable in a trade policy context given that in many national legislatures, including the US Congress, it is legal for lobby groups to make contributions but illegal to link

[3]A larger number of smaller firms are more likely to find that their lobbying efforts are undermined by a coordination problem. As a result, introducing a coordination asymmetry that favors large firms tends to increase the likelihood of a trade agreement being signed. A more conventional terms-of-trade based motive for a trade agreement would arise if the interests of consumers were incorporated into the model. The incorporation of consumers is discussed further below.

[4]In addition to having been used to model lobbying over trade policy by Hillman (1988), the all-pay auction has been used to model rent-seeking and lobbying activities in general (Hillman and Samet 1987, Hillman and Riley 1989, Baye, Kovenock, and de Vries 1996), competitions for a monopoly position (Ellingsen 1991), and R&D races (Dasgupta 1986). Variations of the all-pay auction have been used to model competitions for multiple prizes (Clark and Riis 1998 and Barut and Kovenock 1998), the effect of lobbying caps (Che and Gale 1998, Kaplan and Wettstein 2006), and R&D races with endogenous prizes (Che and Gale 2003). Siegel (2009) allows the cost-effectiveness of contributions to vary across players, generalizing the idea of an all-pay auction to an all-pay contest. Other models assume a probabilistic relation between players' contributions or efforts and prize allocation. One classic example is Tullock's (1980) contest success function in which each player's probability of winning a single prize is proportional to the player's share of the total expenditures. A second classic example is Lazear and Rosen's (1981) two-player tournaments.

contributions to specific policy outcomes. Hence, it is not possible for a lobby group to write a binding contract with the government that is enforceable in a court of law.[5]

We begin our analysis of trade agreement outcomes by modelling trade restrictions as iceberg transport costs. We will refer to iceberg transport costs as 'non-tariff barriers' (NTBs) to clarify that they are determined by the government (not geography) but raise no revenue. Focusing on an agreement over NTBs will serve as a useful benchmark in two ways. First, most of the literature that examines protectionism in the Melitz model assumes iceberg transport costs because the model becomes more tractable when the revenue considerations of trade policy are ignored. So this approach will be familiar from past work. Second, it turns out that with iceberg transport costs and firm productivities distributed according to the Pareto distribution, the contest between lobby groups conforms to an all-pay auction with homogeneous valuations. That is, the aggregate profit loss of adopting an agreement by firms who favor autarky is identical to the aggregate profit gain by firms who favor trade liberalization. So the value of successfully swinging the outcome in their favor is the same to the pro-trade lobby as it is to the anti-trade lobby; valuations are homogeneous in other words. This feature holds regardless of the extent of trade liberalization under the proposed trade agreement.[6]

In equilibrium, the governments propose an NTB agreement that entails free trade because this extracts the largest average contributions from firms. A novel feature of the equilibrium is that the trade agreement does not go ahead with certainty. This outcome rests on the characterization of lobbying as an all-pay auction. It can readily be verified that there exists no pure strategy Nash equilibrium in a contest that has an underlying all-pay auction structure. Say that in equilibrium one lobby group adopts the pure strategy of contributing at some positive level. Then the probability that the other lobby wins rises discontinuously if it contributes at or above that level, and so the

[5] Thus there is an asymmetry in that the government can commit to adopt the policy preferred by the highest contributor, but contributors cannot commit to a specific level of contribution. This set-up reflects a legal framework wherein the government is allowed to accept contributions, and wants to create incentives for these to be as large as possible, but is precluded by law from linking contributions to specific policy outcomes. It has been argued in the prior literature that lobbying can be characterized as a repeated game wherein an implicit contract can be reached between lobby groups and the government. However this requires a sufficiently low discount rate which may fail to exist, especially when there is a high degree of uncertainty as to when a given administration's period in office may end. Here we are effectively assuming a discount rate that is too high to support an implicit contract.

[6] The homogeneity of valuations rests on the fact that firm productivities follow the Pareto distribution. Under alternative distributions, valuations across lobby groups will be heterogeneous. Rather than examine the implications of other distributions, in this paper we will focus on how heterogeneity of valuations arises under tariff instruments. This analysis will show that the forces illustrated by our benchmark example of homogeneous valuations arise under heterogeneous valuations as well.

other lobby will bid at less than that level with zero probability. But then the first lobby would be better off making less than its original contribution since the probability of winning would be the same. Hence a pure strategy cannot in fact arise in equilibrium. Consequently, lobbies randomize their contributions and this in turn introduces a degree of uncertainty into the outcome. Note that we do not interpret the lobbies' randomization over contributions as literally implying that they use a randomization device to decide on their contribution levels. We interpret this as reflecting the fact that they have no way to commit to a given contribution level. In this setting, the anti-trade lobby has the same probability of making a larger contribution as the pro-trade lobby and if its contribution is larger then the agreement will fail to go ahead. Therefore small firms, represented by the anti-trade lobby in an agreement over NTBs, are not necessarily too small to protect. The uncertainty in the outcome is appealing in that it stems from the realistic feature that a lobby group cannot write an enforceable contract with the government over policy outcomes.

We then redo the analysis of trade agreement formation but instead of NTBs we assume that trade restrictions are modelled as (ad valorem) tariffs. Tariffs alter the analysis of NTBs in two ways. First, tariffs generate revenue for the government. All else equal, this makes the outcome of a trade agreement with positive non-prohibitive tariffs more attractive to the government because it receives not just the lobbying revenue from the pro-trade lobby but the tariff revenue as well. This enhanced value of the trade agreement to the government creates a so-called 'head start' for the pro-trade lobby.[7] Second, ad valorem tariffs affect the extensive margin differently than NTBs (specifically iceberg transport costs) in models of monopolistic competition (Cole 2011, Schröder and Sørensen 2011, Besedeš and Cole 2013). With this set-up, starting from autarky, a move toward (but not including) free trade would yield lower aggregate profits for exporters because export profits are more adversely affected by foreign tariffs than NTBs. So under positive tariffs, differently than NTBs, the pro-trade lobby group puts a lower valuation on seeing the agreement go ahead than the anti-trade lobby group puts on preventing it. The all-pay auction literature shows that when one lobby group puts a lower valuation on success in a contest than the other they compete less aggressively and the government raises less revenue from the contest (Hillman and Riley 1989).[8] One might have expected the pro-trade lobby's head start to make it compete

[7]Head starts in all-pay auctions have been analyzed by Konrad (2002), Kaplan, Luski and Wettstein (2003) and Siegel (2014) among others.

[8]When a lobby group competes less aggressively this implies that the expected value of their contribution is lower.

more aggressively, offsetting the tendency created by its lower valuation of success. But a surprising insight from the literature is that the agent who receives a head start, in this case the pro-trade lobby, tends to compete less aggressively (Siegel 2014). Putting these insights together, both of these features tend to reduce the pro-trade lobby's aggressiveness in the competition. We show that a proposed agreement that entails both lobbying and tariff revenue raises less revenue for the government than one that entails free trade, where both lobby groups value the outcome equally. Therefore, drawing on the all-pay auction literature, a surprising and simplifying result to come out of our analysis is that the governments would rather choose an agreement that maximizes lobbying revenue at free trade than one that raises both lobbying and tariff revenue.[9] In an agreement that entails free trade, as in the NTB case, the agreement may not go ahead. The fact that the agreement may fail to go ahead means that under tariffs, as under NTBs, small firms are again not necessarily too small to protect.

An interesting feature of the all-pay auction literature with head starts that carries over to the present setting occurs where one lobby group's head start is so large that the other lobby group has no hope of winning and therefore drops out of the contest. In our model, the tariff may be sufficiently high that tariff revenue is greater than the anti-trade lobby's gain from stopping the agreement. In that case, the anti-trade lobby cannot profitably make a contribution large enough to compensate the government for its loss of tariff revenue if it stops the agreement. And so the anti-trade lobby does not contribute at all. In other words, the pro-trade lobby's head start is so large that the anti-trade lobby drops out of the contest, and so the pro-trade lobby drops out too and the agreement goes ahead with certainty. We are able to show in the paper that tariff revenue is increasing at the point where the anti-trade lobby drops out. This implies that at the revenue maximizing tariff neither lobby group makes a contribution. So the foregoing analysis implies that the governments' decision over what trade agreement to propose simplifies dramatically to choosing one of two alternatives: an agreement that maximizes lobby revenue at free trade but implies no tariff revenue, or an agreement that maximizes tariff revenue but implies no lobby revenue.

Following Grossman and Helpman (1994), the use of a menu auction to model the process of

[9]Note that here we are drawing on insights of separate analyses of all-pay auctions with heterogeneous valuations (Hillman and Riley 1989) and head starts (Siegel 2014) respectively. As far as we are aware, there is only one prior instance in the literature where heterogeneous valuations and head starts are incorporated in the same framework: that of Kaplan et al 2003. Since their setting is different from ours, our analysis may be regarded as an extension of the literature on all-pay auctions as well.

lobbying over trade policy has become standard in the literature on the political economy of tariff setting. Therefore, it is important to explain how a trade agreement with heterogeneous firms would differ when lobbying takes place according to a menu auction. To understand this consider Chang and Willmann (2014), who do use a menu auction framework to study lobbying over trade agreement formation in a Melitz model.[10] Chang and Willmann's framework is similar to ours in modelling a conflict of interest between firms within a single sector based on the Melitz model. However, there are three key differences. First, in their framework, governments take account of consumers. Second, trade agreement formation and lobbying occur concurrently in their framework, whereas in ours the firms that lobby are determined prior to trade agreement formation taking place. Third, they focus on an agreement over tariffs and vary the groups that can lobby: only exporting firms can lobby; only import-competing firms can lobby; both types of firm (exporting and import-competing) can lobby. This contrasts with our approach of varying the trade policy instrument under consideration between NTBs and tariffs but always allowing both types of firm to lobby. Their equilibrium outcome where both types of firm can lobby has two similar features to ours with NTBs in that a trade agreement involves free trade and the government collects all the surplus. Our approach brings to light two additional possibilities: first, the agreement may not go ahead in equilibrium; second, the agreement over tariffs may not involve free trade but the revenue-maximizing tariff instead.[11]

Although our paper focuses on large versus small firms within a sector, it contributes to several strands of the literature on trade agreements where the tension is between export- and import-competing sectors.[12] One strand highlights a commitment problem faced by governments wishing to liberalize (Staiger and Tabellini 1987, Staiger 1995, Maggi and Rodriguez-Clare 1998). Trade agreements can serve as a means through which a government can commit to trade liberalization in the face of opposition from protectionist forces, say industrial interest groups, within its own nation. Maggi and Rodriguez-Clare (2007) have shown that when capital is mobile between sectors

[10] As far as we are aware, Chang and Willmann (2014) is the only other paper to consider trade agreements in a setting where firms' interests over a trade agreement conflict within a sector rather than between sectors.

[11] Chang and Willmann's main focus is different from ours. In the literature that tests Grossman and Helpman's (1994) model, the estimates on the value of campaign contributions to the government relative to social welfare have generally been regarded to be implausibly low. Chang and Willmann show that modelling competing interests over a trade agreement as being between firms within a sector reveals a downward bias on the estimated value to the government of campaign contributions in the conventional between-sector approach.

[12] See Bagwell, Bown and Staiger (2014) for a comprehensive review of the literature on the economics of international trade agreements.

the government wants to set a tariff cap rather than a fixed tariff as part of the agreement. This then leaves scope for export interests to lobby ex post for further liberalization after capital has moved to the export sector. Their model predicts that the easier it is for capital to move between sectors the deeper trade liberalization will be. In our model, by contrast, the more costly it is for firms to become exporters the larger contributions will be. The reason is that free trade provides greater profits for firms who can become exporters, eliciting relatively large contributions from them in support of the agreement, and imposes greater losses on those who cannot, eliciting relatively large contributions from them as well.

Our paper also relates to the literature on the terms-of-trade driven theory of trade agreements. In that theory, trade policy has the features of a terms-of-trade driven prisoner's dilemma: each government provides excessive protection in order to improve their own nation's terms-of-trade, shifting the cost of protection onto trade partners. In that setting, Bagwell and Staiger (1999) demonstrate that the sole purpose of a trade agreement is to escape from a terms-of-trade driven prisoner's dilemma. This is true even if politics plays a role: even if lobby groups can seek to sway the outcome of an agreement through financial contributions. Our model shares some features with the terms-of-trade driven theory. Countries are large and can improve their terms of trade through tariff setting, and the terms-of-trade improvement shifts some of the costs of tariff setting onto the foreign country. This implies that there is a terms-of-trade based incentive to set tariffs too high in our model and hence a terms-of-trade driven motive for a trade agreement, just as in Bagwell and Staiger's (1999) conventional 'export versus import-competing sector' set-up. But we neutralize this motivation by leaving consumers out of the model and by assuming that a government only cares about firm profits if they are donated to the government through lobbying. So although terms-of-trade gains exist in our model, it is politics that play a central role in driving the trade agreement towards free trade.[13]

[13] A further difference between our approach and that of Bagwell and Staiger (1999) is that our agreement escapes from a Nash equilibrium involving autarky whereas theirs escapes from a Nash equilibrium that involves positive but sub-optimal trade flows. We could begin our analysis at the same starting point as Bagwell and Staiger at the cost of significant additional complexity, because we would need to consider tariffs in the initial equilibrium as well as the outcome of the agreement. We establish our new approach to the analysis of trade agreements based on the simpler initial starting point of autarky, and then discuss the extension to a starting point involving positive trade flows in the concluding section. On a different note, observe the parallel in our equilibrium outcome to Grossman and Helpman's (1994) common agency approach in which interest groups compete over trade policy. But in that setting the government has no incentive to commit to free trade as it may well extract more rents in the political equilibrium. The reason for the difference is that in Grossman and Helpman lobbying takes place over the level of trade policy whereas in ours it takes place over whether or not a proposed agreement goes ahead.

The third strand of the literature on trade agreements concerns profit-shifting theories. Governments can shift profits from foreign firms to the home country in an environment where barriers to entry allow firms to make positive profits in equilibrium. Mrazova (2011) develops a profit shifting rationale for trade agreements, based on the notion that an agreement helps to internalize an international profit-shifting externality. An agreement can play a similar role in our setting, but if the government implements this it is as a result of lobbying inducements rather than out of a concern for national welfare.[14]

The paper proceeds as follows. Section 2 sets out the model. It begins with a brief recapitulation of the Melitz model that links firm productivity to profits, and then goes on to provide microfoundations for lobbying over a trade agreement as an all-pay auction based on firm profits. Section 3 then examines the scope for reaching a trade agreement, first for NTBs and then for tariffs. Conclusions are drawn in Section 4. The Appendix presents proofs of the results and a full formalization of lobbying over tariff agreement formation as an all-pay auction.

2 The Model

There are two countries in the model, Home and Foreign. Variables pertaining to Foreign are denoted with a superscript '$*$'. There are two goods. Good X comprises a continuum of differentiated varieties indexed by i. As standard in the Melitz model, these varieties are produced by a continuum of monopolistically competitive firms, correspondingly indexed by i, each using an increasing returns to scale technology. Each variety of good X may face a trade barrier as it crosses the international border. There is free entry in sector X but entry is costly, so profits are zero in expectation but positive ex post. Good Y is homogeneous, produced under constant returns to scale, perfect competition and free trade. Also, good Y is the numeriare and, since it is freely traded, world prices and domestic prices of this good can be normalized to 1. Each unit of good Y is produced from just one unit of labor, and so the wage is normalized to 1 as well. The role of good Y in the model is to capture the general equilibrium effects of trade agreement formation.

[14]The delocation theory of trade agreements due to Ossa (2011) is also related. He considers a monopolistically competitive setting in which firms producing differentiated products compete for sales in both the home and foreign markets under conditions of free entry, and where exporting the product abroad involves shipping costs. In Ossa's model each country could enjoy savings in transport costs if it could have more of the world's firms located locally rather than abroad, generating a delocation externality, and the purpose of a trade agreement is to internalize this.

For this purpose, we will assume that each country is endowed with a sufficient quantity of labor that enough of good Y is produced to clear the trade account in equilibrium. In terms of economic structure, the two countries are identical to one another but will produce different varieties in a trading equilibrium.

There are two time periods. In period 1 there is autarky in both sectors of both countries, with trade restrictions set at prohibitive levels. Producers of good X undertake entry decisions and production. Then markets clear for period 1 and consumption takes place. Autarky constitutes a Nash equilibrium in trade policies in the sense that national welfare would decline as a result of unilateral trade liberalization. In period 2 the governments may announce a proposed trade agreement, which would entail free trade in good Y, and a symmetrical reduction in trade restrictions for good X to a level that would allow at least some trade in good X. Governments are able to communicate with each other over the proposed trade agreement before announcing it, and each has veto power over the agreement until it has been implemented. For simplicity we will assume that this announcement is unanticipated. Once the proposed trade agreement is announced, each firm predicts (perfectly) the amount of profit it would earn under the alternatives of autarky and the trade agreement and hence which policy it would prefer. There are two lobby groups, one named L_A that supports the status quo of autarky in sector X and one named L_T that supports the trade agreement. Each firm makes a contribution to one of the lobby groups, depending on the regime under which its profits would be maximized. We will assume that within the group of firms making contributions to a particular lobby group the collective action problem is resolved. Aggregate contributions to L_A total l_A and aggregate contributions to L_T total l_T. Once collected, these contributions are passed by the lobby groups to the government and cannot be retrieved thereafter. The policy that receives the greatest financial support, autarky or the trade agreement, is then endorsed by the government. If the agreement is endorsed by both governments then it is enacted. Otherwise autarky is maintained. If the trade agreement goes ahead, the government resets tariffs in accordance with the agreement. Finally, conditional on tariffs, consumption takes place and markets clear.

Although we will undertake separate treatments of trade agreements over NTBs and tariffs, our development of the model will incorporate both instruments simultaneously. This will provide a parsimonious representation of the model and make it possible to compare the effects of the two restrictions. The remainder of this section specifies the details of the model.

2.1 Consumers

Let the within-period utility function for the representative agent in Home take the following form:

$$U = \mu \ln (X) + Y \tag{1}$$

where

$$X = \left(\int_{i \in \Omega} x(i)^{\theta} di \right)^{\frac{1}{\theta}},$$

$0 < \theta < 1$, $\varepsilon = 1/(1-\theta)$ is the elasticity of substitution between varieties of X, and Ω is the set of varieties (potentially) available to the consumer. Demand for each good by a consumer in Home is

$$x(i) = \frac{p(i)^{-\varepsilon} \mu}{\mathcal{P}^{1-\varepsilon}}$$

where $p(i)$ is the price of variety i sold in Home, and

$$\mathcal{P}^{1-\varepsilon} = \int_{i \in \Omega} p(i)^{1-\varepsilon} di$$

is the aggregate price index in Home. An analogous set of equations holds for the foreign country where, by assumption, $\mu^* = \mu$.

2.2 Heterogeneous Firms

Firms considering entry to sector X face a one time sunk market entry cost f_E (measured in units of labor). If this cost is paid, the firm then draws a constant marginal cost coefficient a from the Pareto distribution

$$G(a) = \left(\frac{a}{a_U} \right)^k, \ 0 < a < a_U$$

where the shape parameter k is assumed to take values $k > (\varepsilon - 1)$. We will denote by a_i the marginal cost drawn by firm i. Once this is observed, a firm decides whether or not to undertake production. If it chooses to produce, it must incur an additional fixed cost f_D paid each period. If trade restrictions are at a level that admits trade, a firm must pay an additional fixed cost

$f_X = \gamma f_D > f_D$ in order to serve the foreign market. Production exhibits constant returns to scale with labor as the only factor of production.

The decision of whether or not to undertake production and whether to export depends on the associated profits. Given the wage equal to 1, the per-period operating profit of firm i facing marginal cost a_i and selling only domestically is

$$\begin{aligned} \pi_D(i,t,\tau) &= [p(i) - a_i]\, Q_D(i) - f_D \\ &= \left[\frac{[p(i) - a_i]\mu}{\mathcal{P}^{1-\varepsilon}} \right] p(i)^{-\varepsilon} - f_D. \end{aligned}$$

A firm selling domestically will charge a price equal to a constant markup over marginal cost, $p(i) = \frac{a_i}{\theta}$. Therefore, the operating profit function for a purely domestic firm is

$$\pi_D(i,t,\tau) = a_i^{1-\varepsilon} B - f_D \tag{2}$$

where

$$B = \frac{1}{\varepsilon \theta^{1-\varepsilon}} \left(\frac{\mu}{\mathcal{P}^{1-\varepsilon}} \right).$$

We have expressed (2) here as a function of tariffs, t, and NTBs, τ, to allow for the possibility that policy variables affect the price index, $\mathcal{P}$. In due course, we will provide a solution for $\mathcal{P}$ that makes explicit the effects of t and τ.

In order to reach the foreign market, in addition to the fixed cost γf_D, firm i incurs a per-unit cost arising either from t or τ: $t > 1$ is ad valorem and $\tau > 1$ takes iceberg form. For tractability we assume that any tariff revenue is used by the government to consume only the numeraire.[15] Thus the additional operating profit from exporting for a firm that exports is

$$\pi_X(i,t,\tau) = \frac{(t\tau a_i)^{1-\varepsilon} B^*}{t} - \gamma f_D, \tag{3}$$

[15] Tariff revenue can only affect demand for good X if: (1) the government's preferences are not, in fact, encompassed by the representative consumer and the government demands some of good X; (2) tariffs are not symmetric across countries and thus income is being shifted from one country to the other (however this is not an issue with quasi-linear utility) or; (3) the government simply throws the revenue away (this is not an issue with quasi-linear utility as long as a positive amount of the numeraire is produced and consumed). We do not allow for these three possibilities in our model.

where

$$B^* = \frac{1}{\varepsilon\theta^{1-\varepsilon}} \left(\frac{\mu}{\mathcal{P}^{*1-\varepsilon}} \right).$$

Note that the tariff affects export profits differently to the NTB. The reason can be understood by first appreciating that an NTB is modelled as an iceberg transport cost. Through its monopolistic power, a firm is able to recoup a portion of its losses in transport by charging a markup over marginal cost, whereas tariff revenue is completely captured by the domestic government. This set-up is standard in the literature on heterogenous firms.

2.3 Government

The government's objective is to maximize income, which is derived entirely from tariff revenue and contributions by lobby groups. The government consumes only the numeraire good. Therefore, the only effect of government policy on sector X comes through the distortions created by t and τ. In period 2, when governments may propose a trade agreement, the Home government's problem is formalized as follows:

$$\max_{t,\tau} \{l_A(t,\tau), l_T(t,\tau) + TR(t,\tau)\}, \tag{4}$$

where $TR(t,\tau)$ represents tariff revenue for given t and τ. (Recall that in period 1 when there is autarky the government earns no income). The Foreign government's problem is analogous. The government chooses whichever policy receives the greatest financial support: if $l_A(t,\tau) < l_T(t,\tau)+TR(t,\tau)$ then the agreement goes ahead while if $l_A(t,\tau) > l_T(t,\tau)+TR(t,\tau)$ then autarky is maintained. If $l_A(t,\tau) = l_T(t,\tau)+TR(t,\tau)$ then a coin toss determines whether or not the trade agreement goes ahead. This specification shows the sense in which tariff revenue provides the pro-trade lobby with a 'head start'.

Although the problem here is defined over NTBs and tariffs, τ and t, by assumption governments will make an agreement over only one instrument. That is, in the next section tariffs will be set to zero ($t = 1$) while we first consider an agreement over NTBs. After that, NTBs will be set to zero ($\tau = 1$) while we consider an agreement over tariffs. Until we consider each type of agreement in turn, we will continue to develop the model in terms of both τ and t together.

2.4 Lobby Groups

Based on the assumption that contributions to the government cannot be retrieved once they have been made, we can model the lobbying process as a first price all-pay auction (with complete information). Lobbies contest the government's decision over whether or not to adopt a trade agreement. The contribution that each lobby group makes to the government may be regarded as a (non-negative) sealed bid. The 'value to L_A of maintaining autarky', denoted v_A, is given by

$$v_A = v_A(t,\tau) = N_E \int_{\varphi}^{a_A} \left(\pi_A(a,t,\tau) - \pi_T(a,t,\tau)\right) dG(a), \tag{5}$$

where N_E is the number (mass) of entrepreneurs taking a draw from the productivity distribution, φ is the firm that is just indifferent between autarky and the trade agreement, and a_A is the least efficient firm in autarky. Also, $\pi_A(a,t,\tau)$ and $\pi_T(a,t,\tau)$ are the respective profit levels of firm $a \in [\varphi, a_A]$ under autarky and the trade agreement: $\pi_A(a,t,\tau) = \pi_D(a,t,\tau)$ since all firms serve only the domestic market in autarky; $\pi_T(a,t,\tau) = \pi_D(a,t,\tau) + \pi_X(a,t,\tau)$ where $\pi_X(a,t,\tau) = 0$ if under the agreement firm a serves only the domestic market.[16] The 'value to L_T of the trade agreement being adopted' is

$$v_T = v_T(t,\tau) = N_E \int_{0}^{\varphi} \left(\pi_T(a,t,\tau) - \pi_A(a,t,\tau)\right) dG(a). \tag{6}$$

On this basis, the value to each lobby group of swaying the government's decision in its favor depends on the underlying profits made under the respective policies of autarky and the trade agreement. The payoff to lobby L_A is given by

$$u_A(t,\tau) = \begin{cases} -l_A(t,\tau) & l_A(t,\tau) < l_T(t,\tau) + TR(t,\tau) \\ \frac{v_A(t,\tau)}{2} - l_A(t,\tau) \quad \text{if} & l_A(t,\tau) = l_T(t,\tau) + TR(t,\tau) \\ v_A(t,\tau) - l_A(t,\tau) & l_A(t,\tau) > l_T(t,\tau) + TR(t,\tau) \end{cases}. \tag{7}$$

The payoff function for L_T is analogous. This specification of the payoff function makes clear that each lobby group must pay their contribution whether or not they are successful in the contest.

[16] These terms will be defined explicitly when we characterize the equilibrium.

3 Trade Agreements with Heterogenous Firms

We will begin by characterizing firm behavior under autarky and a trade agreement. We then use these characterizations to analyze trade agreement formation under NTBs and then under tariffs.

3.1 Autarky

Use subscript-A to denote autarky. There exists a firm whose marginal cost, a_A, makes it just indifferent between supplying the domestic market and exiting. Using equation (2), a_A is characterized by:

$$f_D = \frac{\mu}{\varepsilon}\left(\frac{a_A}{\theta \mathcal{P}_A}\right)^{1-\varepsilon}. \tag{8}$$

There is free entry, so that an entrepreneur will pay to take a draw from the productivity distribution as long as the present value of average profits $\bar{\pi}$ is positive. We assume that in period 1 firms fully discount profits that they expect to make in period 2, and so take into account only the expected profit in the current period when making their entry decision.[17] The free entry condition is

$$V(a_A)B_A - G(a_A)f_D = f_E \tag{9}$$

where

$$V(z) = \int_0^z a^{1-\varepsilon} dG(a) = \frac{k}{k-\varepsilon+1}\left(\frac{z}{a_U}\right)^k z^{1-\varepsilon}.$$

Conditions (8) and (9) close the model and pin down a_A and the number (mass) of entrepreneurs taking a draw from the productivity distribution, N_E. We can now provide a solution for $\mathcal{P}$:

$$\mathcal{P} = \left(\left[\frac{k}{k+1-\varepsilon}\right]\left[N_D\left(\frac{a_D}{\theta}\right)^{1-\varepsilon} + N_X\left(\frac{t\tau a_X}{\theta}\right)^{1-\varepsilon}\right]\right)^{\frac{1}{1-\varepsilon}}.$$

From this we can see that $\mathcal{P}$ depends on t and τ; the price index is increasing in trade frictions.

Since we restrict entry to period 1, our only equilibrium conditions in that period are the non-negative profit conditions of that period. This set of assumptions would only be restrictive if more entrepreneurs wanted to take a productivity draw under the trade agreement than under autarky.

[17] This assumption does not affect the qualitative results, but affects the number of firms taking a draw, N_E.

In that case we would need to account for these new entrants and how they might lobby.[18] Our approach of fixing the mass of entrants is not unlike others in the literature such as Eaton and Kortum (2005), Arkolakis (2008), Chaney (2008), and Do and Levchenko (2009). Since we have assumed firm productivity is distributed Pareto, we can find closed form solutions for these two variables:

$$a_A = \left[\frac{\psi f_E}{f_D}\right]^{\frac{1}{k}} a_U,$$
$$N_E = \frac{\theta\mu}{k f_E},$$

where

$$\psi \equiv \frac{[k-(\varepsilon-1)]}{(\varepsilon-1)} > 0.$$

Note that all parameters are identical across countries, so it follows that $a_A = a_A^*$. Furthermore, in order to rule out corner solutions (i.e. to ensure that $a_A \leq a_U$), we make the simplifying assumption that the shape parameter k is bounded from above as well: $k \leq (\varepsilon-1)[f_E + f_D]$. Finally, we are left with ex post aggregate per period industry profits in autarky equal to

$$\begin{aligned}\Pi_A &= N_E \int_0^{a_A} \pi_A(a,t,\tau)\, dG(a) \\ &= N_E\left[V(a_A)B_A - G(a_A)f_D\right] = \frac{\theta\mu}{k}. \qquad (10)\end{aligned}$$

This solution for Π_A gives us a simple parametric expression that will be used to underpin the values to L_A and L_T of swinging the government's decision in their respective favors.

3.2 Basics of a Trade Agreement

To develop the framework needed to analyze equilibrium under a trade agreement, begin by assuming that a trade agreement proposed by the two governments has been adopted. We can now derive the expressions needed to evaluate the payoffs. Under the trade agreement, a given firm

[18] Since productivity is distributed Pareto, this is not an issue as it can be shown that

$$V(a_D)B_T - G(a_D)f_D + \frac{V(a_X)(t\tau)^{1-\varepsilon}B_T}{t} - G(a_X)f_X \leq f_E.$$

will continue to produce domestically if it makes nonnegative profits from doing so. There exists a cutoff productivity level a_D which represents the productivity of the firm that is indifferent under the trade agreement between supplying the domestic market and exiting:

$$\frac{\mu}{\varepsilon}\left(\frac{a_D}{\theta \mathcal{P}_T}\right)^{1-\varepsilon} = f_D. \tag{11}$$

A firm will export if the profits from doing so are positive. The cutoff productivity level a_X for becoming an exporter is given by the following condition:

$$\frac{\mu}{\varepsilon t}\left(\frac{\tau t a_X}{\theta \mathcal{P}_T^*}\right)^{1-\varepsilon} = \gamma f_D. \tag{12}$$

Since we are considering a symmetric trade agreement in which trade restrictions are at the same level in both countries, with aggregate expenditure on good X identical across both countries and equal to μ, we will have a symmetric equilibrium under an agreement and henceforth drop the $*$-superscripts. Thus, these two conditions (11) and (12) close the model under the trade agreement. Since firm productivity is distributed Pareto, we can find closed form solutions for the two cutoffs:

$$\begin{aligned} a_D &= \left[\frac{\gamma^{\psi}\tau^k t^{\frac{k}{\theta}}}{\gamma^{\psi}\tau^k t^{\frac{k}{\theta}}+t}\frac{\psi f_E}{f_D}\right]^{\frac{1}{k}} a_U; \\ a_X &= \left[\frac{1}{\gamma^{\psi}\tau^k t^{\frac{k}{\theta}}+t}\frac{\psi f_E}{\gamma f_D}\right]^{\frac{1}{k}} a_U. \end{aligned}$$

Using these cut-offs, we are able to calculate ex-post aggregate industry profits in an equilibrium under the trade agreement, Π_T:

$$\begin{aligned} \Pi_T(t,\tau) &= N_E \int_0^{a_D} \pi_T(a)\, dG(a) \\ &= \frac{\theta\mu}{k}\left[\frac{\gamma^{\psi}\tau^k t^{\frac{k}{\theta}}+1}{\gamma^{\psi}\tau^k t^{\frac{k}{\theta}}+t}\right] \end{aligned} \tag{13}$$

Since by (10), $\Pi_A = \theta\mu/k$, we can write Π_T as

$$\Pi_T = \Pi_A\left[\frac{\gamma^{\psi}\tau^k t^{\frac{k}{\theta}}+1}{\gamma^{\psi}\tau^k t^{\frac{k}{\theta}}+t}\right]. \tag{14}$$

For the firm φ that is just indifferent between autarky and the trade agreement,

$$\pi_T(\varphi, t, \tau) - \pi_A(\varphi, t, \tau) = 0.$$

We can now obtain the following solution for φ:

$$\varphi(t, \tau) = \left(\frac{1}{\gamma}\left[(1 + \tau^{1-\varepsilon}t^{-\varepsilon}) - H^{1-\varepsilon}\right]\right)^{\frac{1}{\varepsilon-1}} a_D = \lambda a_D \tag{15}$$

where

$$H \equiv \left[\frac{\gamma^{\psi}\tau^{k}t^{\frac{k}{\theta}}}{\gamma^{\psi}\tau^{k}t^{\frac{k}{\theta}} + t}\right]^{\frac{1}{k}},$$

and

$$\lambda = \left(\frac{1}{\gamma}\left[(1 + \tau^{1-\varepsilon}t^{-\varepsilon}) - H^{1-\varepsilon}\right]\right)^{\frac{1}{\varepsilon-1}}.$$

We can now use these expressions for aggregate profits and related variables to micro-found the lobbying process.

To relate the difference in lobbying revenues to the difference in aggregate profits, use (5), (6), (10) and (14):[19]

$$\begin{aligned} v_A(t, \tau) - v_T(t, \tau) &= N_E \int_0^{a_A} \left(\pi_A(a, t, \tau) - \pi_T(a, t, \tau)\right) dG(a) \\ &= \Pi_A - \Pi_T(t, \tau). \end{aligned} \tag{16}$$

Therefore, the difference in the valuations of success in lobbying to L_A and L_T is determined by the difference in aggregate profits under the two outcomes.

3.3 Trade Agreement Over NTBs

In this subsection we remove the effect of tariffs by setting $t = 1$. With NTBs, any agreement that the governments reach raises no revenue; $TR(1, \tau) = 0$. So the home government's problem (4) may be simplified to

$$\max_{\tau} \{l_A(1, \tau), l_T(1, \tau)\}.$$

[19]This derivation uses the fact that $\pi_T(a, t, \tau) = 0$ for $a \in [a_D, a_A]$ to rewrite equation (14) as $\Pi_T = N_E \int_0^{a_A} \pi_T(a, t, \tau)\, dG(a)$.

By inspection of (14), if we set $t = 1$ then ex-post aggregate industry profits in autarky and the trade agreement are always equal for any proposed agreement, i.e. $\Pi_T(1, \tau) = \Pi_A = \theta\mu/k$ for any τ. Therefore, by (16), $v_A(1, \tau) = v_T(1, \tau)$.[20] Intuitively, this means that the aggregate gains from adopting the trade agreement exactly offset the aggregate losses; for any given proposed level of τ, the value to L_A of maintaining autarky is the same as the value to L_T of adopting the trade agreement. In the language of the literature on all-pay auctions, we can say that in an agreement over NTBs the lobby groups have 'homogenous valuations'. Taking the agreement NTB level as given at τ, we can then analyze the interaction between the lobby groups as a game that takes the form of an all-pay auction. The Nash equilibrium outcome of a two-player all-pay auction with homogeneous valuations is known from Hillman and Riley (1989) and Baye, Kovenock and de Vries (1996). Their characterization of equilibrium is adapted to the present context in the following lemma.

Lemma 1. *In an agreement over NTBs, taking the level of τ as given, the Nash equilibrium is unique and symmetric. The value to lobby group L_A of remaining in autarky is the same as the value to L_T of moving to a proposed trade agreement for any τ between (and including) autarky and free trade. Therefore, the equilibrium outcome is one in which both L_A and L_T randomize their respective contributions l_A and l_T continuously on the interval $[0, v_A(1, \tau)]$. Since the lobby groups randomize continuously, the highest contribution wins with probability one. Each lobby group earns an expected payoff of zero in equilibrium. The expected payoff to the government is $v_A(1, \tau) = v_T(1, \tau)$.*

This equilibrium characterization has the feature that there can be no equilibrium in pure strategies. In fact this logic applies not just to pure strategies, as described in the Introduction, but also to mixed strategies where either lobby group tends to favor a contribution at a particular contribution level in the interval $[0, v_A(1, \tau)]$ but does not contribute at that level with certainty. As a benchmark, consider the situation where a given lobby group does not favor any particular contribution level within the interval but instead randomizes continuously over the interval $[0, v_A(1, \tau)]$. Then with a continuous probability distribution the probability that any particular contribution

[20] But note that while Π_A and Π_T do not vary with τ, $v_A(1, \tau)$ and $v_T(1, \tau)$ do. We will explore the properties of $v_A(1, \tau)$ and $v_T(1, \tau)$ further in due course. These depend on the Pareto distribution and the iceberg transport cost assumptions. However, the tensions described are present in general.

level is chosen is zero. On the other hand, if a particular contribution level $l_g \in [0, v_A(1,\tau)]$ is favored by L_g, $g \in \{A, T\}$, then we say that L_g 'puts mass' at l_g; that is, it contributes l_g with positive probability. But if L_g puts mass at $l_g \in [0, v_A(1,\tau))$ then L_h, $h \neq g$ has an incentive to shift mass to a point above l_g such that it outbids L_g on average and increases its expected payoff at L_g's expense. Once one group puts mass at $v_A(1,\tau)$, the tendency is for the other group to put mass at a lower contribution level, and so on. Consequently, in equilibrium it must be the case that both lobby groups randomize continuously on the entire interval $[0, v_A(1,\tau)]$, competing each others' expected profits from the contest to zero in the process. The reason the government's expected revenue is v_A is because each lobby group expects to contribute $v_A/2 = v_T/2$ on average. The reason expected payoffs to the lobby groups are zero is because each group expects to win half the time.

So far we have only said that the governments may propose to reduce NTBs from autarky in a symmetrical trade agreement. We can go further and predict the trade agreement that the governments will propose. To do so, first recall that both lobby groups randomize continuously over $[0, v_A(1,\tau)]$. So in order to maximize the lobby groups' average contributions, each government needs to choose τ in such a way as to maximize the value of autarky $v_A(1,\tau)$, to L_A. For this, the government needs to choose τ in such a way that it makes the firms who would lobby for autarky as badly off as possible under the trade agreement:[21]

$$\max_{\tau} \; v_A(1,\tau) = \max_{\tau} \; N_E \left[\int_{\varphi}^{a_A} \pi_A(a,1,\tau)\, dG(a) - \int_{\varphi}^{a_X} \pi_X(a,1,\tau) dG(a) - \int_{\varphi}^{a_D} \pi_D(a,1,\tau)\, dG(a) \right]. \tag{17}$$

To characterize the problem, first note that when τ changes the following also change: the firm that is indifferent between autarky and trade, φ; the least efficient exporter, a_X; the profits from exporting, $\pi_X(a,1,\tau)$; the least efficient purely domestic firm, a_D; and the domestic profits of firms that would prefer autarky to trade. By definition, it follows that $\pi_X(a_X,1,\tau) = 0$, $\pi_D(a_D,1,\tau) = 0$ and $\pi_A(\varphi,1,\tau) - \pi_T(\varphi,1,\tau) = 0$. Therefore, the tension we are concerned with is how the decrease in domestic profits that occur when the government reduces τ weigh against the increase in profits

[21] Identically, the government could maximize $v_T(1,\tau)$ by choosing τ to make the firms who lobby for the trade agreement as well of as possible and arrive at the same revenue maximizing policy.

from exporting. In formal terms:

$$\frac{dv_A(1,\tau)}{d\tau} = -N_E \left[\int_{\varphi}^{a_X} \frac{d\pi_X(a,1,\tau)}{d\tau} dG(a) + \int_{\varphi}^{a_D} \frac{d\pi_D\,(a,1,\tau)}{d\tau} dG(a) \right]. \tag{18}$$

The first term in brackets captures the effect of a change in τ on the sum of export profits $\pi_X\,(a,1,\tau)$ of the group of firms that export but would be better off in autarky, $a \in (\varphi, a_X)$. The second term in brackets captures the effect of a change in τ on domestic profits of those same firms plus the domestic profits of the firms that only serve the domestic market. As NTBs are lowered, $\pi_D\,(a,t,\tau)$ is reduced for all firms. But at the same time, as NTBs are lowered, this increases the return that a given firm would make from exports. Lowering NTBs also changes the set of firms that would find it profitable to export. The lower the NTB in the trade agreement, the larger the mass of firms that would prefer the trade agreement to autarky; φ increases. These are the tensions that the government faces in setting τ. The balance of these tensions is revealed in the following result.

Lemma 2. *The NTB policy that maximizes lobby revenue for the government is free trade:* $\tau = 1$.

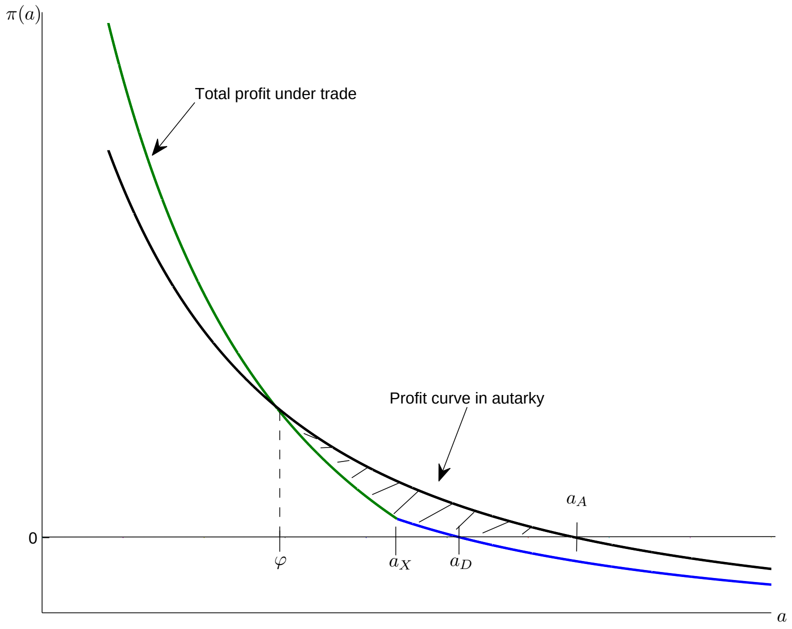
The proof shows that

$$\frac{dv_A(1,\tau)}{d\tau} = -\frac{N_E k f_D}{\tau\psi} \left[\gamma^{\psi}\tau^{k+1-\varepsilon} - 1\right] \lambda^{k+1-\varepsilon} \left(\frac{a_D}{a_U}\right)^k \epsilon_{a_D}^{\tau} < 0 \quad \forall\ \tau \geq 1$$

where $\epsilon_{a_D}^{\tau}$ is the elasticity of the marginal purely domestic firm. This is negative because $\gamma > 1$, $\tau \geq 1$, and $k > \varepsilon - 1$. This means that the value of autarky to the firms in L_A monotonically increases as τ is reduced. So by lowering the NTB associated with the agreement the government increases the interval $[0, v_A\,(1,\tau)]$ over which firms continuously randomize their contributions, hence increasing the size of the (average) contribution by each lobby group. By combining Lemmas 1 and 2 we can say that, under an agreement involving NTBs, the governments will propose an agreement that involves free trade: $\tau = 1$.

Proposition 1. *Assume an agreement over NTBs. The governments propose a free trade agreement, wherein $\tau = 1$. Both L_A and L_T randomize their respective contributions l_A and l_T continuously on the interval $[0, v_A\,(1,\tau)]$ and the highest contribution wins with probability one. Each lobby group earns an expected payoff of zero in equilibrium. The expected payoff to the government is $v_A\,(1,1) = v_T\,(1,1)$.*

Figure 1: Profits Under Two Regimes: Autarky and Trade



Further intuition about the characterization of equilibrium can be gleaned from the graphical illustration in Figure 1. With a lower NTB domestic profits decrease, as illustrated by a downward shift of the blue line. However, at the same time profits from exporting increase thereby shifting the green line to the right. As explained above, the government's problem can be formalized as seeking to maximize aggregate contributions by L_A. Our formalization shows that this is equivalent to maximizing the difference between profits under autarky and the trade agreement, or maximizing the shaded region in Figure 1. Under NTBs, this area is maximized under free trade, $\tau = 1$.

Given that lobby group valuations are homogenous, $v_A(1,1) = v_T(1,1)$, the government expects to receive contributions equal to the valuation $v_A(1,1)$. This is similar to Grossman and Helpman's (1994, p. 846) Example 2 in which the whole population is represented by one interest group or another. Competition between interest groups is maximized in this situation and all surplus is transferred to the government through competition over contributions. A similar process is at work in our setting except that here the lobby groups are competing over the outcome of a trade agreement whereas in Grossman and Helpman (1994) the setting is the determination of trade policy in a single small country.

This observation serves to reinforce the sense in which the outcome of a trade agreement is different in our paper from that of Bagwell and Staiger (1999). In their setting, the initial non-cooperative Nash equilibrium is determined in part by competition between the lobbies. It is the fact that the countries are large that underpins the terms-of-trade driven prisoner's dilemma in which each is excessively protectionist. Then the purpose of the trade agreement is to neutralize the prisoner's dilemma. In our setting, it is the trade agreement itself that generates competition between the two lobby groups. And it is a desire to maximize the size of contributions that drives the agreement to free trade. As we will see, this effect is central to the outcome of a tariff agreement as well.

3.4 Trade Agreement Over Tariffs

Next we consider an agreement over tariffs, t, while setting $\tau = 1$. Tariff revenue is raised by any outcome other than autarky and free trade, and so equation (4) is the appropriate expression of the Home government's problem. For this form of agreement we need to calculate explicitly how much tariff revenue the government would receive for any tariff level t in a proposed agreement:

$$\begin{aligned} TR(t,1) &= (t-1) \times \text{imports value} \\ &= (t-1) N_E \int_0^{a_X} p(a)x(a)dG(a) \\ &= \left[\frac{(t-1)}{\gamma^{\psi} t^{\frac{k}{\theta}} + t} \right] \mu. \end{aligned} \tag{19}$$

To calculate aggregate profits of the firms that prefer the trade agreement we set $\tau = 1$ in (13) to obtain

$$\Pi_T(t,1) = \frac{\theta\mu}{k} \left[\frac{\gamma^{\psi} t^{\frac{k}{\theta}} + 1}{\gamma^{\psi} t^{\frac{k}{\theta}} + t} \right]. \tag{20}$$

We can now calculate, for any trade agreement involving positive but non-prohibitive tariffs $t > 1$, the difference between the aggregate profits of firms that prefer autarky and those that prefer the trade agreement. This is calculated by taking the difference between (10) (with $\tau = 1$) and (20):

$$0 \leq \Pi_A - \Pi_T(t,1) = \left[\frac{(t-1)}{\gamma^{\psi} t^{\frac{k}{\theta}} + t} \right] \frac{\theta\mu}{k} = \eta TR(t,1) \leq TR(t,1), \tag{21}$$

where $\eta = \theta/k < 1$. Interestingly, the aggregate profits of firms that gain from the trade agreement are less than the aggregate profits of those who would prefer autarky. This is because of a standard rent-shifting effect whereby the foreign government shifts profits away from exporting firms. However, from (10) (with $\tau = 1$) and (20) we can also say that the combined aggregate profits of firms that prefer the trade agreement plus tariff revenue is greater than the aggregate profits of firms that prefer autarky:

$$\Pi_T(t,1) \leq \Pi_A \leq \Pi_T(t,1) + TR(t,1). \tag{22}$$

Using equations (16) and (22), we have that

$$v_A(t,1) - TR(t,1) \leq v_T(t,1),$$

holding with equality only at autarky and free trade. So although the value of the agreement net of tariff revenue to L_T is less than the value of remaining in autarky is to L_A, the combined value of the agreement to L_T and the government is greater than (or equal to at free trade and autarky) the value of autarky to L_A: $v_A(t,1) \leq v_T(t,1) + TR(t,1)$. Recall from Section 2.3 that if $l_A < l_T + TR(t,1)$ then the outcome goes in L_T's favor. So the presence of tariff revenue will provide L_T with a head start in the contest over the agreement. The presence of this asymmetry in the agreement over tariffs changes the basic characterization from an all-pay auction with homogenous valuations to one where valuations are heterogeneous and the pro-trade lobby has a head start.

We know from the respective literatures on all-pay auctions with heterogeneous valuations and head starts that, differently from equilibrium with homogeneous valuations, there may be an incentive to put mass at a certain value. We allow for this possibility by introducing the following notation: $\alpha_g(z)$ denotes the mass that lobby $g \in \{A, T\}$ puts at contribution level $l_g = z$;[22] $l_g \sim U[x,y]$ denotes that lobby g randomizes its contribution continuously over the uniform distribution $U[x,y]$. Because the case $\alpha_g(0)$ will feature prominently in our analysis, for brevity we will let $\alpha_g \equiv \alpha_g(0)$.

We can now use similar logic as for a trade agreement with NTBs to determine the ranges over which the respective lobbies will contribute. First let us consider contributions by L_A. As with NTBs, there can be no incentive to bid more than $v_A(t,1)$. But, differently from the NTB case,

[22] That is, $\alpha_g(z)$ denotes the probability with which L_g contributes at the level $l_g = z$.

if $TR(t,1) > 0$ then any $l_A < TR(t,1)$ will lose for sure. In addition, as in the NTB case, there can be no incentive to contribute a positive amount with positive probability because L_T can win for sure with a slightly higher aggregate (i.e. inclusive of TR) contribution. We cannot, however, rule out the possibility that L_A puts mass at 0, i.e. may choose to set $\alpha_A > 0$. If $\alpha_A > 0$ then L_A will, with probability $1 - \alpha_A > 0$, randomize its contribution continuously over the interval $l_A \sim U[TR(t,1), v_A(t,1)]$.

Next let us consider contributions by L_T. As with NTBs, there can be no incentive to bid less than 0. But, differently from the NTB case, any contribution above $v_A(t,1) - TR(t,1)$ would be wasteful given that the government receives $TR(t,1) > 0$ as well if the agreement goes ahead. Therefore, without loss of generality, L_T may put mass at 0, setting $\alpha_T > 0$, and with probability $1 - \alpha_T$ randomize its contribution continuously over the interval $l_T \sim U[0, v_A(t,1) - TR(t,1)]$. The characterization of equilibrium then entails determination of equilibrium values for α_A and α_T.

As the next step towards characterization, observe that the choices of equilibrium strategies by L_A and L_T, together with the corresponding expected payoffs, are completely characterized in Table 1.[23]

Table 1: Expected payoffs

		Autarky Lobby: $l_A = 0$	Autarky Lobby: $l_A \sim U[TR, v_A]$
Trade Lobby	$l_T = 0$	$\mathbb{E}[u_A(t,1)] = 0$ $\mathbb{E}[u_T(t,1)] = v_T$ $\mathbb{E}[u_G(t,1)] = TR$	$\mathbb{E}[u_A(t,1)] = v_A - \left(\frac{v_A+TR}{2}\right)$ $\mathbb{E}[u_T(t,1)] = 0$ $\mathbb{E}[u_G(t,1)] = \left(\frac{v_A+TR}{2}\right)$
Trade Lobby	$l_T \sim U[0, (v_A - TR)]$	$\mathbb{E}[u_A(t,1)] = 0$ $\mathbb{E}[u_T(t,1)] = v_T - \left(\frac{v_A-TR}{2}\right)$ $\mathbb{E}[u_G(t,1)] = TR + \left(\frac{v_A-TR}{2}\right)$	$\mathbb{E}[u_A(t,1)] = \frac{v_A}{2} - \left(\frac{v_A+TR}{2}\right)$ $\mathbb{E}[u_T(t,1)] = \frac{v_T}{2} - \left(\frac{v_A-TR}{2}\right)$ $\mathbb{E}[u_G(t,1)] = \frac{TR}{2} + v_A$

Table 1 represents the fact that L_A will contribute $l_A = 0$ with probability α_A and $l_A \sim U[TR(t,1), v_A(t,1)]$ with probability $1 - \alpha_A$, while L_T will contribute $l_T = 0$ with probability α_T and $l_T \sim U[0, v_A(t,1) - TR(t,1)]$ with probability $1 - \alpha_T$. The payoffs in Table 1 can thus be

[23]For compactness, in Table 1 we omit the functional notation $(t,1)$ from $TR(t,1)$, $v_A(t,1)$, and $v_T(t,1)$ and just write TR, v_A, and v_T.

translated into expected payoff functions for the respective lobbies and for the government. Writing the expected payoff for lobby L_A as $\mathbb{E}[u_A(t,1)]$, we can write L_A's expected payoff function as:

$$\begin{aligned}
\mathbb{E}[u_A(t,1)] &= 0\alpha_A + \left[\left(\frac{v_A(t,1) - TR(t,1)}{2}\right)\alpha_T - \left(\frac{TR(t,1)}{2}\right)(1-\alpha_T)\right](1-\alpha_A) \\
&= \left[\frac{v_A(t,1)}{2}\alpha_T - \frac{TR(t,1)}{2}\right](1-\alpha_A).
\end{aligned} \tag{23}$$

For L_T's expected payoff, we have:

$$\begin{aligned}
\mathbb{E}[u_T(t,1)] &= \left[v_T(t,1)\,\alpha_A + 0(1-\alpha_A)\right]\alpha_T + \\
&\left[\begin{array}{c} \left(v_T(t,1) - \left(\frac{v_A(t,1)-TR(t,1)}{2}\right)\right)\alpha_A \\ + \left(\frac{v_T(t,1)}{2} - \left(\frac{v_A(t,1)-TR(t,1)}{2}\right)\right)(1-\alpha_A) \end{array}\right](1-\alpha_T) \\
&= v_T(t,1)\,\alpha_A(0)\alpha_T(0) \\
&+ \left[\frac{v_T(t,1)}{2}\alpha_A + \frac{v_T(t,1)}{2} - \left(\frac{v_A(t,1) - TR(t,1)}{2}\right)\right](1-\alpha_T)
\end{aligned} \tag{24}$$

Finally, for the government's expected payoff we have:

$$\begin{aligned}
\mathbb{E}[u_G(t,1)] &= \alpha_T\left[\alpha_A\left(\frac{TR(t,1) - v_A(t,1)}{2}\right) + \left(\frac{v_A(t,1) + TR(t,1)}{2}\right)\right] \\
&+ (1-\alpha_T)\left[\left[\frac{2v_A(t,1) + TR(t,1)}{2}\right] - \frac{\alpha_A v_A(t,1)}{2}\right] \\
&= \alpha_T\left(\alpha_A\frac{TR(t,1)}{2} - \frac{v_A(t,1)}{2}\right) + \left[v_A(t,1) + \frac{TR(t,1)}{2}\right] - \alpha_A\frac{v_A(t,1)}{2}
\end{aligned} \tag{25}$$

We now use these payoff functions in the characterization of equilibrium.

3.4.1 Characterizing Equilibrium of an Agreement over Tariffs

Following the same approach as for NTBs, we begin by taking the agreement tariff level as given at t and analyze the interaction between the lobby groups as a game that takes the form of an all-pay auction. First consider the situation where $t = 1$ so that the outcome corresponds to free trade, wherein $TR(1,1) = 0$. In that case, $v_A(1,1) = v_T(1,1)$ and the game is one of homogeneous valuations as in the NTB case. This is analyzed exactly as in the NTB case, so that in equilibrium the expected payoff to L_A and L_T are both equal to zero and the payoff to the government is equal to $v_A(1,1) = v_T(1,1)$, which is the same as characterized in Lemma 1.

Next consider the situation where $t > 1$ and, by assumption for now, both lobbies make positive contributions. We will characterize below the circumstances under which each lobby makes a positive contribution. As before, the equilibrium involves both lobbies playing a mixed strategy. As a result of L_T's head start, $v_T(t,1) + TR(t,1) > v_A(t,1)$, and so L_T can always outbid L_A. As a result, L_A can do no better in equilibrium than if it did not contribute, i.e. $\mathbb{E}[u_A(t,1)] = 0$. But if L_A did not contribute at all then L_T could win with a very small contribution, inviting L_A to make a slightly larger contribution itself. So both lobby groups contribute at positive levels in expectation in equilibrium and, following the same logic as for the NTB game, contribution at a positive level must involve continuous randomization of contribution levels for both lobby groups. Moreover, L_A cannot make a lower return than 0 in equilibrium because it could obtain a higher payoff from playing a pure strategy $l_A = 0$. And it cannot make a higher return than 0 because this would mean that L_T would make a return lower than $v_T(t,1) - (v_A(t,1) - TR(t,1))$, inducing L_T to secure a higher return of $v_T(t,1) - (v_A(t,1) - TR(t,1)) - \epsilon$ by playing a pure strategy $l_T = v_A(t,1) - TR(t,1) + \epsilon$. So in equilibrium L_A chooses α_A such that $\mathbb{E}[u_A(t,1)] = 0$.

Staying with the case where $t > 1$ and both lobbies make positive contributions, and turning to lobby L_T, it can always win for certain by playing a pure strategy $v_T(t,1) = v_A(t,1) - TR(t,1) + \epsilon$, obtaining a payoff of $v_T(t,1) - (v_A(t,1) - TR(t,1)) - \epsilon$. But it can obtain a higher expected payoff $v_T(t,1) - (v_A(t,1) - TR(t,1))$ by randomizing continuously over the interval $[0, v_A(t,1) - TR(t,1)]$ with some probability and making a zero contribution otherwise. L_T cannot obtain a higher expected payoff than $v_T(t,1) - (v_A(t,1) - TR(t,1))$, because this would mean that L_A would make a lower expected return than 0, inducing L_A simply to refrain from making a contribution. So in equilibrium L_T chooses α_T such that $\mathbb{E}[u_T(t,1)] = v_T(t,1) - (v_A(t,1) - TR(t,1))$.

The foregoing observations about equilibrium enable us to solve for α_A and α_T. First, we can use in (23) the fact that $\mathbb{E}[u_A(t,1)] = 0$ in equilibrium to obtain $\alpha_T = \frac{TR(t,1)}{v_A(t,1)}$. In the same way, using the fact that in equilibrium $\mathbb{E}[u_T(t,1)] = v_T(t,1) - (v_A(t,1) - TR(t,1))$, plus the fact that $\alpha_T = \frac{TR(t,1)}{v_A(t,1)}$, we obtain $\alpha_A = 1 - \frac{(v_A(t,1) - TR(t,1))}{v_T(t,1)}$.

We can now determine the government's expected payoff. Using (25) and our solution $\alpha_T =$

$\frac{TR(t,1)}{v_A(t,1)}$, we obtain

$$\mathbb{E}[u_G(t,1)] = v_A(t,1) - \alpha_A \left(\frac{v_A(t,1)^2 - TR(t,1)^2}{2v_A(t,1)} \right). \tag{26}$$

To compare the government's payoff $\mathbb{E}[u_G(t,1)]$ for $t > 1$ to $\mathbb{E}[u_G(1,1)]$ obtained under $t = 1$, we first need to know the effect of t on $v_A(t,1)$. This effect is characterized in the following result.

Lemma 3. *For $t > 1$, taking all other parameters as given, $dv_A(t,1)/dt < 0$ for γ sufficiently large.*

The value of γ affects the number of firms that switch from being purely domestic firms to being exporters as t is reduced towards free trade. If γ is relatively small then the cost of becoming an exporter is relatively low and as t is reduced a relatively large number of firms switch from being purely domestic to exporting as well. Consequently, the value to L_A of remaining in autarky falls, reflected by a reduction in $v_A(t,1)$. If γ is sufficiently large then a relatively large share of firms finds it prohibitively expensive to become an exporter and $v_A(t,1)$ increases with a reduction in t. To obtain clear cut results we will assume that γ is sufficiently large that $dv_A(t,1)/dt < 0$ for all $t > 1$. Consequently, $v_A(t,1) < v_A(1,1)$ for all $t > 1$.

To evaluate the second term of (26), note that lobby groups will have no reason to lobby if $v_A(t,1) \leq TR(t,1)$. As discussed in the Introduction, the government knows it can get $TR(t,1)$ for sure from the agreement, and it is not worth L_A contributing above v_A to try to forestall the agreement. Since L_A does not lobby, there is no reason for L_T to lobby either and the agreement goes ahead for sure. To proceed, it will be useful to recognize that $TR(t,1)$ is increasing in t up to and past the point where $v_A(t,1) = TR(t,1)$. We will denote by $\bar{t}$ the unique value of t at which this holds: $v_A(\bar{t},1) = TR(\bar{t},1)$. We can now say that, for $1 < t < \bar{t}$, in equilibrium $\alpha_A > 0$ and the second term of (26) must be negative. This establishes that $\mathbb{E}[u_G(t,1)] < \mathbb{E}[u_G(1,1)]$, as discussed intuitively in the Introduction.

Although the logic of all-pay auctions plays an important role in determining lobbying behavior in our framework, we have also observed that the tariff may give the pro-trade lobby a sufficiently large head start that the lobbies will not make a contribution at all in equilibrium. From the solutions just obtained, at $TR(\bar{t},1) = v_A(\bar{t},1)$ we have that $\alpha_A = \alpha_T = 1$. This implies that neither L_A nor L_T will make a positive contribution in equilibrium for $t \geq \bar{t}$. We must allow for this possibility in our characterization of equilibrium lobbying behavior, which takes as given a non-prohibitive tariff level t.

Lemma 4. *In an agreement over tariffs, there are three possibilities:*

(i) if $t = 1$ then the equilibrium is symmetric and exactly as characterized in Lemma 1. In particular note that $\mathbb{E}[u_G] = v_A(1,1)$.

(ii) If $1 < t \leq \bar{t}$ then $TR(t,1) > 0$ and the Nash equilibrium is unique and asymmetric. In equilibrium, L_T contributes $l_T = 0$ with probability $\alpha_T = TR(t,1)/v_A(t,1)$, and randomizes its contribution continuously over $l_T \sim U[0, v_A(t,1) - TR(t,1)]$ with probability $1-\alpha_T$. In equilibrium, L_A contributes $l_A = 0$ with probability $\alpha_A = (v_T(t,1) - (v_A(t,1) - TR(t,1))) / (v_T(t,1))$, and randomizes its contribution continuously on $l_T \sim U[TR(t,1), v_A(t,1)]$ with probability $1 - \alpha_A$. L_T's expected payoff in equilibrium is $\mathbb{E}[u_T] = v_T(t,1) - (v_A(t,1) - TR(t,1))$, while L_A's expected payoff in equilibrium is $\mathbb{E}[u_A(t,1)] = 0$, and the government's expected payoff is $\mathbb{E}[u_G(t,1)] = v_A(t,1) - \alpha_A \left(\frac{v_A(t,1)^2 - TR(t,1)^2}{2v_A}\right)$. For γ sufficiently large this implies $\mathbb{E}[u_G(t,1)] < v_A(1,1)$.

(iii) If $t > \bar{t}$, both lobby groups contribute zero and the only payoff to the government is through tariff revenue. The expected payoff to the government inclusive of lobby revenue and tariff revenue is $u_G(t,1) = TR(t,1)$.

Lemma 4 shows that the determination of government expected revenues through strategic interaction between the lobbies will be different depending on whether $t = 1$, $1 < t < \bar{t}$, or $\bar{t} \leq t$. The government's objective is therefore to choose whichever tariff level from among these three brings about the highest expected revenues. The problem is simplified by the fact that if $t = 1$ then $\mathbb{E}[u_G(1,1)] = v_A(1,1)$ whereas if $1 < t \leq \bar{t}$ then $\mathbb{E}[u_G(t,1)] < v_A(1,1)$. Therefore, by Lemma 4, we can rule out the possibility that the government will choose $1 < t \leq \bar{t}$ in equilibrium.

The final step in characterization is to determine whether the government will choose $t = 1$ or some tariff $t \geq \bar{t}$. To facilitate this step, it can be shown that there exists a value of $t \geq \bar{t}$ at which the value of $TR(t,1)$ is maximized: $\hat{t} = \arg\max TR$. Then the government's choice simplifies to a choice between $t = 1$ and $t = \hat{t}$; whichever raises the most revenue. We can now characterize equilibrium accordingly.

Proposition 2. *Assume an agreement over tariffs. The governments propose a trade agreement involving free trade, $t = 1$, if*

$$\begin{aligned} & v_A(1,1) > TR(\hat{t},1) \\ \Leftrightarrow & \left[2 - \left[\frac{1+\gamma^{\psi}}{\gamma^{\psi}}\right]^{\frac{\varepsilon-1}{k}}\right]^{\frac{k}{\varepsilon-1}} > \left[\frac{k(1+\gamma^{\psi})}{\left((k-\theta)\gamma^{\psi}\hat{t}^{\frac{k}{\theta}}\right)}\right] \end{aligned}$$

and propose a tariff $t = \hat{t}$ to maximize TR otherwise. In the case where free trade is proposed, both L_A and L_T randomize their respective contributions l_A and l_T continuously on the interval $[0, v_A(1,1)]$ and the highest contribution wins with probability one. Each lobby group earns an expected payoff of zero in equilibrium. The expected payoff to the government is $\mathbb{E}[u_G(1,1)] = v_A(1,1) = v_T(1,1)$. In the case where $\hat{t}$ is proposed, both L_A and L_T contribute zero and the payoff to the government is $u_G = TR(\hat{t},1)$ with certainty.

This result shows that equilibrium is characterized by one of two possible trade agreement outcomes. Depending on parameter values, either the government will choose free trade or it will choose the revenue maximizing tariff. Lemma 4 showed that, somewhat surprisingly, competition between lobbies is weakened in the presence of tariff revenue and this leads the government to expect a higher payoff from an agreement that entails free trade. Therefore, if an agreement over tariffs comes about in which lobbying plays a role then it entails no tariff revenue. This parallels in an interesting way the equilibrium NTB agreement; the logic of that agreement extends to a setting where tariff revenues play a role. Unlike in the prior literature, it is the government's incentive to increase contributions that propels the agreement to free trade. We now see that this insight holds as much for a tariff agreement as for an agreement over NTBs. But in the equilibrium of an agreement over tariffs there is an alternative possibility. The conventional forces of tariff revenue maximization may dominate, in which case the lobbies do not contribute at all in equilibrium. We will now undertake comparative statics to explore the parameter values under which each respective possible equilibrium outcome is more likely to prevail.

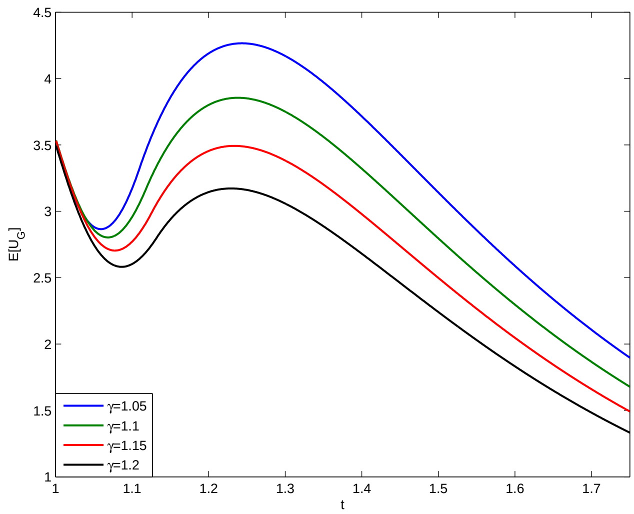
3.4.2 Comparative Statics

We first present a comparative statics result for the parameter γ:

Proposition 3. *The government is more likely to choose free trade the greater the fixed cost to exporting is relative to the fixed cost of serving the domestic market, i.e. the greater is γ.*

This result is based on the same logic as Lemma 3. Intuitively, as the fixed cost of exporting, γ, is increased, fewer firms choose to be exporters which means there is a smaller tax base from which to generate tariff revenue. However, contributions by L_A are determined by two conflicting forces. On the one hand there is less foreign competition in the domestic market as γ is increased, which implies each firm that favors autarky is willing to contribute less to see that it is maintained. But on the other hand, there are more firms that prefer autarky (i.e. $d\varphi/d\gamma < 0$ and $da_D/d\gamma > 0$) which means more firms contribute in support of autarky. The proof shows that the negative effect of an increase in γ on tariff revenue will always outweigh the possibly positive effect on lobby contributions, making free trade more likely to arise in equilibrium.

Figure 2: Expected Government Payoff when $k = 2.5$ and $\theta = 0.4$



The model properties captured by Proposition 3 are illustrated in Figure 2. For values of $k = 2.5$ and $\theta = 0.4$, Figure 2 shows that for $\gamma = 1.05$ or $\gamma = 1.1$, the expected government payoff is maximized at the revenue maximizing tariff. On the other hand, when γ is increased to 1.15 or 1.2, the expected government payoff is maximized at free trade.

We next turn our attention to comparative statics of the shape parameter k. Tariff revenue is decreasing in k:

$$\frac{k}{TR(t,1)}\frac{dTR(t,1)}{dk} = -\left[\frac{k\left[\varepsilon\log(t)+\log(\gamma)\right]\gamma^{\psi}t^{\frac{k}{\theta}}}{(\varepsilon-1)\left(\gamma^{\psi}t^{\frac{k}{\theta}}+t\right)}\right] < 0.$$

As k is increased, the distribution of firms becomes more skewed towards less productive firms. This decreases the ex ante expected profit and thus lowers the number of firms taking a draw. This in turn reduces the number of large exporting firms, lowering the amount of tariff revenue generated from imports. In terms of the value of autarky to anti-trade firms that underpins their lobby contributions, $(v_A(1,1))$, when k is sufficiently large relative to $(\varepsilon-1)$ we have that $dv_A(1,1)/dk < 0$. Since the value from remaining in autarky is lower to anti-trade firms, the amount they are willing to contribute to stay in autarky is correspondingly lower as well.

Figure 3: Expected Government Payoff when $\gamma = 1.1$ and $\theta = 0.4$

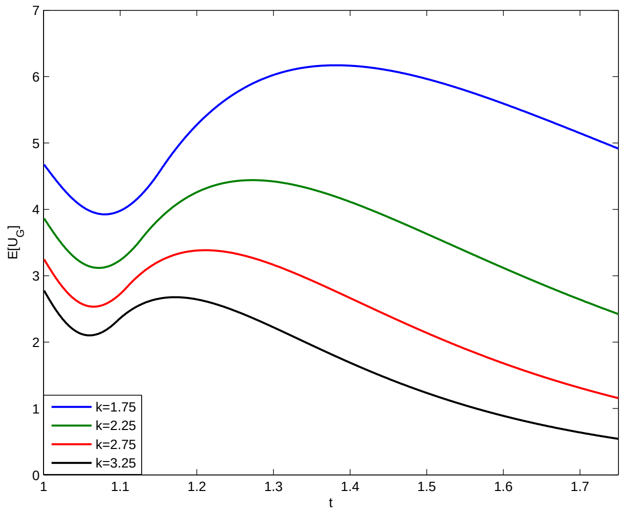


Figure 3 illustrates how, for a sufficiently high value of k, the fall in tariff revenue that results from an increase in k is even greater than the fall in lobbying revenue at free trade and so each government will choose free trade as the proposed policy.

Finally we investigate the effect θ (and consequently the elasticity of substitution ε) has on the proposed trade agreement. The effect on tariff revenue is straightforward and positive:

$$\frac{\theta}{TR(t,1)}\frac{dTR(t,1)}{d\theta} = \left[\frac{k\left[\log(t)+\log(\gamma)\right]\gamma^{\psi}t^{\frac{k}{\theta}}}{\theta\left(\gamma^{\psi}t^{\frac{k}{\theta}}+t\right)^{2}}\right] > 0.$$

As θ increases, so does the elasticity of substitution. This drives down the equilibrium price for each variety but increases the amount sold. Therefore, even though the amount of tariff revenue collected per unit is lower (since the price decreases) the number of units increases resulting in higher tariff revenue. Similarly, as long as k is sufficiently large relative to $(\varepsilon - 1)$, the value to anti-trade firms $(v_A(1,1))$ of autarky increases and hence their lobby contributions also increase with θ. The intuition for this can be gleaned from the fact that aggregate profits in autarky increase (see Figure 1 and note that the black line becomes steeper and shifts up with an increase in θ). This means that the profit loss from going to free trade is now higher despite being spread over fewer firms.

Figure 4: Expected Government Payoff when $\gamma = 1.1$ and $k = 2.5$

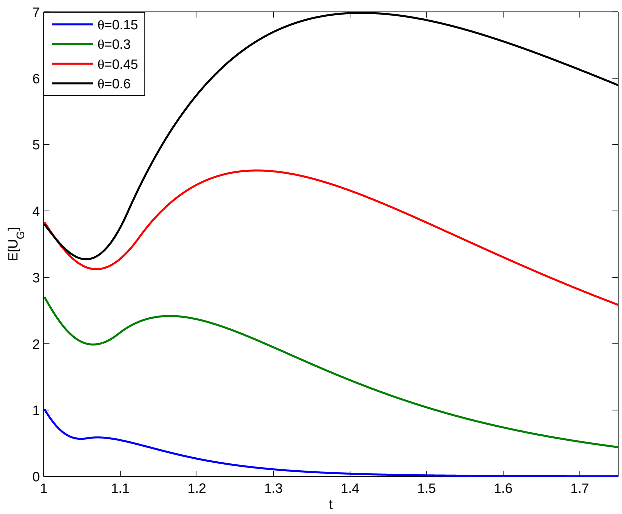


Figure 4 illustrates how, for a sufficiently high value of θ, the increase in tariff revenue that results from an increase in θ is greater than the increase in lobbying revenue at free trade and so each government will choose a tariff policy that maximizes tariff revenue $(\hat{t})$ in equilibrium and lobby contributions will be equal to zero.

4 Conclusion

In this paper we have developed a model of trade agreements where the tension is between larger firms who have an interest in trade liberalization and smaller firms who would prefer autarky.

This contrasts with the standard framework wherein the conflict of interest over a trade agreement is between an export sector and an import competing sector. We model lobbying over trade agreements as an all-pay auction to capture the feature that while it is legal for lobby groups to make contributions to the government it is illegal to tie contribution levels to specific policy outcomes.

Our model of a trade agreement over NTBs forms a benchmark. With that policy instrument, if an agreement comes about then it involves free trade and all the surplus generated by the agreement is transferred to the government. An agreement over tariffs brings tariff revenues into play. Overall, this serves to weaken competition between the two lobby groups by opening up an asymmetry in the valuations across the two groups of successful lobbying and granting a head start to the pro-trade lobby group. In spite of the additional complexities introduced by lobbying over a tariff agreement, the all-pay auction logic implies that the equilibrium outcome of a tariff agreement is surprisingly straight forward. Either it involves free trade or the tariff revenue maximizing tariff. Parameter values relating to the cost of exporting, the productivity distribution across firms, and the elasticity of substitution between products determine the form of agreement that the governments would prefer: free trade or tariff revenue maximizing.

The incentives to lobby over trade policy generated in this model reveal a new motivation for the formation of a trade agreement. Whether the trade policy instrument is a tariff or an NTB, the government has an incentive to adopt free trade in order to maximize lobbying revenues. This complements the motive of forming a trade agreement to escape from the terms-of-trade driven prisoner's dilemma because countries are large in our model. But the motive that we reveal in this paper is driven by the variation in firm productivity within a sector rather than the incentive to set trade policy to influence the terms of trade.

What are the directions for future research? It would be useful to explore both theoretically and empirically how the incentives to form a trade agreement revealed in this paper compare with the terms-of-trade motive that is the main focus of the prior literature. This would involve incorporating consumers in a full utilitarian specification of government payoffs. It would also involve exploring the alternative starting point not at autarky but at the tariffs that maximized the respective governments' terms of trade. The main question would be which matters more in trade agreement formation: the conflict of interest over trade policy between or within sectors?

Perhaps the easiest way to test the present framework econometrically would be by extending the framework of this paper to one of preferential trade agreements. Our two country model would extend naturally to consider economic integration agreements (EIAs) involving larger numbers of countries.[24] Moreover, our theoretical prediction that in forming an agreement governments choose between free trade and the revenue maximizing tariff is consistent with two features of real-world preferential trade agreement formation: first, under Article XXIV, sectors that liberalize must go all the way to free trade; second, some sectors are exempted and the tariffs that remain in place could be characterized as revenue maximizing.

A nascent econometric literature examines the effects on extensive and intensive margins of EIAs across countries and time (Baier, Bergstrand and Feng 2014). This econometric literature has tight links to the theoretical literature on firm exporting behavior in the face of exogenous trade liberalization. Our framework would provide a way to link empirically the exporting behavior of firms to the underlying motivations of governments to form agreements. Do politicians form agreements because of the lobbying revenue or because of the political prestige? If the profits of EIA formation through the extensive margin take longer to materialize and lobbying revenue is important, does this mean that governments tend to propose agreements earlier in their terms rather than later? An extension of our model would provide a framework to examine such questions empirically.

Finally, we have analyzed the incentives of trade agreement formation taking the legal framework as given. What features are omitted from the model of the present paper that might make it optimal for governments to be able to accept contributions but not to be able to tie these to specific policy outcomes? This is a question that we leave to future research.

[24]EIA is the most general term for a trade agreement that may encompass one way and two way agreements, preferential agreements that do not go all the way to free trade, free trade agreements, customs unions, common markets and so on.

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A Appendix

A.1 Proofs of Results

Lemma 2. *The NTB policy that maximizes lobby revenue for the government is free trade; i.e.* $\tau = 1$.

Proof. First note that

$$\begin{aligned}
\frac{d\pi_X(a)}{d\tau} &= \frac{(\varepsilon-1)\gamma f_D}{\tau}\left(\frac{a}{a_X}\right)^{1-\varepsilon}\epsilon^{\tau}_{a_X} = \frac{(1-\varepsilon)\gamma f_D}{\tau}\left(\frac{a}{a_X}\right)^{1-\varepsilon}\left(\frac{\gamma^{\psi}\tau^{k}}{\gamma^{\psi}\tau^{k}+1}\right)\\
\frac{d\pi_D(a)}{d\tau} &= \frac{(\varepsilon-1)f_D}{\tau}\left(\frac{a}{a_D}\right)^{1-\varepsilon}\epsilon^{\tau}_{a_D} = \frac{(\varepsilon-1)f_D}{\tau}\left(\frac{a}{a_D}\right)^{1-\varepsilon}\left(\frac{1}{\gamma^{\psi}\tau^{k}+1}\right),
\end{aligned}$$

where ϵ refers to elasticity. So the first derivative, equation (18), reduces to:

$$\begin{aligned}
\frac{dv_A(1,\tau)}{d\tau} &= \frac{-N_E(\varepsilon-1)f_D}{\tau}\left[\int_{\varphi}^{a_X}\left(\frac{a}{a_X}\right)^{1-\varepsilon}\gamma\epsilon^{\tau}_{a_X}dG(a) + \int_{\varphi}^{a_D}\left(\frac{a}{a_D}\right)^{1-\varepsilon}\epsilon^{\tau}_{a_D}dG(a)\right]\\
&= -\frac{N_E(\varepsilon-1)f_D}{\tau}\left[\frac{\gamma\epsilon^{\tau}_{a_X}}{a_X^{1-\varepsilon}}\left[V(a_X)-V(\varphi)\right] + \frac{\epsilon^{\tau}_{a_D}}{a_D^{1-\varepsilon}}\left[V(a_D)-V(\varphi)\right]\right]\\
&= -\frac{N_E k f_D}{\tau\psi}\left[\gamma^{\psi}\tau^{k+1-\varepsilon}-1\right]\lambda^{k+1-\varepsilon}\left(\frac{a_D}{a_U}\right)^{k}\epsilon^{\tau}_{a_D} < 0 \quad \forall\ \tau \geq 1.
\end{aligned}$$

This is negative because $\gamma > 1$, $\tau \geq 1$, and $k > \varepsilon - 1$. Therefore, the government will choose free trade. $\square$

Lemma 3. *For* $t > 1$*, taking all other parameters as given,* $dv_A(t,1)/dt < 0$ *for* γ *sufficiently large.*

Proof. We begin by obtaining a reduced form expression for $dv_A(t,1)/dt$. First, observe that

$$\begin{aligned}
\frac{d\pi_X(a)}{dt} &= \frac{(\varepsilon-1)\gamma f_D}{a_X}\left(\frac{a}{a_X}\right)^{1-\varepsilon}\frac{da_X}{dt} = \frac{(\varepsilon-1)\gamma f_D}{t}\left(\frac{a}{a_X}\right)^{1-\varepsilon}\epsilon^{t}_{a_X}\\
\frac{d\pi_D(a)}{dt} &= \frac{(\varepsilon-1)f_D}{a_D}\left(\frac{a}{a_D}\right)^{1-\varepsilon}\frac{da_D}{dt} = \frac{(\varepsilon-1)f_D}{t}\left(\frac{a}{a_D}\right)^{1-\varepsilon}\epsilon^{t}_{a_D}
\end{aligned}$$

where ϵ refers to elasticity. Using these, the equation for $dv_A(t,1)/dt$ can be written

$$\frac{dv_A(t,1)}{dt} = -N_E(\varepsilon-1)f_D\left[\int_{\varphi}^{a_X}\left(\frac{a}{a_X}\right)^{1-\varepsilon}\frac{\gamma\epsilon^{t}_{a_X}}{t}dG(a) + \int_{\varphi}^{a_D}\left(\frac{a}{a_D}\right)^{1-\varepsilon}\frac{\epsilon^{t}_{a_D}}{t}dG(a)\right].$$

Simplifying further,

$$\frac{dv_A(t,1)}{dt} = \frac{N_E(1-\varepsilon)f_D}{t}\left[\frac{\gamma\epsilon^{t}_{a_X}}{a_X^{1-\varepsilon}}\left[V(a_X)-V(\varphi)\right] + \frac{\epsilon^{t}_{a_D}}{a_D^{1-\varepsilon}}\left[V(a_D)-V(\varphi)\right]\right] \tag{27}$$

Since $\varepsilon > 1$, the term outside the bracket is negative. Therefore the signs of the terms inside the bracket will determine the sign of the overall expression. What makes the sign ambiguous is the fact that the elasticity of the least efficient exporter, a_X, is negative but the elasticity of the least efficient purely domestic firm, a_D, is positive. The elasticities are

$$\begin{aligned}\epsilon^t_{a_X} &= -\frac{k\gamma^{\psi}t^{\frac{k}{\theta}}+\theta t}{\theta k\left(\gamma^{\psi}t^{\frac{k}{\theta}}+t\right)} < 0\\ \epsilon^t_{a_D} &= \frac{t(k-\theta)}{\theta k\left(\gamma^{\psi}t^{\frac{k}{\theta}}+t\right)} > 0.\end{aligned}$$

Since $\gamma > 1$ and $k > \theta$, the cutoff for the least efficient exporter is more elastic than for the least efficient purely domestic firm. However, this is countered by the fact that

$$[V(a_X) - V(\varphi)] < [V(a_D) - V(\varphi)]$$

and

$$\frac{1}{a_X^{1-\varepsilon}} < \frac{1}{a_D^{1-\varepsilon}}.$$

Therefore, to sign $dv_A(t,1)/dt$ we must reduce our expression for it further:

$$\begin{aligned}\frac{dv_A(t,1)}{dt} &= \frac{N_E(1-\varepsilon)f_D}{t}\left[\frac{\gamma\epsilon^t_{a_X}}{a_X^{1-\varepsilon}}[V(a_X)-V(\varphi)] + \frac{\epsilon^t_{a_D}}{a_D^{1-\varepsilon}}[V(a_D)-V(\varphi)]\right]\\ &= \frac{N_E(1-\varepsilon)f_D}{t}\left[\frac{\gamma\epsilon^t_{a_X}}{\epsilon^t_{a_D}}\left(\frac{1}{\gamma t^{\varepsilon}}\right)[V(a_X)-V(\varphi)] + [V(a_D)-V(\varphi)]\right]\frac{\epsilon^t_{a_D}}{a_D^{1-\varepsilon}}\\ &= \frac{N_E(1-\varepsilon)f_D}{t}\left[[V(a_D)-V(\varphi)] - \frac{k\gamma^{\psi}t^{\frac{k}{\theta}}+\theta t}{t^{1+\varepsilon}(k-\theta)}[V(a_X)-V(\varphi)]\right]\frac{\epsilon^t_{a_D}}{a_D^{1-\varepsilon}}\\ &= -\frac{N_E f_D}{t\psi a_U^k}\left[\left[a_D^{k-(\varepsilon-1)} - \varphi^{k-(\varepsilon-1)}\right] - \frac{k\gamma^{\psi}t^{\frac{k}{\theta}}+\theta t}{t^{1+\varepsilon}(k-\theta)}\left[a_X^{k-(\varepsilon-1)} - \varphi^{k-(\varepsilon-1)}\right]\right]\frac{\epsilon^t_{a_D}}{a_D^{1-\varepsilon}}\\ &= -\frac{N_E f_D}{t\psi a_U^k}\left[\left[1-\lambda^{k-(\varepsilon-1)}\right] - \frac{k\gamma^{\psi}t^{\frac{k}{\theta}}+\theta t}{t^{1+\varepsilon}(k-\theta)}\left[\frac{t^{\varepsilon}}{\gamma^{\psi}t^{\frac{k}{\theta}}} - \lambda^{k-(\varepsilon-1)}\right]\right]\epsilon^t_{a_D}a_D^k\end{aligned}$$

Rewrite the last line as

$$\frac{dv_A(t,1)}{dt} = -\frac{N_E f_D}{t\psi a_U^k}\epsilon^t_{a_D}a_D^k\Theta(t,\gamma)$$

where

$$\Theta(t,\gamma) = \left[1-\lambda^{k-(\varepsilon-1)}\right] - \frac{k\gamma^{\psi}t^{\frac{k}{\theta}}+\theta t}{t^{1+\varepsilon}(k-\theta)}\left[\frac{t^{\varepsilon}}{\gamma^{\psi}t^{\frac{k}{\theta}}} - \lambda^{k-(\varepsilon-1)}\right]$$

At $t = 1$ and $\gamma = 1$,

$$\lim_{t\to 1,\ \gamma\to 1}\Theta(t,\gamma) = -\left[\frac{2\theta}{(k-\theta)}\left[1-\lambda^{k-(\varepsilon-1)}\right]\right] < 0,$$

while for any $t \geq 1$,

$$\lim_{\gamma\to\infty}\Theta(t,\gamma) = 1 > 0.$$

Moreover, for any $t \geq 1$, $\Theta(t,\gamma)$ is monotonically increasing in γ:

$$\frac{d\Theta(t,\gamma)}{d\gamma} = \frac{\psi}{\gamma}\left[\left(\frac{\psi\varepsilon\theta t^{1-\varepsilon}}{\lambda^{2(\varepsilon-1)-k}}\right)\left(\frac{kt^{\varepsilon}((\gamma t)^{\psi}-1)+\theta(1+t^{\varepsilon})}{k\gamma H^{(\varepsilon-1)-k}(k-\theta)\gamma^{\psi}t^{\frac{k}{\theta}}}\right)+\lambda^{k-\varepsilon+1}+\frac{\theta\left(1-\left(\frac{1+t^{-\varepsilon}-H^{1-\varepsilon}}{t^{-\varepsilon}}\right)^{\psi}\right)}{(k-\theta)\gamma^{\psi}t^{\frac{k}{\theta}}}\right] > 0$$

where the inequality follows from the fact that $(\gamma t)^{\psi} > 1$ and $H^{1-\varepsilon} > 1$. Therefore, for any $t > 1$, by the intermediate value theorem there exists a value for γ for which $\Theta(t,\gamma) = 0$. For all γ greater than that value, $\Theta(t,\gamma) > 0$ by the monotonicity of $\Theta(t,\gamma)$ in γ, and consequently $\frac{dv_A(t,1)}{dt} < 0$. □

Proposition 2. *Assume an agreement over tariffs. The governments propose a trade agreement involving free trade, $t = 1$, if*

$$v_A(1,1) > TR(\hat{t},1)$$
$$\Leftrightarrow \left[2-\left[\frac{1+\gamma^{\psi}}{\gamma^{\psi}}\right]^{\frac{\varepsilon-1}{k}}\right]^{\frac{k}{\varepsilon-1}} > \left[\frac{k(1+\gamma^{\psi})}{\left((k-\theta)\gamma^{\psi}\hat{t}^{\frac{k}{\theta}}\right)}\right]$$

and propose a tariff $t = \hat{t}$ to maximize TR otherwise. In the case where free trade is proposed, both L_A and L_T randomize their respective contributions l_A and l_T continuously on the interval $[0, v_A(1,1)]$ and the highest contribution wins with probability one. Each lobby group earns an expected payoff of zero in equilibrium. The expected payoff to the government is $E[u_G(1,1)] = v_A(1,1) = v_T(1,1)$. In the case where $\hat{t}$ is proposed, both L_A and L_T contribute zero and the payoff to the government is $u_G = TR(\hat{t},1)$ with certainty.

Proof. We know from Lemma 3 that the government will choose free trade, $t = 1$, in preference to any t such that $\bar{t} > t > 1$. We are able to solve for the revenue maximizing tariff, $\hat{t}$, at which $TR = \frac{k(1+\gamma^{\psi})}{\left((k-\theta)\gamma^{\psi}\hat{t}^{\frac{k}{\theta}}\right)}$ which occurs for $\hat{t} > \bar{t}$. So the payoff maximizing choice of the government reduces to a choice between $t = 1$ and $t = \hat{t}$. □

Proposition 3. *The government is more likely to choose free trade the greater the fixed cost to exporting is relative to the fixed cost of serving the domestic market, i.e. the greater is γ.*

Proof. Let

$$\Gamma \equiv \left(\frac{1}{\hat{t}^{\varepsilon}}\left[\frac{k}{(k-\theta)}\right]^{\frac{\varepsilon-1}{k}}+1\right)\left[\frac{1+\gamma^{\psi}}{\gamma^{\psi}}\right]^{\frac{\varepsilon-1}{k}}$$

so

$$\frac{d\Gamma}{d\gamma} = -\left[\frac{\psi\theta\varepsilon}{\gamma^{\psi+1}k}\right]\left(\frac{1}{\hat{t}^{\varepsilon}}\left[\frac{k}{(k-\theta)}\right]^{\frac{\varepsilon-1}{k}}+1\right)\left[\frac{1+\gamma^{\psi}}{\gamma^{\psi}}\right]^{\frac{-(k-(\varepsilon-1))}{k}} - \frac{\varepsilon}{\hat{t}^{2\varepsilon}}\left[\frac{k}{(k-\theta)}\right]^{\frac{\varepsilon-1}{k}}\left[\frac{1+\gamma^{\psi}}{\gamma^{\psi}}\right]^{\frac{\varepsilon-1}{k}}\frac{d\hat{t}}{d\gamma}$$
$$= -\left[\frac{\psi\theta\varepsilon}{\gamma k}\right]\left[\frac{1+\gamma^{\psi}}{\gamma^{\psi}}\right]^{\frac{\varepsilon-1}{k}}\left(1+\frac{1}{\hat{t}^{\varepsilon}}\left[\frac{k}{(k-\theta)}\right]^{\frac{\varepsilon-1}{k}}\left[1-\frac{\theta\hat{t}^{1-\varepsilon}}{(k-\theta)(\hat{t}-1)\gamma^{\psi}\hat{t}^{\frac{k-\theta}{\theta}}}\right]\right)$$

It is straightforward to show that

$$\hat{t} > \frac{k}{k-\theta}$$

so

$$\left.\frac{d\Gamma}{d\gamma}\right|_{\hat{t}=\tilde{t}} = -\left[\frac{\psi\theta\varepsilon}{\gamma k}\right]\left[\frac{1+\gamma^{\psi}}{\gamma^{\psi}}\right]^{\frac{\varepsilon-1}{k}}\left(1+\frac{1}{\tilde{t}^{\varepsilon}}\left[\frac{k}{(k-\theta)}\right]^{\frac{\varepsilon-1}{k}}\left[1-\frac{\theta\hat{t}^{1-\varepsilon}}{\theta\gamma^{\psi}\hat{t}^{\frac{k-\theta}{\theta}}}\right]\right)$$

$$= -\left[\frac{\psi\theta\varepsilon}{\gamma k}\right]\left[\frac{1+\gamma^{\psi}}{\gamma^{\psi}}\right]^{\frac{\varepsilon-1}{k}}\left(1+\frac{\tilde{t}^{\frac{\varepsilon-1}{k}}}{\tilde{t}^{\varepsilon}}\left[1-\frac{\hat{t}^{1-\varepsilon}}{\gamma^{\psi}\hat{t}^{\frac{k-\theta}{\theta}}}\right]\right) < 0.$$

The previous inequality follows from the fact that $\frac{d^2\Gamma}{d\gamma d\hat{t}} < 0$. □

A.2 Formalization of Lobbying over Tariff Agreement as All-Pay Auction

This Appendix shows how, when valuations are asymmetric, the game of lobbying over a tariff agreement can be formalized as an all-pay auction where the pro-trade lobby has a head start.

In this case, $1 < t \leq \bar{t}$ and $TR > 0$. We will take t and τ as parametric and so will drop them from functional specifications. We know that each lobby's equilibrium strategy involves a mixed strategy. Therefore, each lobby's contribution level is a realization from a distribution with cumulative distribution function $F_g(l_g)$, $g \in (A, T)$, where $F_g(l_g)$ is continuous over $(0, \infty)$ and the minimum contribution level is 0 for each lobby. The expected payoff for L_T from making a contribution l_T can be written as

$$\mathbb{E}\left[u_T(l_T)\right] = F_A(l_T + TR)\, v_T - l_T \tag{28}$$

where $F_A(l_T + TR)$ is the probability that $l_A \leq l_T + TR$ and hence L_T will win the contest.

The expected payoff for L_A is

$$\mathbb{E}\left[u_A(l_A)\right] = F_T(\max\{0, l_A - TR\})\, v_A - l_A \tag{29}$$

where the term $\max\{0, l_A - TR\}$ reflects the fact that if $l_A \leq TR$ then the government will adopt the agreement for sure.

Since F_A and F_T are continuous over $(0, \infty)$, if l^* is the maximum contribution by L_A, L_T wins for sure if it adopts the pure strategy of $l^* - TR$. Therefore, L_T will never spend strictly more than $l^* - TR$. In addition, L_T's maximum contribution cannot be less than $l^* - TR$, otherwise L_A's maximum contribution would be unnecessarily high.

We will now solve for $F_T(0)$ and $F_A(0)$, showing that both L_T and L_A put mass at zero in equilibrium. Start with $F_T(0)$. In mixed strategy Nash equilibrium each contribution l_i must yield the same expected payoff. Observe from (29) that L_A makes the same expected payoff from $l_A = 0$ as from $l_A = TR$:

$$\mathbb{E}\left[u_A(l_A)\right] = F_T(0)\, v_A - TR = 0.$$

From this it follows that

$$F_T(0) = \frac{TR}{v_A} > 0.$$

Since $F_T(0) = \alpha_T$, we have now shown that L_T puts mass at zero.

Turn now to $F_A(0)$. We have shown above that L_A expects to derive a payoff of zero in equilibrium from all its pure strategies. This holds for its maximum contribution as well, l^*: $v_A - l^* = 0$. So L_A will never contribute more than v_A. This means that L_T can guarantee a payoff of $v_T - (v_A - TR) - \epsilon$ by adopting a pure strategy $l_T = v_A - TR + \epsilon$. To avoid triggering such a deviation by L_T, L_A must set $F_A(0)$ in order to facilitate for L_T the higher payoff $v_T - (v_A - TR)$:

$$
\begin{aligned}
\mathbb{E}\left[u_T\left(l_T\right)\right] &= F_A\left(l_T+TR\right)v_T-\left(l_T-TR\right) \\
&= v_T-\left(v_A-TR\right),\ l_T\in\left[0,l^*-TR\right].
\end{aligned} \tag{30}
$$

Setting $l_T=0$, we obtain $F_A\left(TR\right)=1-\frac{v_A-TR}{v_T}$. Since L_A stands no chance of winning for $l_A\leq TR$, we can substitute $F_A\left(0\right)$ for $F_A\left(TR\right)$ from which we obtain

$$F_A\left(0\right)=1-\frac{v_A-TR}{v_T}=\alpha_A>0,$$

thus establishing that L_A puts mass at zero as well.

Now we can solve for equilibrium mixed strategies. Since in equilibrium $\mathbb{E}\left[u_A\left(l_A\right)\right]=0$,

$$F_T\left(l_A-TR\right)=\frac{l_A}{v_A},\ l_A\in\left[TR,v_A\right].$$

Because any contribution by L_T is augmented by TR, a contribution by L_A must be greater than l_T by TR to be regarded as equivalent: $l_A=l_T+TR$. Using this in F_T, we have the equilibrium strategy for L_T:

$$F_T\left(l_T\right)=\frac{TR}{v_A}+\frac{l_T}{v_A},\ l_T\in\left[0,v_A-TR\right].$$

For L_A, by (30),

$$F_A\left(l_T+TR\right)=1-\frac{v_A-TR}{v_T}+\frac{l_T}{v_T}.$$

Therefore in equilibrium, using $l_A=l_T+TR$,

$$F_A\left(l_A\right)=1-\frac{v_A}{v_T}+\frac{l_A}{v_T},\ l_A\in\left[TR,v_A\right].$$

We can now use the equilibrium strategies to calculate expected contribution levels. Starting with L_T,

$$
\begin{aligned}
\mathbb{E}\left\{l_T\right\} &= \int_0^{v_T} l_T dG_T \\
&= \left[\frac{l_T^2}{2v_A}\right]_0^{v_T}
\end{aligned}
$$

But since L_T does not contribute above v_A-TR, we can replace the upper limit v_T with v_A-TR. From this we obtain

$$\mathbb{E}\left\{l_T\right\}=\frac{\left(v_A-TR\right)^2}{2v_A}. \tag{31}$$

Now for L_A,

$$\mathbb{E}\left\{l_A\right\}=\int_{TR}^{v_A} l_A dG_A,$$

noting that since any contribution $l_A \leq TR$ yields no revenue we use TR as the lower limit. Then

$$\begin{aligned}\mathbb{E}\{l_A\} &= \left[\frac{l_A^2}{2v_T}\right]_{TR}^{v_A} \\ &= \frac{v_A^2 - TR^2}{2v_T} \qquad (32)\end{aligned}$$

Now we can calculate the government's expected income:

$$\mathbb{E}[u_G] = \mathbb{E}\{l_A + l_T\} + \frac{1}{2}[1 + \alpha_A - \alpha_T + \alpha_A\alpha_T]\,TR,$$

where the coefficient on TR is the probability that the agreement goes ahead so that the second term is expected tariff revenue. Summing (31) and (32), expected contributions come to

$$\mathbb{E}\{l_A + l_T\} = v_A\left(\frac{v_A + v_T}{2v_T}\right) - TR - \left(\frac{TR^2}{2v_T} - \frac{TR^2}{2v_A}\right). \qquad (33)$$

Expected tariff revenue is

$$\frac{1}{2}[1 + \alpha_A - \alpha_T + \alpha_A\alpha_T]\,TR = TR - \frac{TR\left(v_A^2 - TR^2\right)}{2v_Av_T}.$$

Summing the previous two expressions, we obtain the government's expected payoff,

$$\mathbb{E}[u_G(t,1)] = v_A\left(\frac{v_A(t,1) + v_T(t,1)}{2v_T(t,1)}\right) - \left(\frac{TR^2}{2v_T(t,1)} - \frac{TR^2}{2v_A(t,1)}\right) - \frac{TR\left(v_A(t,1)^2 - TR^2\right)}{2v_A(t,1)\,v_T(t,1)}. \qquad (34)$$

which simplifies to (26).

Note that the expected contribution levels derived above, $\mathbb{E}\{l_T\}$ and $\mathbb{E}\{l_A\}$, take into account the size of the mass placed at zero in equilibrium. To show that these are consistent with the expected contribution levels shown in Table 1, we need to multiply the values shown in Table 1 by the corresponding probability that this expected contribution is made:

$$\begin{aligned}\mathbb{E}\{l_T\} &= (1-\alpha_T)\left(\frac{v_A - TR}{2}\right) = \left(\frac{v_A - TR}{v_A}\right)\left(\frac{v_A - TR}{2}\right) = \frac{(v_A - TR)^2}{2v_A}; \\ \mathbb{E}\{l_A\} &= (1-\alpha_A)\left(\frac{v_A + TR}{2}\right) = \left(\frac{v_A - TR}{v_T}\right)\left(\frac{v_A + TR}{2}\right) = \frac{v_A^2 - TR^2}{2v_T}.\end{aligned}$$

Thus the contribution levels reported in Table 1 correspond to the contribution levels derived in (31) and (32). Finally, substituting equilibrium values for α_A and α_T into (25) obtains (34) as required.