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Marginal reforms to facilitate climate change mitigation: An assessment of costs and abilities to abate of different countries.

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Abstract

Marginal reforms to facilitate climate change mitigation:
An assessment of costs and abilities to abate of different countries.

Employing the by-production approach to modelling emission-generating technologies proposed in Murty, Russell, Levkoff [2012] and Murty [2015] and adapting the policy reform approach to taxation policy in public economics due to Feldstein [1975] and Guesnerie [1977], we propose a methodology to identify profit-increasing and emission non-increasing input and sequestration reforms and to compute marginal abatement costs (MACs) of countries. The proposed formulae are based purely on data that defines the current status-quo and gain significance when long-run abatement plans based on future projections of the economy are implemented in a piecemeal manner. We show that the so derived MAC of any country is the ratio of its reduction in profit (RIP) and its ability to abate (ATA) at the status-quo. The RIP and the ATA measure, respectively, the decrease in profit and the reduction in emission at the status-quo when the abatement strategy that results in the greatest proportional decrease in emission is adopted. While the RIP depends on factors such as the profitability of various inputs and the costs of sequestration at the status-quo, the ATA depends on factors such as the extents of usage of fossil-fuels and carbon sequestration efforts. The formulae derived are used to compute the RIP, ATA, and the MACs of a sample of 118 countries. A wide international variation is found in the estimates, indicating potentials for gains from international trading in emission given the current state of the world economy and its level of technological development. In particular, countries that have submitted the highest emission-reduction targets under the Intended Nationally Determined Contributions (INDC) scheme to the UNFCCC are also among the countries with the highest MACs in our sample, while high fossil-fuel using countries that have thus far shown caution in submitting their INDCs have low MACs. Carbon sequestration efforts such as afforestation contribute towards lowering MACs in countries that do undertake them.

JEL classification codes: Q5, Q54, Q58, H23, H21, H20
Keywords: marginal abatement costs, policy reforms, by-production model of emission generating technology, ability to abate, reduction in profit.

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1. Introduction.

The main obstacle for reducing current emissions to curb anthropogenic climate change is that abatement is a drain on our resources. A sound knowledge of the costs involved is, hence, an important ingredient in the design of climate policies that balance correctly the costs and benefits of abatement. The marginal abatement cost (MAC) measures the increments to our cost (or, equivalently, the decrease in our payoff) per unit increase in abatement (or, equivalently, per unit decrease in emission). There are many contexts in which marginal abatement costs (MACs) become relevant and several different methodologies are employed to compute them.[1] It is well-known that international differences in MACs can be exploited to minimise domestic and global abatement costs. Computable general equilibrium (CGE) models compute regional and global MACs numerically for studying the scope and patterns of international emission-trading and comparing the abatement costs of alternative emission scenarios.[2] MACs are also computed at sectoral, industry, or firm levels. Here, MAC is often derived as the shadow price of emission, which is obtained by estimating models of emission-generating technologies and computing the technological trade-off between the intended ("good") output and emission.[3] Or it can be estimated in more informal ways by identifying, estimating, and aggregating additions to costs incurred under various cost headings by an economic unit when it complies to environmental regulations.

In this paper, we propose a novel way of computing MACs. We adapt and employ the classic policy-reform methodology developed in public economics for designing welfare improving tax reforms to derive explicit formulae for computing international MACs, which are based on data that summarises the current status-quo. We apply these formulae to compute the MACs of 118 OECD and non-OECD countries.

The policy reform literature in taxation theory is in contrast to the literature on optimal taxation. Both are developed in the context of second-best economies. Both take the view that the current status-quo is a tax equilibrium, which can generally be expected to be suboptimal. The literature on optimal taxation prescribes tax formulae

[1] See Fischer and Morgenstern (2006) and references there in.

[2] See, *e.g.,* Ellerman and Decaux (1998), EPPA model of Paltsev et al (2005), and Eyckmans et al (2001).

[3] See, *e.g.,* Färe, Grosskopf, Lovell, Yaisawarng (1193), Färe, Grosskopf, and Weber (2006), and Murty and Kumar (2002), where an emission generating technology, based on the assumptions of weak disposability and null-jointness, is employed to estimate the shadow price of emission.

that characterise a second-best Pareto optimum.[4] However, moves to attain the optimum through immediate implementation of these formulae at a suboptimal status-quo may require big changes in the existing fiscal and political systems. At the same time they also require considerable confidence in the accuracy, reliability, and coverage of all the relevant factors that go into the derivation of these formulae and their computation. Hence, policy makers are reluctant to implement the required large changes in policy instruments that are implied by these formulae. Rather, in reality, policy changes tend to be "slow and piecemeal." As argued by Feldstein (1975), emphasis has to be shifted from "tax design," which is the issue studied in the literature on optimal taxation to "tax reform" that aims to study small changes in policies that, starting from a sub-optimal status-quo, enhance the objectives of the policy maker. In a pioneering article, Guesnerie (1977) formalises the notion of a tax reform. Starting from an initial tax equilibrium (the status-quo), the aim is to identify small changes in policy instruments (called tax reforms) that are equilibrium-preserving (*i.e.,* lead to tax equilibria in the local neighbourhood of the status-quo) and Pareto improving.[5] The article also shows that the informational requirements needed to compute such reforms are readily available in the form of data that defines the current status-quo. Empirical applications of these results can be found in the influential works of Stern and Ahmad (1984) and Murty and Ray (1989).

In the same vein, in our context of measuring abatement costs, we can contrast the CGE approach to measuring MACs with our policy-reform-based approach. CGE models assume that all economic agents are optimising. Though enhancing certain economic objectives is considered desirable by economic agents, in reality, the status-quo attained will seldom reflect their optimal choices, *i.e.,* economic inefficiencies may be expected to prevail at the status-quo. In this paper, we assume that increasing the profitability of its production sector subject to its emission target is an important objective of any country, as this profit accrues to its consumers and impinges on their welfare. In this regards, the current status-quo could be allocatively inefficient (*i.e.,* not profit maximising subject to the current level of emission) given the current level of input prices, costs of cleaning-up (carbon sequestration) activities, and stocks of long-term assets in the economy.[6] Indeed, we find this to be true of all the 118 countries in our sample. None maximises profit given its current emission level. Parametric uncertainties and imperfect knowledge about

[4] See pioneering articles by Diamond and Mirrlees (1971 I and II).

[5] See also Weymark (1978, 1979), Myles (1995), Blackorby and Brett (2000), and Murty and Russell (2005).

[6] Several other forms of inefficiencies such as technical inefficiencies could also be prevalent. There is a big literature that measures technical inefficiencies of production units generating emissions. See, *e.g.,* Färe, Grosskopf, Lovell, and Pasurka (1989), Färe, Grosskopf, Noh, and Weber (2005), and Murty, Kumar, and Dhavala (2007), and Murty, Russell, and Levkof (2012). In this paper, we allow only allocative inefficiencies when measuring MACs.

globally feasible technological options and other institutional bottlenecks may hinder the identification and implementation of the big leap to the profit maximising production allocation starting from the status-quo. Rather, as in the tax policy reform approach, what may be more practical and feasible are small (incremental) changes in the levels of inputs and sequestration efforts (input and sequestration policy reforms) that, starting from a status-quo, lead to allocations in a local neighbourhood of it and increase the profit of the economy without increasing its emission level. Since none of the countries in our sample is profit maximising, for each country such profit-increasing and emission non-increasing reforms exist. Consider the set of all such reforms for a given country. Intuitively, Theorem 1 of this paper, derives the formula to compute the reform in this set that leads to the maximum increase in profit of the country, based on its status-quo data on levels of inputs, carbon sequestration, emission, input prices, sequestration costs, and stocks of long-term assets.

Long-run plans for stabilisation of green-house gas (GHG) concentrations and measurement of costs associated with them are based (in *e.g.,* CGE models) on assumptions regarding the magnitudes of and the links between several economic and climate variables and future projections about emissions, technological change, economic growth, resource availability, *etc.* Thus, they are fraught with several uncertainties. The uncertainty in meeting long-run emission targets can be minimised by implementing long-run abatement plans in a piecemeal manner.[7] Piecemeal implementation of a long-run plan leaves scope for adjustments at every stage where the realised abatement levels fall below the ex-ante projected levels. Adjustments at each point include local policy reforms based not on future projections but on the current state of the economy and the level of its technological development, which seek to remove existing inefficiencies and to exploit cost-effective opportunities available immediately at that point for increasing abatement levels. Thus, local emission policies that are based on the current state of the economy and its technology and long-run goals of stabilising GHG concentration levels that are based on various economic and technological projections are complementary to one another.

The current level of emission can be interpreted as a cap on its emission that is voluntarily chosen by the country. The production sector employs inputs such as labour, capital, fossil fuels, and renewable energy inputs. Usage of fossil fuels contributes to the generation of emission into the atmosphere, while carbon sequestration activities such as afforestation reduce it. To compute the MAC employing the reform approach, assume that the emission constraint is tightened slightly at the status-quo, *i.e.,* there is a slight

[7] For example, UK plans to cut down its emission levels by 80% of 1990 levels by 2050. The Committee on Climate Change, UK, studies its implementation in stages, with careful plans about how abatement will be implemented in each stage based on projections regarding development of clean technologies over the years. See www.theccc.org.uk.

reduction in the nation-wide permissible level of emission relative to the status-quo level. It is intuitive that the maximum increase in profit that is made possible by the set of reforms that lead to allocations in the local neighbourhood of the status-quo and are permissible given the reduced emission cap will, in general, be lower than in the case when the emission cap was unchanged from its status-quo level. Hence, intuitively, our formulae compute the MAC as the reduction in the local maximum profit per unit reduction in the emission cap from its status-quo level. International differences in MACs computed in this manner are based solely on current economic differences between countries as indicated by current data and signal immediate gains from emission trading. In particular, they signal countries having long-run emission-reduction targets but with high current MACs to adopt emission-trading as a short-run cost minimising abatement strategy in order to minimise pressures on their own domestic energy sectors.

The MACs have to be derived in the context of a model of an emission-generating production technology, which includes all factors such as fossil fuels, alternative sources of energy, carbon sequestration efforts such as afforestation, *etc.,* that are considered pertinent for emission-generation, its mitigation, and production of the intended output of the production unit. Such a model should clearly delineate alternative abatement strategies available to the production unit. Abatement strategies could include, *e.g.,* a pure increase in afforestation, a pure decrease in usage of a fossil-fuel, a pure increase in usage of renewable energy sources, a mixture of afforestation and reduction in usage of fossil fuels, inter-fuel substitution, *etc.* The by-production approach to modelling emission generating technologies that is proposed in Murty, Russell, Levkoff (2012) and Murty (2015) is suitable for this purpose. Under this approach, an emission generating technology is parametrized by two sets of production relations – the first represents a standard neo-classical technology that shows how inputs of the production unit are transformed into its intended output, while the second represents how various emission-causing inputs employed by the production unit are transformed into emission and how sequestration measures such as afforestation help in its mitigation. This approach is quite consistent with the guidelines laid down in IPCC (2006), where emission-factors for each type of fossil fuel are provided indicating a linear relation between emission generation and fossil-fuel usage. It also gives guidelines for computing carbon sequestration factors of different types of forests. In this paper, we have employed country reports of Global Forest Resources Assessment (FRA) to compute carbon sequestration factors for individual countries. Net emission, in our paper, is hence a linear function of fossil fuels and abatement measures such as afforestation. This function is quite distinct from the function that represents how all inputs (including energy inputs) are transformed into the intended output.

The measure of MAC defined above turns out to be not independent of the units of measurement of all inputs. By considering reforms in terms of proportionate changes in (as opposed to changes in the levels of) inputs and sequestration at the status-quo and employing the elasticity analogues of the arguments made above, we obtain another measure of the MAC that does not face this problem. It turns out that each of our measures of MAC can be interpreted as the ratio of two quantities that we call the *reduction in profit* (RIP) and the *ability to abate* (ATA) of a country. Intuitively, the RIP measures the reduction in profit at the status-quo due to the adoption of the reform vector (say, q_Ψ) that leads to the maximum proportionate reduction in emission, while the ATA measures the decrease in emission due to the adoption of vector of policy reforms q_Ψ. Ceteris paribus, the higher is the ATA of a country or the lower is its RIP, the lower is its MAC. But the MAC of a high ATA (respectively, high RIP) country can also be high (respectively, low) if its RIP (respectively, ATA) is disproportionately higher. We show that the ATA is higher (i) the higher is the usage of fossil fuels and the higher is the afforestation level at the status-quo and (ii) the more the country uses those fossil fuels, whose emission factors are relatively high or the more is its afforestation if the sequestration factor of its forests is high. The RIP is higher (i) the higher is the profitability of fossil fuels or the higher is the cost of afforestation at the status-quo and (ii) the more the vector q_Ψ discourages the use of those fossil fuels whose profitability is high or the more it encourages afforestation when its cost is high. All these theoretical conclusions are borne out by our empirical results and we obtain a wide variation in MACs in our sample of countries. This has distinct implications for the patterns of mutually beneficial emission trades between countries.

In Section 2, we lay out the model employed in this paper. In Section 3, we formally define the concept of an input policy reform, and what we mean by profit increasing, permissible, and emission non-increasing input reforms. Both differential and non-differential versions of these definitions are presented and it is shown that differential changes in profit due to policy reforms can be rigorously interpreted as derivatives of the profit function along linear paths of abatement strategies defined by the reforms. We show that if there exists a policy reform vector that is differentially emission non-increasing and profit increasing at the status-quo, then there exist reforms that are emission non-increasing and lead to non-differential increases in the profit. In Section 4, we derive formulae for computing optimal profit increasing and emission-non increasing directions of reforms. We provide first an intuitive definition of short-run MAC, followed by a more formal definition. Defined this way, formulae for computing the MAC, which are based entirely on data that is available at the status-quo, can be obtained. We distinguish between and characterise the cases when MAC is zero and when it is positive. We show that our definition of the MAC captures the reduction in profit per unit reduction in emission when a reform that leads to the maximum possible reduction in emission is adopted. The formulae for MAC derived in Section 4 are not independent of the units of measurement of all inputs. In Section 5, we derive another measure of MAC based on elasticities of the profit and emission functions with respect to various inputs that does no face this problem. We show that the MAC of a country can be expressed as a ratio of its RIP and ATA. We discuss the factors affecting the RIP and the ATA. Section 6 describes the data employed in the empirical application of the theory developed in the previous sections. Section 7 presents the results of our empirical analysis, where we study international differences in reductions in profits (RIPs), abilities to abate (AsTA), and the MACs generated by data. We also study the role of afforestation policy in lowering MACs of countries by studying cases with and without afforestation as a policy for reform. We conclude in Section 8. Proofs are relegated to the appendix and the analysis is supported by several diagrams and tables.

2. The model.

Data considerations were crucial for the design of the model presented below. The model presented below can be extended and improved along many dimensions as data sets become richer and richer over time covering many more pertinent factors. The production function is given by a mapping $F : \mathbf{R}^4_+ \longrightarrow \mathbf{R}_+$ with image

$$y = F(k, l, e, o), \tag{2.1}$$

where y is the level of the country's intended output, and k, l, e, and o denote, respectively, the levels of capital, labour, electrical energy, and energy from oil.[8] It is assumed that coal, gas, and renewables are employed mainly to generate electrical energy. Thus, total electrical energy is the sum of electrical energy levels generated by coal, gas, and renewables, denoted by c, g, and r, respectively:

$$e = c + g + r. \tag{2.2}$$

We assume that oil is a major source of non-electrical energy (denoted by o), *e.g.,* energy required by the transportation sector.[9] The stock of forest is affected by levels of afforestation and deforestation. Government incurs expenditure on forests. This could be on maintenance of the stock of forest and also on afforestation efforts to increase the stock of forests. Government's expenditure function is given by the mapping $G : \mathbf{R}^2_+ \longrightarrow \mathbf{R}_+$ with image

$$\zeta = G\left(f + a - d, a\right), \tag{2.3}$$

[8] Details about how we define output y and how function F, which includes energy as an input, is estimated from available data can be found in the Section 6.5.

[9] This approach is suggested by data, where we find that although individual entries for energy from coal, gas, and renewables could be zero for countries in the sample, the aggregate electrical energy and energy from oil are positive for all countries. Hence, in the empirical part, we assume that aggregate electrical energy and non-electrical energy from oil are essential inputs in the production of the intended output.

where f, a, and d denote, respectively, the country's inherited stock of forest and afforestation and deforestation levels planned for the current period. Hence, $f + a - d$ is the stock of forest at the end of the planning period if no reforms in afforestation and deforestation policies take place.

In this paper, we study only one carbon sequestration policy, namely, afforestation. This is because data on other methods of carbon sequestration for all countries and estimates of their costs are not readily available in the public domain.[10] Net emission is the difference between the gross emission produced (carbon released) by the combustion of emission-causing fossil fuels and the carbon sequestered by the net change in the stock of forests. It is given by the function $Z : \mathbf{R}^4_+ \longrightarrow \mathbf{R}_+$ with image

$$Z(c, g, o, a - d) := \alpha_c c + \alpha_g g + \alpha_o o - s[a - d], \tag{2.4}$$

where, for $i = c, g, o$, $\alpha_i > 0$ is the amount of carbon released by the generation of one unit of energy by combusting fossil fuel i. $s > 0$ is the sequestration of carbon per unit increase in the volume of forest stock. Thus, if the net increase in forest stock $a - d$ is negative, *i.e.,* deforestation is more than afforestation, then there is negative sequestration of carbon, *i.e.,* there is a net positive release of carbon due to burning of forest cover by deforestation activities.

Let w denote the wage rate. The prices in the energy sector are given by the vector $p = \langle p_c, p_g, p_r, p_o \rangle \in \mathbf{R}^4_{++}$.

3. Policy reforms, profit increasing, permissible, and emission non-increasing input reforms.

3.1. Permissible policies and the status-quo.

We will maintain the assumption that prices of all inputs, the stocks of capital and forest, and the deforestation level are fixed (at least in the short run) and form a part of the inherited environment within which policies about other variables are reformed. The vector that summarises the levels of economic variables that are fixed in the short-run is denoted by $\Theta = \langle w, p, k, f, d \rangle \in \mathbf{R}^5_+$.

The active policy variables, *i.e.,* those policy variables which are subjected to immediate reforms, are the levels of labour, energy inputs, and afforestation. Thus, an active policy vector is denoted by $\nu = \langle l, c, g, r, o, a \rangle \in \mathbf{R}^6_+$. There is also a quantity constraint on emission, denoted by $z \in \mathbf{R}_+$, which is implemented in the form of a cap on emission. This is, however, assumed to be a sporadic policy, in the sense that active policy changes

[10] With increasing availability of and access to such data, the model can easily be extended to incorporate other sequestration and cleaning-up methods.

are planned as a response to sporadic policy changes. A policy vector comprises of both active and sporadic policies: $\langle \nu, z\rangle \in \mathbf{R}^7_+$.

The profit function of the economy, conditional on an environment that is fixed in the short-run, Θ, is defined by $\Pi : \mathbf{R}^{13}_+ \longrightarrow \mathbf{R}_+$ with image

$$\pi = \Pi(l, c, g, r, o, a, \Theta) = F(k, l, c+g+r, o) - wl - p_c c - p_g g - p_r r - p_o o - G(f+a-d, a). \quad (3.1)$$

Define the function $\mathcal{Z} : \mathbf{R}^{14}_+ \longrightarrow \mathbf{R}_+$ with image

$$\mathcal{Z}(l, c, g, r, o, a, z, \Theta) := Z(c, g, o, a-d) - z. \quad (3.2)$$

Given a sporadic policy level z and an environment Θ that is fixed in the short-run, an active policy vector $\nu \in \mathbf{R}^6_+$ is *permissible* if $\mathcal{Z}(\nu, z, \Theta) \leq 0$, *i.e.,* if the emission actually generated under the given vector of active policies, $\nu = \langle l, c, g, r, o, a\rangle$, is less than or equal to the existing emission-cap, z, *i.e.,* $Z(c, g, o, a-d) \leq z$. It is *tightly permissible* if $\mathcal{Z}(\nu, z, \Theta) = 0$, *i.e.,* if the actual emission generated under ν is exactly equal to the emission cap. We will maintain the following assumptions throughout the analysis:

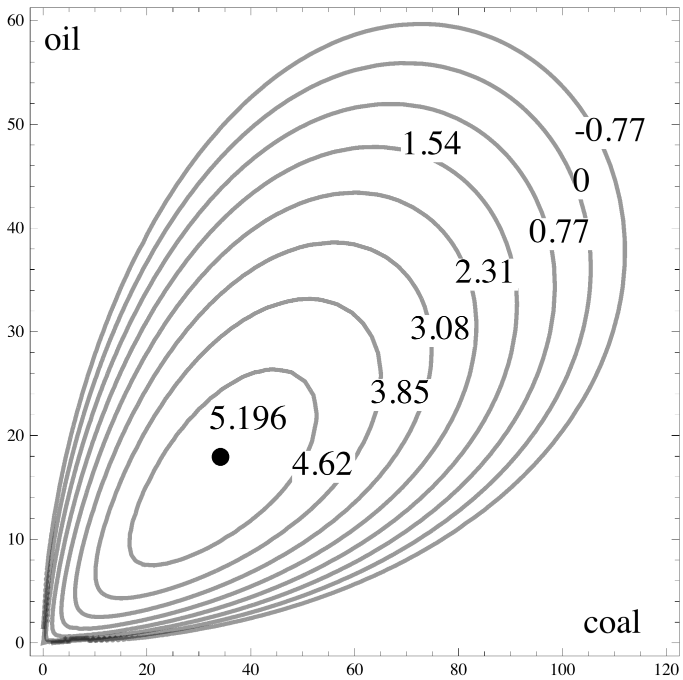
Assumption 1: Π is continuously differentiable in the interior of the domain of its definition. It is strictly concave in ν and has a global maximum with respect to ν.

Assumption 2: The vector $\langle \alpha_c, \alpha_g, \alpha_o, s\rangle \gg 0^T_4$, *i.e.,* $\nabla_v \mathcal{Z} \neq 0_6$.[11]

To illustrate Assumption 1 and many other concepts in this paper in a two-dimensional space, we will consider a simpler version of our model with only two inputs (active policies) coal and gas. Suppose production function F in (2.1) is Cobb-Douglas in the two inputs, and takes the form: $y = 2c^{0.5}o^{0.4}$. Suppose $p_c = .4$ and $p_o = .5$. In this case, the profit function in (3.1) becomes $\pi = 2c^{0.5}o^{0.4} - .4c - .5c$. It can be verified that this profit function is strictly concave and has a global maximum when coal and oil usage are 32.48 and 15.59, respectively. The maximum profit is equal to 5.196. Some level curves of this profit function are shown in Figure 1.

A *status-quo* is an inherited policy environment within which reforms are designed. It consists of the inherited levels of the economic variables that are fixed in the short run as well as inherited levels of active and sporadic policy variables such that the active policy vector is permissible, given the inherited sporadic policy variable and the economic variables that are fixed in the short run. It is, hence, denoted by vectors such as $\langle \nu_0, z_0, \Theta_0\rangle \in \mathbf{R}^{12}_+$, such that given z_0 and Θ_0, the active policy vector ν_0 is permissible. A status-quo $\langle \nu_0, z_0, \Theta_0\rangle$ where the active policy vector ν_0 is tightly permissible given z_0 and Θ_0 is called a tight status-quo.

[11] 0_n denotes a zero vector in $\mathbf{R}^n$. Given any function $f : \mathbf{R}^n \longrightarrow \mathbf{R}$ with image $f(x)$, $\nabla_x f$ denotes the gradient of function f with respect to x.



oil
coal
-0.77
0
0.77
1.54
2.31
3.08
3.85
4.62
5.196
0
10
20
30
40
50
60
0
20
40
60
80
100
120

Figure 1

3.2. Profit increasing, permissible, and emission non-increasing input reforms.

Given a status-quo, $\langle \nu_0, z_0, \Theta_0 \rangle \in \mathbf{R}^{12}_{+}$, a reform in active policies, denoted by $\delta = \langle \delta_l, \delta_c, \delta_g, \delta_r, \delta_o, \delta_a \rangle \in \mathbf{R}^6$, is a vector of changes in active policies starting from the status-quo. A reform in the sporadic policy, starting from the status-quo, is a change in the emission cap, denoted by $\delta_z \in \mathbf{R}$. It is clear that, starting from the status-quo, reform δ in active policies increases the profit if

$$\Pi(\nu_0 + \delta, \Theta_0) - \Pi(\nu_0, \Theta_0) > 0. \tag{3.3}$$

Suppose $\langle \nu_0, z_0, \Theta_0 \rangle \in \mathbf{R}^{12}_{+}$ is a tight status-quo. Starting from the status-quo, given a sporadic policy reform δ_z in the emission cap, a reform δ in active policies is a *permissible reform* if $\nu_0 + \delta$ is permissible given the emission cap $z_0 + \delta_z$ or, equivalently, if the change in net emission induced by reform δ is less than or equal to the change in the sporadic policy δ_z, *i.e.,* if the following holds:[12]

$$\begin{aligned} \mathcal{Z}(\nu_0 + \delta, z_0 + \delta_z, \Theta_0) \leq 0 \iff & \mathcal{Z}(\nu_0 + \delta, z_0 + \delta_z, \Theta_0) - \mathcal{Z}(\nu_0, z_0, \Theta_0) \leq 0 \\ \iff & Z(c_0 + \delta_c, g_0 + \delta_g, o_0 + \delta_o, a_0 + \delta_a - d_0) - Z(c_0, g_0, o_0, a_0 - d_0) \leq \delta_z. \end{aligned} \tag{3.4}$$

A particular special case of interest is one where reforms in active policies take place with no change in the sporadic policy, *i.e.,* when $\delta_z = 0$. In that case, starting from a tight status-quo $\langle \nu_0, z_0, \Theta_0 \rangle \in \mathbf{R}^{12}_{+}$, we will also call a permissible reform δ in active policies an *emission non-increasing* reform:

$$\begin{aligned} \mathcal{Z}(\nu_0 + \delta, z_0, \Theta_0) \leq 0 \iff & \mathcal{Z}(\nu_0 + \delta, z_0, \Theta_0) - \mathcal{Z}(\nu_0, z_0, \Theta_0) \leq 0 \\ \iff & Z(c_0 + \delta_c, g_0 + \delta_g, o_0 + \delta_o, a_0 + \delta_a - d_0) - Z(c_0, g_0, o_0, a_0 - d_0) \leq 0. \end{aligned} \tag{3.5}$$

3.3. Profit increasing, permissible, and emission non-increasing input reforms: A differential characterisation.

Below, we present a differential characterisation of profit increasing, permissible, and emission non-increasing input reforms.

Suppose $\langle \nu_0, z_0, \Theta_0 \rangle \in \mathbf{R}^{12}_{+}$ is a tight status-quo. Employing the gradient of the profit function with respect to active policies evaluated at the status-quo (denoted by $\nabla_\nu \Pi(\nu_0, \Theta_0)$), a reform δ in active policies is *differentially profit increasing* if[13]

$$\nabla_\nu \Pi(\nu_0, \Theta_0) \cdot \delta > 0, \tag{3.6}$$

[12] Recall, by the definition of a tight status-quo, we have $\mathcal{Z}(\nu_0, z_0, \Theta_0) = 0$.

[13] The dot product of the two vectors x and $\bar{x}$ in $\mathbf{R}^n$ leads to a scalar given by $x \cdot \bar{x} = \|x\| \|\bar{x}\| \cos\theta$, where θ is the angle between vectors x and $\bar{x}$. Using matrix notation, $x \cdot \bar{x} = x^\top \bar{x}$, where $x^\top$ denotes the transpose of vector x.

where $\nabla_\nu \Pi(\nu_0, \Theta_0) \cdot \delta$ is an approximation of the left-side of (3.3).

Similarly, employing the gradient of the emission function evaluated at the status-quo, given a reform in the sporadic policy δ_z, a reform δ in active policies is *differentially permissible* if[14]

$$\nabla_\nu \mathcal{Z}(\nu_0, z_0, \Theta_0) \cdot \delta + \nabla_z \mathcal{Z}(\nu_0, z_0, \Theta_0)\delta_z \leq 0 \iff \nabla_\nu \mathcal{Z}(\nu_0, z_0, \Theta_0) \cdot \delta - \delta_z \leq 0, \quad (3.7)$$

where $\nabla_\nu \mathcal{Z}(\nu_0, \Theta_0) \cdot \delta + \nabla_z \mathcal{Z}(\nu_0, z_0, \Theta_0)\delta_z$ is an approximation of the left-side of (3.4). If there is no change in the sporadic policy, *i.e.,* $\delta_z = 0$, then reform δ in active policies is *differentially emission non-increasing* if

$$\nabla_\nu \mathcal{Z}(\nu_0, \Theta_0) \cdot \delta \leq 0, \quad (3.8)$$

3.4. Active policy paths defined by reforms, derivatives and directional derivatives of profit and emission functions.

Below, we show that the differential changes in profit and emission levels (discussed in Section 3.3) that approximate the true changes in profit and emission levels due to reforms in active and sporadic policies (discussed in Section 3.2) can be more rigorously interpreted as the derivatives of the profit and emission functions, evaluated at the status-quo and along linear policy paths defined by the policy reforms.

A vector of reforms in active policies $\delta \in \mathbf{R}^6$ defines a *linear* path of active policies starting from the status-quo as a vector-valued function $\varphi^\delta : \mathbf{R}_+ \longrightarrow \mathbf{R}^6_+$ with image

$$\nu = \varphi^\delta(t) \equiv \nu_0 + \delta t. \quad (3.9)$$

Hence, the gradient of φ^δ is

$$\dot{\nu} := \nabla_t \varphi^\delta(t) = \delta \qquad \forall t \geq 0. \quad (3.10)$$

Different reforms in active policies define different active policy paths. Starting from the status-quo, (3.9) implies that the distance travelled along the path φ^δ is a function of t. In particular, it is given by $\|\dot{\nu}\|t = \|\delta\|t$, where $\|\dot{\nu}\| = \|\delta\|$ is called the magnitude of the reform vector δ.[15] In general, reform vectors can vary with respect to both their magnitudes and directions. We will be concerned also with the set of reforms in active policies whose magnitudes are the same but which differ with respect to their directions. For this purpose, we define a *direction of reform in active policies* as any reform in active policies whose magnitude is one.[16]

14 Note, from (3.2), $\nabla_z \mathcal{Z}(\nu_0, z_0, \Theta_0) = -1$.

15 Given a vector $x \in \mathbf{R}^n$, $\|x\| = \sqrt{x \cdot x}$ is its Euclidean norm.

16 Thus, the set of directions of reforms is the set $\{\delta \in \mathbf{R}^6 \mid \|\delta\| = 1\} \equiv \{\delta \in \mathbf{R}^6 \mid \delta \cdot \delta = 1\}$.

Given a status-quo $\langle \nu_0, z_0, \Theta_0 \rangle$ and an active policy reform δ, the level of profit along the path of active policies $\nu = \varphi^{\delta}(t)$ is given by $\Pi(\varphi^{\delta}(t), \Theta_0)$. From the chain rule of differentiation, it follows that the differential change in profit due to reform δ in active policies can be interpreted as the derivative of the profit function with respect to t along the path of active policies φ^{δ}, evaluated at the status-quo (*i.e.,* when $t = 0$):

$$\frac{\partial \Pi(\varphi^{\delta}(0), \Theta_0)}{\partial t} = \nabla_{\nu} \Pi(\nu_0, \Theta_0) \cdot \delta = \nabla_{\nu} \Pi(\nu_0, \Theta_0) \cdot \dot{\nu}. \tag{3.11}$$

Hence, if δ is a differentially profit-increasing reform (*i.e.,* if (3.6) is true), then it implies that the above derivative of the profit function is positive.

If δ is a direction of reform, *i.e.,* if $\|\delta\| = 1$, then $\nabla_{\nu} \Pi(\nu_0, \Theta_0) \cdot \delta$ is called the *directional derivative of the profit function* evaluated at the status-quo in the direction δ of active policies.[17]

Suppose $\langle \nu_0, z_0, \Theta_0 \rangle$ is a tight status-quo. A sporadic policy reform, $\delta_z \in \mathbf{R}$, defines, starting from the status-quo, a linear path for the sporadic policy given by:

$$z = \varphi^{\delta_z}(t) = z_0 + \delta_z t \quad \Longrightarrow \quad \dot{z} := \nabla_t \varphi^{\delta_z}(t) = \delta_z. \tag{3.12}$$

It follows from the chain rule of differentiation that the derivative of the emission function with respect to t along the vector of active and sporadic policy paths, $\langle \varphi^{\delta}, \ \varphi^{\delta_z} \rangle$, and evaluated at the status-quo, is

$$\begin{aligned} \frac{\partial \mathcal{Z}\left(\varphi^{\delta}(0), \varphi^{\delta_z}(0), \Theta_0\right)}{\partial t} &= \nabla_{\nu} \mathcal{Z}(\nu_0, z_0, \Theta_0) \cdot \delta + \nabla_z \mathcal{Z}(\nu_0, z_0, \Theta_0) \cdot \delta_z \\ &= \nabla_{\nu} \mathcal{Z}(\nu_0, z_0, \Theta_0) \cdot \dot{\nu} + \nabla_z \mathcal{Z}(\nu_0, z_0, \Theta_0) \cdot \dot{z}. \end{aligned} \tag{3.13}$$

Hence, given a sporadic reform δ_z, if δ is a differentially permissible reform in active policies (*i.e.,* if (3.7) is true), then the above derivative of the emission function is non-positive. Recalling (3.8), it follows that if $\delta_z = 0$ then the above derivative of the emission function is non-positive if and only if the active policy reform δ is differentially emission non-increasing.

4. Deriving the marginal abatement cost and the optimal profit-increasing and emission non-increasing active policy reform.

We first provide an intuitive definition of the MAC before deriving the formulae that can be used to compute it employing data that exists at a status-quo.

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[17] If δ is a direction of reform, then the distance travelled along the path φ^{δ} is exactly equal to t as $\|\delta\| t = t$. Hence, the directional derivative of the profit function is also the derivative of the profit function with respect to the distance travelled along the path φ^{δ}.

4.1. An intuitive definition of the marginal abatement cost.

In what follows below, by a "reform" we will mean, by default, a reform in active policies. We will continue referring to a reform in the emission cap as a sporadic policy reform. Let $\langle \nu_0, z_0, \Theta_0 \rangle$ be a tight status-quo. Consider the set of all emission non-increasing reforms that, starting from the status-quo, lead to active policies in a local neighbourhood of ν_0. The following programme searches in this set for the reform that leads to the maximum increase in profit. The local neighbourhood of active policies around ν_0 is chosen to be of radius one.

$$\max_{\delta \in \mathbf{R}^6} \left\{ \Pi(\nu_0 + \delta, \Theta_0) - \Pi(\nu_0, \Theta_0) \;\middle|\; \mathcal{Z}(\nu_0 + \delta, z_0, \Theta_0) \leq 0,\; \delta \cdot \delta \leq 1 \right\}. \tag{4.1}$$

Suppose the solution to above problem (4.1) is $\overset{*}{\delta}$ and the maximum increase in profit is positive, *i.e.,* $\Pi(\nu_0 + \overset{*}{\delta}, \Theta_0) - \Pi(\nu_0, \Theta_0) > 0$. Then this implies that, at the status-quo, profit is not maximised when the emission cap is z_0. There exist reforms that lead to active policies in the local neighbourhood of status-quo that increase profit without increasing the emission level. The active policy vector that maximises profit in the local neighbourhood around ν_0 is $\nu_0 + \overset{*}{\delta}$, and the maximum profit is $\Pi(\nu_0 + \overset{*}{\delta}, \Theta_0)$.

Suppose the emission constraint is tightened at the status-quo by a sporadic reform $\delta_z < 0$, which reduces the emission cap to $z_0 + \delta_z$. Consider the set of all permissible reforms under the new emission cap that, starting from the status-quo, result in active policies in a local neighbourhood of radius one around ν_0. The following programme searches in this set for the reform that leads to the maximum increase in profit.

$$\Delta\Pi(\nu_0, z_0, \Theta_0, \delta_z) := \max_{\delta \in \mathbf{R}^6} \left\{ \Pi(\nu_0 + \delta, \Theta_0) - \Pi(\nu_0, \Theta_0) \;\middle|\; \mathcal{Z}(\nu_0 + \delta, z_0 + \delta_z, \Theta_0) \leq 0,\; \delta \cdot \delta \leq 1 \right\}. \tag{4.2}$$

Suppose the solution to the above problem is $\hat{\delta}(\nu_0, z_0, \Theta_0, \delta_z)$. The maximum increase in the profit is given by $\Delta\Pi(\nu_0, z_0, \Theta_0, \delta_z) = \Pi(\nu_0 + \hat{\delta}(\nu_0, z_0, \Theta_0, \delta_z), \Theta_0) - \Pi(\nu_0, \Theta_0)$. Given the sporadic reform δ_z, the active policy vector that maximises profit in the local neighbourhood around ν_0 is $\nu_0 + \hat{\delta}(\nu_0, z_0, \Theta_0, \delta_z)$, and the maximum profit is $\Pi(\nu_0 + \hat{\delta}(\nu_0, z_0, \Theta_0, \delta_z), \Theta_0)$.

In particular, note that programme (4.1) is a special case of programme (4.2), where $\delta_z = 0$, *i.e.,* $\overset{*}{\delta} = \hat{\delta}(\nu_0, z_0, \Theta_0, 0)$. It is intuitive that the tightening of the emission constraint from z_0 to $z_0 + \delta_z$, where $\delta_z < 0$, will generally be costly for the producing unit, *i.e.,* it will not increase its profit. Intuition suggests that, in most cases, tightening of the emission cap will reduce profit. One can expect that the best that the producing unit can do in the local neighbourhood of its status-quo with the existing emission cap will be greater than or at least equal to the best that it can do in that local neighbourhood when the emission-cap is reduced. Hence,

$$\Pi(\nu_0 + \hat{\delta}(\nu_0, z_0, \Theta_0, 0), \Theta_0) > \Pi(\nu_0 + \hat{\delta}(\nu_0, z_0, \Theta_0, \delta_z), \Theta_0).$$

This leads to the following intuitive definition of the marginal abatement cost (MAC) as the change in the local maximum profit per unit change in the emission cap:

$$m(\nu_0, z_0, \Theta_0) = \lim_{\delta_z \longrightarrow 0} \frac{\Pi(\nu_0 + \hat{\delta}(\nu_0, z_0, \Theta_0, \delta_z), \Theta_0) - \Pi(\nu_0 + \hat{\delta}(\nu_0, z_0, \Theta_0, 0), \Theta_0)}{\delta_z}. \quad (4.3)$$

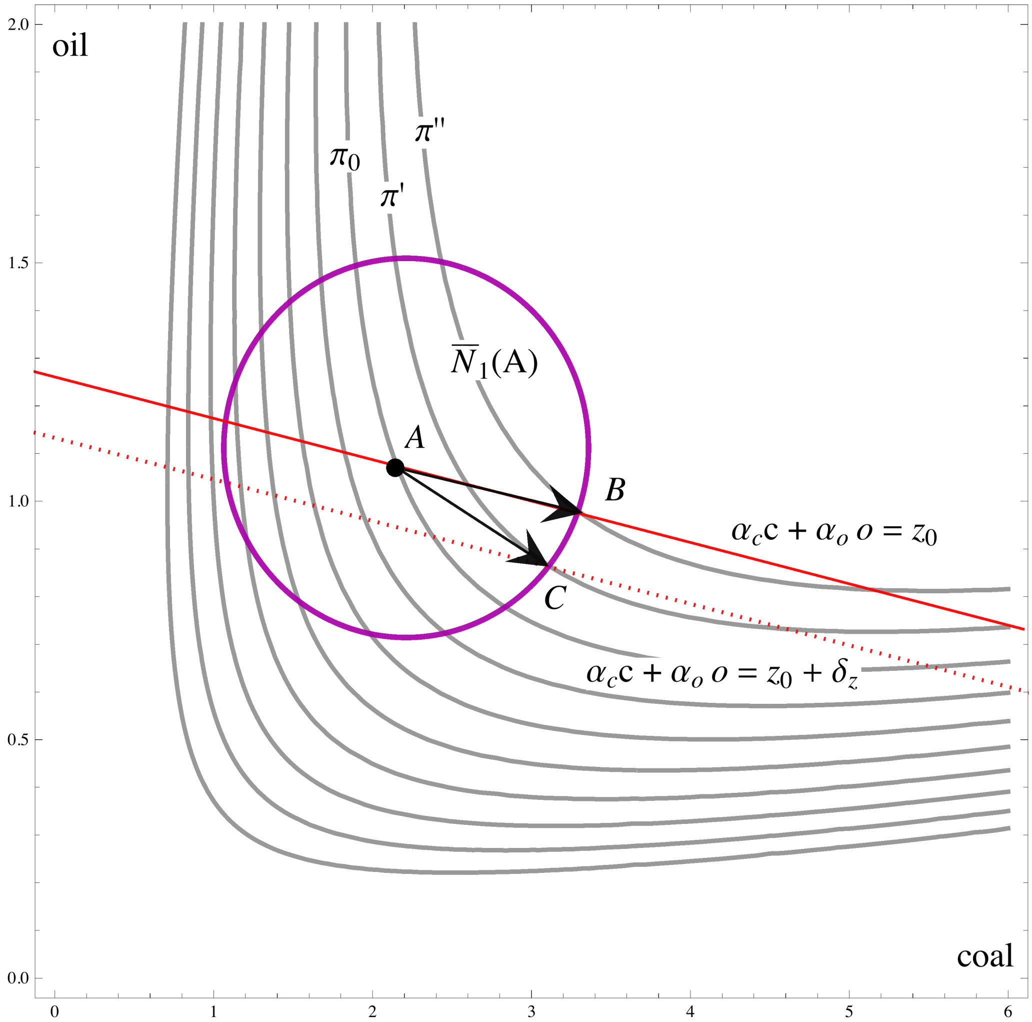
Adding and subtracting $\Pi(\nu_0, \Theta_0)$ to the numerator of the above, MAC can be re-written as

$$m(\nu_0, z_0, \Theta_0) = \lim_{\delta_z \longrightarrow 0} \frac{\Delta\Pi(\nu_0, z_0, \Theta_0, \delta_z)}{\delta_z} - \lim_{\delta_z \longrightarrow 0} \frac{\Delta\Pi(\nu_0, z_0, \Theta_0, 0)}{\delta_z}. \quad (4.4)$$

Thus, the MAC can be measured as the change in the maximum possible increase in profit in a local neighbourhood of the status-quo per unit change in the emission cap.

Programmes (4.1) and (4.2) are illustrated in Figure 2, which shows various contours of the profit function in the space of two inputs, coal and oil. The emission cap at the status-quo is z_0. Recalling (2.4) in the context of this two-inputs case, the line that satisfies equation $z_0 = \alpha_c c + \alpha_o o$ depicts combinations of coal and oil inputs that are exactly equal to z_0. All points on or below this line are permissible under the existing emission cap. A sporadic reform $\delta_z < 0$ decreases the emission cap. All points on or below the dashed line that satisfy equation $z_0 + \delta_z = \alpha_c c + \alpha_o o$ are permissible under the emission cap $z_0 + \delta_z$. Point A is assumed to be the status-quo. It is clear that A is permissible given emission cap z_0. The status-quo level of profit is π_0. Active policies that lie in the local neighbourhood of A are indicated in the figure by the disc $\bar{N}_1(A)$.[18] The intersection of set $\bar{N}_1(A)$ with the set of all combinations of coal and oil that are permissible under emission cap z_0 is the constraint set of problem (4.1). The figure shows that the reform vector that solves problem (4.1) is $\overrightarrow{AB}$. The intersection of set $\bar{N}_1(A)$ with the set of all combinations of coal and oil that are permissible under emission cap $z_0 + \delta_z$ is the constraint set of problem (4.2). Note that the constraint set of (4.2) is a subset of the constraint set of (4.1). The reform vector that solves problem (4.2) is $\overrightarrow{AC}$. It is clear from the figure that, at the status-quo, profit is not maximised when the emission cap is z_0. For example, reform $\overrightarrow{AB}$ in active policies leads to policies in the local neighbourhood of A that increase profit to π'' without changing the emission level. The tightening of the emission cap from z_0 to $z_0 + \delta_z$ implies that the maximum profit obtained under problem (4.2), which is π', is lower than the maximum profit obtained under problem (4.1). The cost of tightening the

[18] For $x \in \mathbf{R}^n$ and $\gamma > 0$, the neighbourhood of x of radius γ relative to $\mathbf{R}^n$ is the open set $N_\gamma(x) := \{x' \in \mathbf{R}^n \mid \|x' - x\| < \gamma\}$. The set, $\bar{N}_\gamma(x) := \{x' \in \mathbf{R}^n \mid \|x' - x\| \leq \gamma\}$, is its closure.



oil
coal
π_0
π'
π''
$\overline{N}_1(\mathrm{A})$
A
B
C
$\alpha_c c + \alpha_o\, o = z_0$
$\alpha_c c + \alpha_o\, o = z_0 + \delta_z$
2.0
1.5
1.0
0.5
0.0
0
1
2
3
4
5
6

Figure 2

emission constraint is, hence, the difference $\pi'' - \pi'$, or the change in the local maximum profit due to a change in emission cap equal to $\delta_z < 0$ is $\pi' - \pi'' < 0$. This corresponds to the numerator of (4.3). The difference $\pi' - \pi''$ can be re-written as $(\pi' - \pi_0) - (\pi'' - \pi_0)$, which is the maximum increase in profit computed by problem (4.2) minus the maximum increase in profit computed by problem (4.1). This corresponds to the numerator of (4.4).

4.2. Marginal abatement cost: A formal definition, computation, and an alternative interpretation.

For the purpose of empirically computing the MAC, it will be more helpful to re-define this concept in the context of differential versions of problems (4.1) and (4.2) by employing the differential characterisations of profit increasing, permissible, and emission non-increasing input reforms developed in Section 3.3. As discussed in Section 3.4, differential changes in profit due to such reforms can be interpreted as the derivatives of the profit function along linear paths of active policies defined by such reforms. Suppose there is a differentially emission non-increasing reform that, starting from the status-quo, leads to a differential increase in profit. Then this implies that the derivative of the profit function, evaluated at the status-quo along the policy path defined by the reform, is positive. We will show that this implies that there are vectors of emission non-increasing changes in active policies that, starting from the status-quo, non-differential increase profit. We will also show that the MAC can be interpreted as the reduction in profit per unit reduction in emission when a direction of reform in active policies that leads to the maximum differential reduction in emission is selected.

4.2.1. A formal definition.

Recalling (3.10) and (3.12), in what follows, we will often use notation $\dot{\nu}$ instead of δ to denote a reform and notation $\dot{z}$ instead of δ_z to denote a sporadic policy reform.

Let $\langle \nu_0, z_0, \Theta_0 \rangle$ be a tight status-quo and let $\dot{z}$ be a sporadic policy reform. Given $\dot{z}$, consider the set of all permissible reforms with magnitudes no bigger than one. The following programme, which is the differential analogue of programme (4.2), searches in this set for the reform that leads to the greatest differential increase in profit.

$$\dot{\mathbf{\Pi}}(\nu_0, z_0, \Theta_0, \dot{z}) := \max_{\dot{\nu}} \left\{ \nabla_\nu \Pi(\nu_0, \Theta_0) \cdot \dot{\nu} \;\middle|\; \nabla_\nu Z(\nu_0, z_0, \Theta_0) \cdot \dot{\nu} \le \dot{z} \;\wedge\; \dot{\nu} \cdot \dot{\nu} \le 1 \right\}. \quad (4.5)$$

Define the Lagrangian of problem (4.5) as

$$L\left(\nu_0, z_0, \Theta_0, \dot{z};\ \mu, \lambda\right) = \nabla_\nu \Pi(\nu_0, \Theta_0) \cdot \dot{\nu} - \mu\left[\nabla_\nu \mathcal{Z}(\nu_0, z_0, \Theta_0) \cdot \dot{\nu} - \dot{z}\right] - \lambda[\dot{\nu} \cdot \dot{\nu} - 1], \quad (4.6)$$

where μ and λ are the Lagrange multipliers for the two constraints of problem (4.5). In the case when the given sporadic policy reform $\dot{z}$ is zero, *i.e.,* there is no change in the emission cap at the status-quo, problem (4.5) becomes

$$\dot{\mathbf{\Pi}}(\nu_0, z_0, \Theta_0, 0) = \max_{\dot{\nu}} \left\{ \nabla_\nu \Pi(\nu_0, \Theta_0) \cdot \dot{\nu} \;\middle|\; \nabla_\nu \mathcal{Z}(\nu_0, z_0, \Theta_0) \cdot \dot{\nu} \leq 0 \;\wedge\; \dot{\nu} \cdot \dot{\nu} \leq 1 \right\}. \tag{4.7}$$

This is the differential analogue of programme (4.1). Suppose the image of the solution mapping and the mappings of the Lagrange multipliers evaluated at the optimum of problem (4.5) is the vector

$$\left\langle \dot{\nu}\,(\nu_0, z_0, \Theta_0, \dot{z})\,,\; \mu\,(\nu_0, z_0, \Theta_0, \dot{z})\,,\; \lambda\,(\nu_0, z_0, \Theta_0, \dot{z}) \right\rangle. \tag{4.8}$$

This implies that the maximum differential increase in profit at the status-quo is

$$\dot{\mathbf{\Pi}}(\nu_0, z_0, \Theta_0, \dot{z}) = \nabla_\nu \Pi(\nu_0, \Theta_0) \cdot \dot{\nu}\,(\nu_0, z_0, \Theta_0, \dot{z})\,. \tag{4.9}$$

In particular, when $\dot{z} = 0$, a solution vector of problem (4.7) and the Lagrange multipliers evaluated at the optimum are denoted by

$$\langle \overset{*}{\dot{\nu}}, \overset{*}{\mu}, \overset{*}{\lambda} \rangle \in \left\langle \dot{\nu}\,(\nu_0, z_0, \Theta_0, 0)\,,\; \mu\,(\nu_0, z_0, \Theta_0, 0)\,,\; \lambda\,(\nu_0, z_0, \Theta_0, 0) \right\rangle. \tag{4.10}$$

Suppose the maximum increase in profit $\dot{\mathbf{\Pi}}(\nu_0, z_0, \Theta_0, 0) > 0$ and the magnitude of the solution vector $\overset{*}{\dot{\nu}}$ is less than one, *i.e.,* $\|\overset{*}{\dot{\nu}}\| < 1$. Then the reform $\hat{\dot{\nu}} := \frac{\overset{*}{\dot{\nu}}}{\|\overset{*}{\dot{\nu}}\|}$ also satisfies the constraints of problem (4.7) and $\nabla_\nu \Pi(\nu_0, z_0, \Theta_0) \cdot \hat{\dot{\nu}} > \dot{\mathbf{\Pi}}(\nu_0, z_0, \Theta_0, 0)$, which is a contradiction to $\overset{*}{\dot{\nu}}$ being the solution vector. This leads to the following remark.

Remark 1. *If* $\dot{\mathbf{\Pi}}(\nu_0, z_0, \Theta_0, 0) > 0$ *then* $\overset{*}{\dot{\nu}} \cdot \overset{*}{\dot{\nu}} = 1$, *i.e., the solution vector* $\overset{*}{\dot{\nu}}$ *is a direction of reform and* $\dot{\mathbf{\Pi}}(\nu_0, z_0, \Theta_0, 0) = \nabla_\nu \Pi(\nu_0, \Theta_0) \cdot \overset{*}{\dot{\nu}}$ *is the directional derivative of the profit function in the direction of reform* $\overset{*}{\dot{\nu}}$.

In panel (a) of Figure 3, the gradient vector $\nabla_\nu \Pi$, which is evaluated at the status-quo A, is orthogonal to the line tangent to the iso-profit curve passing through A and corresponding to profit level π_0, while the gradient vector $\nabla_\nu \mathcal{Z}$ is also evaluated at the status-quo A and is orthogonal to both the lines corresponding to emission constraints z_0 and $z_0 + \delta_z$. Panel (b) of this figure is drawn in the space of $\dot{c}$ and $\dot{l}$ (reforms in coal and oil). The disc $\bar{N}_1(0)$ is the set of all reforms with magnitude less than or equal to one. The set of all emission non-increasing reforms form non-positive dot product with the gradient vector $\nabla_\nu \mathcal{Z}$, *i.e.,* they satisfy the condition $\nabla_\nu \mathcal{Z} \cdot \dot{\nu} \leq 0$.[19] This implies that these reforms form obtuse or right angles with $\nabla_\nu \mathcal{Z}$. The intersection of this set with $\bar{N}_1(0)$

[19] Given vectors x and $\bar{x}$ in $\mathbf{R}^n$, $x \cdot \bar{x} = \|x\|\|\bar{x}\| \cos\theta$, where θ is the angle between vectors x and $\bar{x}$. Hence, $x \cdot \bar{x} \leq 0$ implies that the angle between vectors x and $\bar{x}$ is obtuse (lies between 90° and 180°).

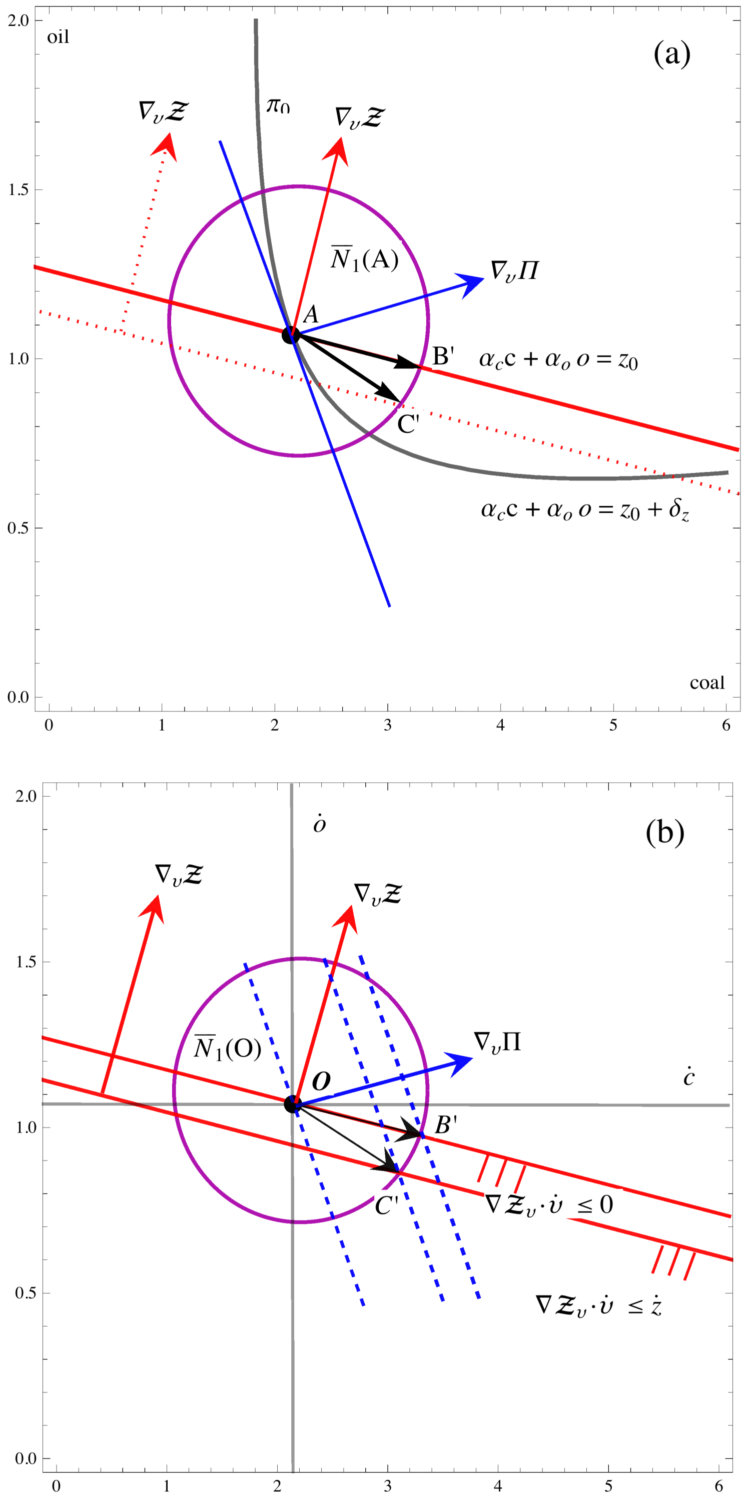

(a)
oil
coal
π_0
$\nabla_\upsilon \mathcal{Z}$
$\nabla_\upsilon \mathcal{Z}$
$\overline{N}_1$(A)
$\nabla_\upsilon \Pi$
A
B'
C'
$\alpha_c c + \alpha_o o = z_0$
$\alpha_c c + \alpha_o o = z_0 + \delta_z$
(b)
$\dot{o}$
$\dot{c}$
$\nabla_\upsilon \mathcal{Z}$
$\nabla_\upsilon \mathcal{Z}$
$\overline{N}_1$(O)
$\nabla_\upsilon \Pi$
O
B'
C'
$\nabla \mathcal{Z}_\upsilon \cdot \dot{\upsilon} \leq 0$
$\nabla \mathcal{Z}_\upsilon \cdot \dot{\upsilon} \leq \dot{z}$

Figure 3

forms the constraint set of problem (4.7), when coal and oil are the only active policies. The level curves of the linear objective function of problem (4.7) are indicated by the dotted lines. The vector $\nabla_\nu \Pi$ is orthogonal to each of this, as it is also the gradient of the objective function of problem (4.7). The solution vector (the optimal reform) of problem (4.7) is $\overrightarrow{OB'}$. The figure shows that the magnitude of vector $\overrightarrow{OB'}$ is one, and hence it is a direction of reform. When the emission constrained is tightened by $\delta_z = \dot{z} < 0$, then the set of permissible reforms satisfy the condition $\nabla_\nu \mathcal{Z} \cdot \dot{\nu} \leq \dot{z}$ is as shown in panel (b). The intersection of this set with $\bar{N}_1(0)$ forms the constraint set of problem (4.5). It is a subset of the constraint set of problem (4.7). The reform that solves problem (4.5) is $\overrightarrow{OC'}$. Note that the maximal differential increase in profit in problem (4.7) (where the emission cap is held at the status-quo level) is higher than in problem (4.5) (where the emission cap is tightened). This motivates the definition of the MAC given below: The formal definition of MAC adopted in this paper is

$$\mathcal{MAC}(\nu_0, z_0, \Theta_0) = \frac{\partial \dot{\mathbf{\Pi}}(\nu_0, z_0, \Theta_0, \dot{z})}{\partial \dot{z}}\Big|_{\dot{z}=0}.$$

This is the differential analogue of the intuitive definition of the MAC derived in (4.4) in Section 4.1. Precisely, it is the reduction in the maximum differential increase in profit in a local neighbourhood of the status-quo per unit reduction in the emission cap. It approximates the reduction in local maximum profit per unit reduction in the emission cap. Employing the envelope theorem after recalling (4.6) and (4.10), we obtain

$$\begin{aligned}\mathcal{MAC}(\nu_0, z_0, \Theta_0) = \frac{\partial \dot{\mathbf{\Pi}}(\nu_0, z_0, \Theta_0, \dot{z})}{\partial \dot{z}}\Bigg|_{\dot{z}=0} &= \frac{\partial L\left(\nu_0, z_0, \Theta_0, \dot{z};\ \mu(\nu_0, z_0, \Theta_0, \dot{z}), \lambda(\nu_0, z_0, \Theta_0, \dot{z})\right)}{\partial \dot{z}}\Bigg|_{\dot{z}=0} \\ &= \mu\left(\nu_0, z_0, \Theta_0, \dot{z}\right)\big|_{\dot{z}=0} = \overset{*}{\mu}.\end{aligned} \tag{4.11}$$

4.2.2. Using status-quo data to compute the marginal abatement cost and some additional results.

We note first that (2.4), (3.1), and (3.2) imply

$$\begin{aligned}\nabla_\nu \Pi &= \langle \Pi_l,\ \Pi_c,\ \Pi_g,\ \Pi_r,\ \Pi_o,\ \Pi_a \rangle \\ &= \langle F_l - w,\ F_e - p_c, F_e - p_g, F_e - p_r, F_o - p_o, -[G_f + G_a] \rangle \\ \nabla_\nu \mathcal{Z} &= \langle 0, \alpha_c, \alpha_g, 0, \alpha_o, -s \rangle\end{aligned} \tag{4.12}$$

These gradients can be evaluated at the status-quo employing status-quo data on all inputs, afforestation, prices, and stocks of capital and forests.

To compute the value that the MAC takes at the status-quo, we derive the Kuhn-Tucker first-order conditions of problem (4.7). To ease notation, with an abuse of notion, in what follows, we will denote

$$\nabla_\nu \Pi := \nabla_\nu \Pi(\nu_0, \Theta_0) \quad \wedge \quad \nabla_\nu \mathcal{Z} := \nabla_\nu \mathcal{Z}(\nu_0, z_0, \Theta_0). \tag{4.13}$$

The Kuhn-Tucker first-order conditions of problem (4.7) are

$$\begin{aligned} \nabla_\nu \Pi - \mu \nabla_\nu \mathcal{Z} - 2\lambda \dot{\nu} = 0_6 \\ \nabla_\nu \mathcal{Z} \cdot \dot{\nu} \leq 0, \quad \mu \geq 0, \quad \mu \left[\nabla_\nu \mathcal{Z} \cdot \dot{\nu} \right] = 0 \\ \dot{\nu} \cdot \dot{\nu} \leq 1, \quad \lambda \geq 0, \quad \lambda [\dot{\nu} \cdot \dot{\nu} - 1] = 0. \end{aligned} \tag{4.14}$$

Theorem 1, below, derives the formulae for computing MAC defined in (4.11) using the status-quo data. It first states that the maximum increase in profit that is possible with reforms that lead to points in a local neighbourhood of the status-quo is non-negative. In particular, it is zero when profit is maximised at the status-quo.[20] Part (1) of the theorem gives the formula for MAC for the case when the status-quo is not profit maximising. Since the MAC is defined as the Lagrange multiplier $\overset{*}{\mu}$, it can be computed employing formulae derived in (i) and (ii) of part (1) of the theorem, where the gradients $\nabla_\nu \mathcal{Z}$ and $\nabla_\nu \Pi$ are fixed as they are evaluated using data at the status-quo. Part (2) of the theorem considers the case when the status-quo is profit maximising. In this case, it is obvious that 0_6 is an optimal reform (*i.e.,* no change in policies starting from the status-quo are recommended). In this case, the formula for $\overset{*}{\mu}$ derived in (i) of part (1) provides only an estimate of the true MAC.

Theorem 1: *Let $\langle \nu_0, z_0, \Theta_0 \rangle$ be a tight status-quo. Then $\nabla_\nu \Pi(\nu_0, z_0, \Theta_0) \cdot \overset{*}{\dot{\nu}} \geq 0$.*

(1) *Suppose $\nabla_\nu \Pi \cdot \overset{*}{\dot{\nu}} > 0$. Then the following are true.*

(i) if $\nabla_\nu \mathcal{Z} \cdot \overset{*}{\dot{\nu}} = 0$ then

$$\mathcal{MAC}(\nu_0, z_0, \Theta_0) = \overset{*}{\mu} = \left(\nabla_\nu \mathcal{Z} \cdot \nabla_\nu \mathcal{Z} \right)^{-1} \left(\nabla_\nu \mathcal{Z} \cdot \nabla_\nu \Pi \right)$$
$$\wedge \quad \overset{*}{\lambda} = \frac{1}{2} \sqrt{\left[\nabla_\nu \Pi - \overset{*}{\mu} \nabla_\nu \mathcal{Z} \right] \cdot \left[\nabla_\nu \Pi - \overset{*}{\mu} \nabla_\nu \mathcal{Z} \right]} \quad \wedge \quad \overset{*}{\dot{\nu}} = \frac{\nabla_\nu \Pi - \overset{*}{\mu} \nabla_\nu \mathcal{Z}}{\sqrt{\left[\nabla_\nu \Pi - \overset{*}{\mu} \nabla_\nu \mathcal{Z} \right] \cdot \left[\nabla_\nu \Pi - \overset{*}{\mu} \nabla_\nu \mathcal{Z} \right]}}.$$

(ii) if $\nabla_\nu \mathcal{Z} \cdot \overset{*}{\dot{\nu}} < 0$ then

$$\mathcal{MAC}(\nu_0, z_0, \Theta_0) = \overset{*}{\mu} = 0 \quad \wedge \quad \overset{*}{\lambda} = \frac{1}{2} \sqrt{\nabla_\nu \Pi \cdot \nabla_\nu \Pi} \quad \wedge \quad \overset{*}{\dot{\nu}} = \frac{\nabla_\nu \Pi}{\sqrt{\nabla_\nu \Pi \cdot \nabla_\nu \Pi}}.$$

(2) *Suppose $\nabla_\nu \Pi \cdot \overset{*}{\dot{\nu}} = 0$. Then*

$$0_6 \in \dot{\nu}(\nu_0, z_0, \Theta_0, 0) \quad \wedge \quad \overset{*}{\lambda} = 0 \quad \wedge \quad \nabla_\nu \Pi = \overset{*}{\mu} \nabla_\nu \mathcal{Z}.$$

[20] See also Remark 2 below.

Remark 2. *Suppose $\langle \nu_0, z_0, \Theta_0 \rangle$ is a tight status-quo and $\nabla_\nu \Pi \cdot \overset{*}{\nu} = 0$. Then ν_0 solves the problem*

$$\max_{\nu'} \left\{ \Pi(\nu', \Theta_0) \;\middle|\; \mathcal{Z}(\nu', z_0, \Theta_0) \geq 0 \right\}, \tag{4.15}$$

i.e., profit is maximised at the status-quo subject to the emission cap z_0. There are no reforms in active policies at the status-quo which increase profit without increasing the emission level.

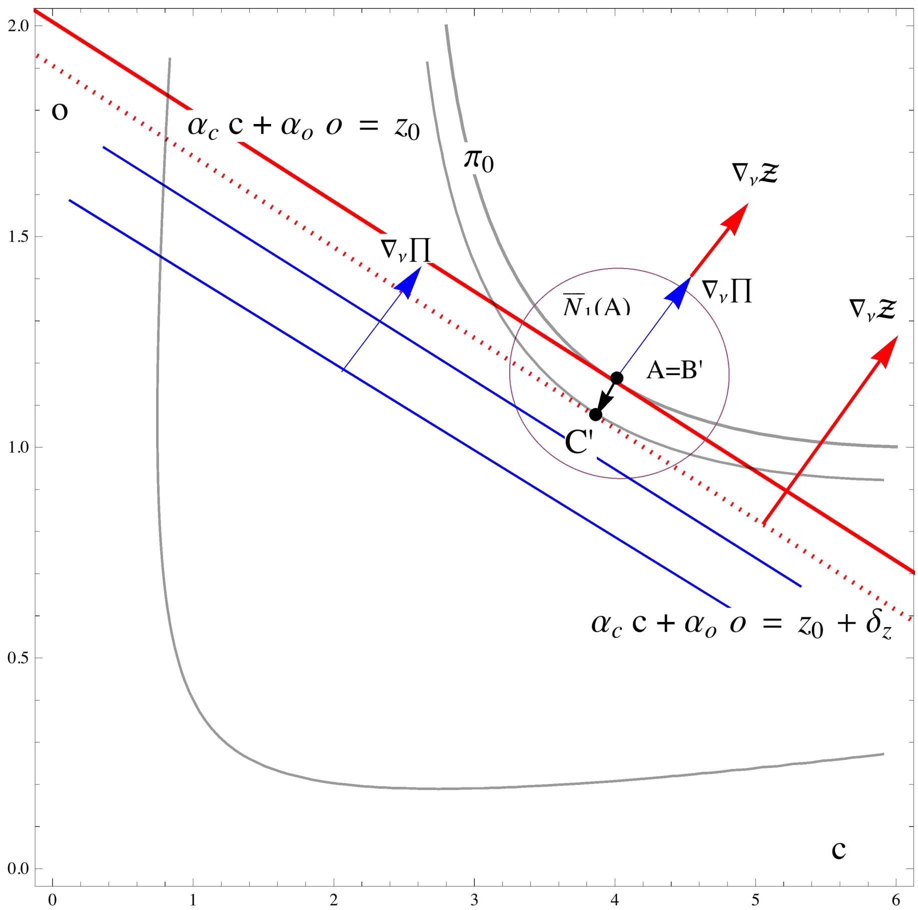
Figure 4 illustrates the above remark, where $\nabla_\nu \Pi$ and $\nabla_\nu \mathcal{Z}$ are proportional to each other. This is consistent with the profit maximisation condition $\nabla_\nu \Pi = \overset{*}{\mu} \nabla_\nu \mathcal{Z}$ in part (2) of Theorem 1.[21] Point A is the status-quo. It is profit maximising given the emission cap z_0. The level curves of the objective function of problem (4.7) are straight lines that are orthogonal to the vector $\nabla_\nu \Pi$. As can be seen, there are no permissible reform vectors that increase profit in neighbourhood $N_1(A)$ given emission cap z_0.

Part (ii) of part (1) of Theorem 1 shows that the MAC can be zero. Theorem 2, below, provides a characterisation of this situation in terms of data. It shows that the MAC is zero if and only if the dot product of the gradients of the emission and profit function evaluated at the status-quo is non-positive, *i.e.,* if and only if the angle between these two gradients lies between 90° and 180°.

Figure 5 illustrates this case. The status-quo is point A. The iso-profit curve passing through A is associated with a profit level π_0. The tangent of this curve passing through A is orthogonal to gradient vector $\nabla_\nu \Pi$. Three level curves of the objective function of problem (4.7) (which include the tangent through A) are shown. They are all orthogonal to gradient vector $\nabla_\nu \Pi$. Vector $\overrightarrow{AB'}$ is the solution to problem (4.7). Note that the emission constraint is non-binding at the optimum, in that $\nabla_\nu \mathcal{Z} \cdot \overrightarrow{AB'} < 0$ (the angle between $\nabla_\nu \mathcal{Z}$ and $\overrightarrow{AB'}$ is obtuse). A tightening of the emission constraint, *i.e.,* a reduction in the emission cap from z_0 to $z_0 + \dot{z}$, where $\dot{z} < 0$, leads to no change in the optimal reform. Solution to problem (4.5) is vector $\overrightarrow{AC'} = \overrightarrow{AB'}$. There is no differential change in local maximum profit when emission constraint is tightened. Hence, the MAC is zero.

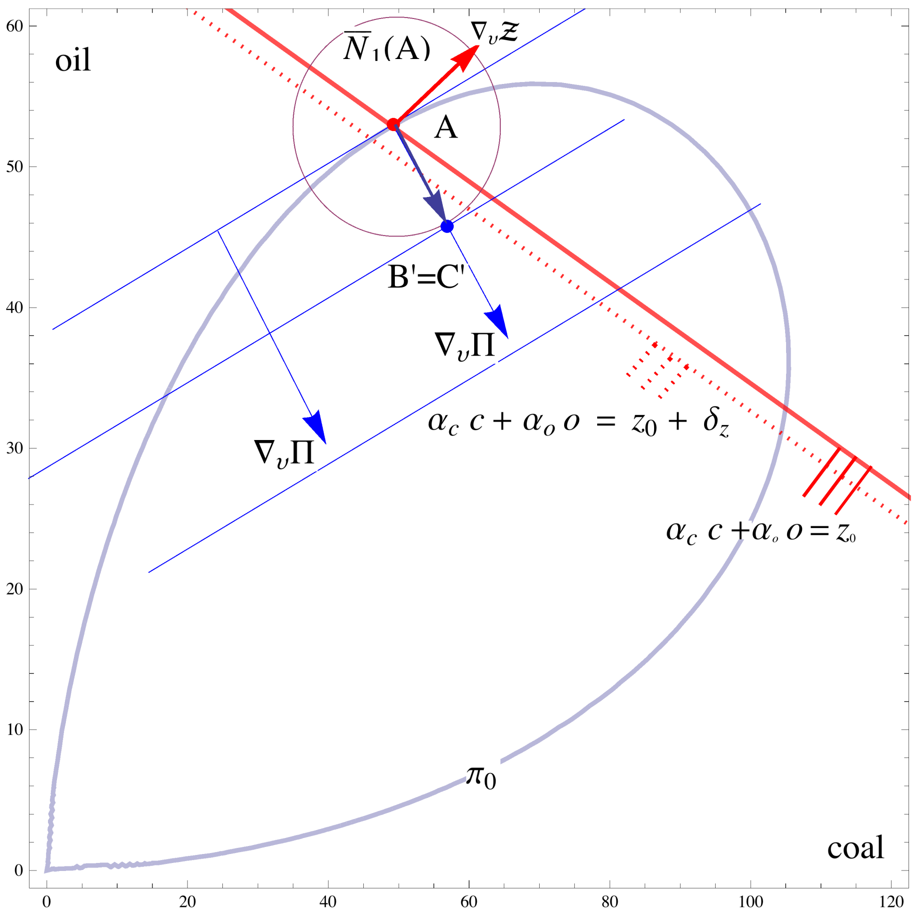
Contrast this with panel (a) of Figure 3, where the angle between $\nabla_\nu \mathcal{Z}$ and $\nabla_\nu \Pi$ is acute (between 0° and 90°). Hence, $\nabla_\nu \mathcal{Z} \cdot \nabla_\nu \Pi > 0$. The solution vector of problem (4.7) is $\overrightarrow{AB'}$. The emission constraint of problem (4.7) is binding in that $\nabla_\nu \mathcal{Z} \cdot \overrightarrow{AB'} = 0$. When the emission cap is reduced to $z_0 + \dot{z}$, where $\dot{z} = \delta_z < 0$, then the solution vector of problem (4.5) is $\overrightarrow{AC}$. The emission constraint of problem (4.5) is also binding, in that $\nabla_\nu \mathcal{Z} \cdot \overrightarrow{AC} = 0$. There is a differential decrease in local maximum profit when emission constraint is tightened. Hence, the MAC is positive.

[21] This forms the first-order conditions for problem (4.15).



o
α_c c + α_o o = z_0
π_0
$\nabla_\nu \mathcal{Z}$
$\nabla_\nu \prod$
$\overline{N}_1$(A)
$\nabla_\nu \prod$
$\nabla_\nu \mathcal{Z}$
A=B'
C'
α_c c + α_o o = z_0 + δ_z
c
2.0
1.5
1.0
0.5
0.0
0
1
2
3
4
5
6

Figure 4



oil
$\overline{N}_1(\mathrm{A})$
$\nabla_\upsilon \mathcal{Z}$
A
B'=C'
$\nabla_\upsilon \Pi$
$\nabla_\upsilon \Pi$
$\alpha_c\ c + \alpha_o\ o\ =\ z_0 +\ \delta_z$
$\alpha_c\ c + \alpha_o\ o = z_0$
π_0
coal

Figure 5

Theorem 2: *Let $\langle \nu_0, z_0, \Theta_0 \rangle$ be a tight status-quo with $\nabla_\nu \Pi \cdot \overset{*}{\nu} > 0$. Then $\overset{*}{\mu} = 0$ if and only if $\nabla_\nu \mathcal{Z} \cdot \nabla_\nu \Pi \leq 0$.*

Theorem 3 shows that if there exists a differentially emission non-increasing reform that results in a differential increase in profit evaluated at the status-quo, then there is a reform which is non-emission increasing that increases profit at the status-quo.

Theorem 3: *Let $\langle \nu_0, z_0, \Theta_0 \rangle$ be a tight status-quo with $\nabla_\nu \Pi \cdot \overset{*}{\nu} > 0$. Then there exists $\overset{*}{\delta} \in \mathbf{R}^6$ and $\overset{*}{t} > 0$ such that*

$$\Pi(\nu_0 + \overset{*}{\delta}, \Theta_0) > \Pi(\nu_0, \Theta_0) \quad \wedge \quad \mathcal{Z}(\nu_0 + \overset{*}{\delta}, z_0, \Theta_0) \leq 0 \quad \wedge \quad \overset{*}{\delta} = \overset{*}{t}\, \overset{*}{\nu}.$$

4.2.3. An interpretation of the marginal abatement cost.

Suppose $\langle \nu_0, z_0, \Theta_0 \rangle$ is a tight status-quo. Starting from this status-quo, every direction of change that leads to a decrease in emission defines an abatement strategy. Thus, $\dot{\nu}$ is an abatement strategy at the status-quo if $\|\dot{\nu}\| = 1$ and the directional derivative of the emission function in the direction of reform $\dot{\nu}$ is negative (see (3.13) with $\dot{z} = 0$):

$$\nabla_\nu \mathcal{Z}(\nu_0, z_0, \Theta_0) \cdot \dot{\nu} < 0,$$

There can be several abatement strategies at the status quo. For example, a direction of change where there is an increase in afforestation, $\dot{a} = 1$, but no change in any other active policies is a particular abatement strategy. A direction of change where there is a decrease in usage of coal, $\dot{c} = -1$, but no change in any other active policies is another abatement strategy.

Any abatement strategy $\dot{\nu}$ defines a directional derivative of the profit function given by $\nabla_\nu \Pi(\nu_0, \Theta_0) \cdot \dot{\nu}$ (see (3.11)). Denote the ratio of the directional derivatives of the profit function and the emission function defined by an abatement strategy $\dot{\nu}$ as

$$\Upsilon(\nu_0, z_0, \Theta_0, \dot{\nu}) := \frac{\nabla_\nu \Pi(\nu_0, \Theta_0) \cdot \dot{\nu}}{\nabla_\nu \mathcal{Z}(\nu_0, z_0, \Theta_0) \cdot \dot{\nu}}.$$

Intuitively, $\Upsilon(\nu_0, z_0, \Theta_0, \dot{\nu})$ captures the change in profit per unit change in emission as one moves, starting from the status-quo, an infinitesimal distance along the abatement path $\varphi^{\dot{\nu}}$ (see (3.9)).

In particular, note that the abatement strategy that leads to the maximum reduction in emission is given by $\dot{\nu}_{\mathcal{Z}} = -\frac{\nabla_\nu \mathcal{Z}(\nu, z, \Theta)}{\|\nabla_\nu \mathcal{Z}(\nu, z, \Theta)\|}$.[22] In that case, the change in the profit per

[22] That is, recalling the definition of a dot product of two vectors, $\dot{\nu}_{\mathcal{Z}}$ solves the problem

$$\min_{\dot{\nu} \in \mathbf{R}^6} \{\nabla_\nu \mathcal{Z} \cdot \dot{\nu} \mid \dot{\nu} \cdot \dot{\nu} = 1\}.$$

unit change in emission brought about by an infinitesimal move along the active policy path defined by the abatement strategy $\dot{\nu}_{\mathcal{Z}}$ is given by

$$\Upsilon\left(\nu_0, z_0, \Theta_0, \dot{\nu}_{\mathcal{Z}}\right) = \frac{\nabla_\nu \Pi(\nu_0, \Theta_0) \cdot \dot{\nu}_{\mathcal{Z}}}{\nabla_\nu \mathcal{Z}(\nu_0, z_0, \Theta_0) \cdot \dot{\nu}_{\mathcal{Z}}} = \frac{\nabla_\nu \Pi \cdot \nabla_\nu \mathcal{Z}}{\nabla_\nu \mathcal{Z} \cdot \nabla_\nu \mathcal{Z}}. \tag{4.16}$$

It is clear that Υ is positive if and only if the change in profit due to abatement strategy $\dot{\nu}_{\mathcal{Z}}$ (*i.e.,* $\nabla_\nu \Pi(\nu_0, \Theta_0) \cdot \dot{\nu}_{\mathcal{Z}}$) is positive. The latter is true if and only if $\nabla_\nu \Pi \cdot \nabla_\nu \mathcal{Z}$ is positive. From Theorems 1 and 2 and the definition of the MAC in (4.11), it follows that

$$\begin{aligned} \mathcal{MAC}(\nu_0, z_0, \Theta_0) &= \Upsilon\left(\nu_0, z_0, \Theta_0, \dot{\nu}_{\mathcal{Z}}\right) \quad && \text{if} \quad \nabla_\nu \Pi \cdot \nabla_\nu \mathcal{Z} > 0 \\ &= 0 && \text{if} \quad \nabla_\nu \Pi \cdot \nabla_\nu \mathcal{Z} \leq 0. \end{aligned} \tag{4.17}$$

Remark 3: *Intuitively, the marginal abatement cost is the change in profit per unit change in emission when the abatement strategy that leads to the maximum reduction in emission is adopted and the change in profit that it induces is positive. However, when the change in profit induced by this abatement strategy is negative, then the MAC is zero.*[23]

5. A measure of marginal abatement cost that is independent of the units of measurement of inputs.

If profit is measured in dollars, then the marginal abatement cost is measured in dollars per unit of carbon. The measure of marginal abatement cost, $\mathcal{MAC}(\nu_0, z_0, \Theta_0)$, derived in (4.11), is not free of units in which the inputs (which correspond to the active policy variables) are measured. Consider the case when at the status-quo, $\nabla_\nu \Pi \cdot \overset{*}{\nu} > 0$ and $\nabla_\nu \Pi \cdot \nabla_\nu \mathcal{Z} > 0$. Recalling (4.12), from Theorems 1 and 2 it follows that

$$\begin{aligned} \mathcal{MAC}(\nu_0, z_0, \Theta_0) &= \frac{\nabla_\nu \Pi \cdot \nabla_\nu \mathcal{Z}}{\nabla_\nu \mathcal{Z} \cdot \nabla_\nu \mathcal{Z}} \\ &= \frac{[F_e - p_c]\alpha_c + [F_e - p_g]\alpha_g + [F_o - p_o]\alpha_o - [G_f + G_a]s}{\alpha_c^2 + \alpha_g^2 + \alpha_o^2 + s^2}. \end{aligned} \tag{5.1}$$

It is clear that the denominator of the right-side of above equation is not independent of the units of measurement of coal, gas, oil, and forests, *e.g.,* the value of MAC obtained if afforestation is measured in meter-cube will be different from the value obtained if afforestation is measured in kilometer-cube.[24] We now derive a measure of MAC that is independent of the units of measurement of all inputs. This is the measure that is computed in our empirical analysis.

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[23] (4.17) computes the MAC as defined in (4.11) exactly when the status-quo is not profit maximising, *i.e.,* when $\nabla_\nu \Pi \cdot \overset{*}{\nu} > 0$. If profit is maximised at the status-quo, then as stated in Theorem 1, the gradients $\nabla_\nu \Pi$ and $\nabla_\nu \mathcal{Z}$ are proportional, and (4.17) finds only an estimate of the true MAC.

[24] When the unit is changed from meter-cube to kilometre-cube, the sequestration factor s has to be multiplied by a factor .001.

5.1. Proportionate changes in active policies, profit, and emission level starting from a tight status-quo and the elasticity analogue of problem (4.5).

We will now consider the space of logarithmic transformations of active policies. We will show that a path in this space, starting from the vector of logarithmic transformation of the status-quo levels of active policies, defines also a path in the space of active policies and determines how the profit and emission levels change along this path. The resulting changes in the profit and emission levels are independent of the units in which inputs are measured.

Consider the profit function and its double-log form when $\pi \neq 0$ and $\nu = \langle l, c, g, r, o, a \rangle \gg 0_6$:[25]

$$\pi = \Pi\left(l, c, g, r, o, a, z, \Theta\right) \qquad \Longleftrightarrow \qquad \ln \|\pi\| = \ln \left\| \Pi\left(\mathbf{e}^{\ln l}, \mathbf{e}^{\ln c}, \mathbf{e}^{\ln g}, \mathbf{e}^{\ln r}, \mathbf{e}^{\ln o}, \mathbf{e}^{\ln a}, \Theta\right) \right\|,$$

We define the vector $\hat{\nu} = \langle \hat{l}, \hat{c}, \hat{g}, \hat{r}, \hat{o}, \hat{a} \rangle := \langle \ln l, \ln c, \ln g, \ln r, \ln o, \ln a \rangle$ and $\hat{\pi} := \ln \|\pi\|$. We rewrite the double-log form of the profit function as

$$\hat{\pi} = \ln \left\| \Pi\left(\mathbf{e}^{\hat{l}}, \mathbf{e}^{\hat{c}}, \mathbf{e}^{\hat{g}}, \mathbf{e}^{\hat{r}}, \mathbf{e}^{\hat{o}}, \mathbf{e}^{\hat{a}}, \Theta\right) \right\|. \tag{5.2}$$

Thus, $\hat{\nu}$ is the vector of logarithmic transformations of active policy variables and $\hat{\pi}$ is the logarithmic transformation of the absolute level of profit. (5.2) implies that the latter is a function of the vector of logarithmic transformations of active policy variables

Let $\langle \nu_0, z_0, \Theta_0 \rangle$ be a tight status-quo, where $z_0 > 0$, $\nu_0 = \langle l_0,\ c_0,\ g_0,\ r_0,\ o_0,\ a_0 \rangle \in \mathbf{R}^6_{++}$, and $\pi_0 \neq 0$. The vector of logarithmic transformations of active policies evaluated at the status-quo is $\hat{\nu}_0 = \langle \hat{l}_0, \hat{c}_0, \hat{g}_0, \hat{r}_0, \hat{o}_0, \hat{a}_0 \rangle = \langle \ln l_0,\ \ln c_0,\ \ln g_0,\ \ln r_0,\ \ln o_0,\ \ln a_0 \rangle$. A linear path in the space of logarithmic transformation of active policy variables, starting from $\hat{\nu}_0$, parametrized by variable t, and defined by the vector $q = \langle q_l, q_c, q_g, q_r, q_o, q_a \rangle \in \mathbf{R}^6$ is a function $\hat{\varphi}^q : \mathbf{R}_+ \longrightarrow \mathbf{R}^6_+$ with image

$$\hat{\nu} = \langle \hat{l}, \hat{c}, \hat{g}, \hat{r}, \hat{o}, \hat{a} \rangle = \hat{\varphi}^q(t) = \hat{\nu}_0 + qt. \tag{5.3}$$

This implies that

$$\dot{\hat{\nu}} = \langle \dot{\hat{l}},\ \dot{\hat{c}},\ \dot{\hat{g}},\ \dot{\hat{r}},\ \dot{\hat{o}},\ \dot{\hat{a}} \rangle = \nabla_t \hat{\varphi}^q = q. \tag{5.4}$$

The path $\hat{\varphi}^q$ implies that the logarithmic transformations of the active policies (comprising vector $\hat{\nu}$) are functions of variable t. The path $\hat{\varphi}^q$ defines also a path $\varphi : \mathbf{R}_+ \longrightarrow \mathbf{R}^6_+$ with image $\nu = \langle l, c, g, r, o, a \rangle = \varphi(t)$ and $\varphi(0) = \nu_0$ in the space of active policies starting from the status-quo, $\langle \nu_0, z_0, \Theta_0 \rangle$. *E.g.,* from (5.3) it follows that $\hat{l} \equiv \ln l = \hat{l}_0 + q_l t$, which implies $l = \mathbf{e}^{\hat{l}_0 + q_l t}$. Hence, evaluated at the status-quo (which corresponds to $t = 0$), we

[25] $\mathbf{e}^x$ denotes the natural exponential function of x.

have $\dot{l} = l_0 q_l$. Employing (5.4), we obtain $\frac{\dot{l}}{l_0} = q_l = \dot{\hat{l}}$. Hence, evaluating the active policies at the status-quo, we obtain

$$\dot{\hat{\nu}} = \langle \dot{\hat{l}}, \dot{\hat{c}}, \dot{\hat{g}}, \dot{\hat{r}}, \dot{\hat{o}}, \dot{\hat{a}} \rangle = q \equiv \langle q_l,\ q_c,\ q_g,\ q_r,\ q_o,\ q_a \rangle = \left\langle \frac{\dot{l}}{l_0},\ \frac{\dot{c}}{c_0},\ \frac{\dot{g}}{g_0},\ \frac{\dot{r}}{r_0},\ \frac{\dot{o}}{o_0},\ \frac{\dot{a}}{a_0} \right\rangle \tag{5.5}$$

Hence, q can be interpreted as the vector of proportional changes in the active policy variables evaluated at the status-quo.

(5.2) and (5.3) imply that the logarithmic transformation of profit $\hat{\pi}$ changes along the path $\hat{\varphi}^q$, *i.e.,* $\hat{\pi}$ a function of t. (5.2) and (5.5) imply

$$\begin{aligned} \dot{\hat{\pi}} &= \varepsilon_{\pi l}\dot{\hat{l}} + \varepsilon_{\pi c}\dot{\hat{c}} + \varepsilon_{\pi g}\dot{\hat{g}} + \varepsilon_{\pi r}\dot{\hat{r}} + \varepsilon_{\pi o}\dot{\hat{o}} + \varepsilon_{\pi a}\dot{\hat{a}} \\ &= \varepsilon \cdot q \qquad \text{with} \qquad \varepsilon := \langle \varepsilon_{\pi l}, \varepsilon_{\pi c}, \varepsilon_{\pi g}, \varepsilon_{\pi r}, \varepsilon_{\pi o}, \varepsilon_{\pi a} \rangle, \end{aligned} \tag{5.6}$$

where *e.g.,* $\varepsilon_{\pi l} = \frac{\partial \hat{\pi}}{\partial \hat{l}} = \Pi_l \frac{l_0}{\pi_0}$ is the elasticity of profit with respect to labour. It is the proportional change in profit per unit proportional change in labour at the status-quo. Since q is the vector of proportional changes in the active policy variables evaluated at the status-quo, (5.6) implies that $\varepsilon \cdot q$ measures, starting from the status-quo, the proportional change in profit due to the vector of proportional changes q in active policies. Precisely, we can show that

$$\dot{\hat{\pi}} = \frac{\dot{\pi}}{\pi_0} = \varepsilon_{\pi l}\frac{\dot{l}}{l_0} + \varepsilon_{\pi c}\frac{\dot{c}}{c_0} + \varepsilon_{\pi g}\frac{\dot{g}}{g_0} + \varepsilon_{\pi r}\frac{\dot{r}}{r_0} + \varepsilon_{\pi o}\frac{\dot{o}}{o_0} + \varepsilon_{\pi a}\frac{\dot{a}}{a_0} = \varepsilon \cdot q. \tag{5.7}$$

Starting from the tight status-quo, the vector q of proportional changes in active policies is a *direction of proportionate changes in active policies* if $\|q\| = 1$. If the magnitude of vector q is one, then $\varepsilon \cdot q$ is the directional derivative of $\hat{\pi}$ in the direction of proportional changes q in active policies. Further, evaluated at the status-quo, (5.7) implies that the *derivative of the profit function due to a direction of proportional changes in active policies* q is

$$\dot{\pi} = \pi_0\ \varepsilon \cdot q. \tag{5.8}$$

Defined in this manner, the derivative of the profit function is independent of the units of measurement of all inputs as elements of vectors ε and q are elasticities of profit with respect to inputs and proportionate changes inputs, respectively, and hence are independent of units.

Similarly, we can derive the double-log form of the emission function when $z > 0$ and $\nu = \langle l, c, g, r, o, a \rangle \gg 0_6$:

$$z = \mathcal{Z}(\nu, z, \Theta) \quad \Longleftrightarrow \quad \hat{z} = \ln\left(\alpha_c \mathbf{e}^{\hat{c}} + \alpha_g \mathbf{e}^{\hat{g}} + \alpha_o \mathbf{e}^{\hat{o}} - s\mathbf{e}^{\hat{a}}\right),$$

where $\hat{z} = \ln z$. The directional derivative of $\hat{z}$, evaluated at the status-quo, given the direction of proportionate changes in active policies q, is

$$\begin{aligned}\dot{\hat{z}} &= \psi_{zc}\dot{\hat{c}} + \psi_{zg}\dot{\hat{g}} + \psi_{zo}\dot{\hat{o}} + \psi_{za}\dot{\hat{a}} \\ &= \psi_{zc}\frac{\dot{c}}{c_0} + \psi_{zg}\frac{\dot{g}}{g_0} + \psi_{zo}\frac{\dot{o}}{o_0} + \psi_{za}\frac{\dot{a}}{a_0} \\ &= \Psi \cdot q \qquad \text{with} \qquad \Psi := \langle 0, \psi_{zc}, \psi_{zg}, 0, \psi_{zo}, \psi_{za} \rangle \,,\end{aligned} \tag{5.9}$$

where, for $i = c, g, o$, we have $\psi_{zi} = \frac{\partial \hat{z}}{\partial \hat{c}} = \mathcal{Z}_i \frac{i_0}{z_0} = \frac{\alpha_i i_0}{z_0}$ is the elasticity of emission with respect to fossil-fuel i, and $\psi_{za} = \frac{\partial \hat{z}}{\partial \hat{a}} = \mathcal{Z}_a \frac{a_0}{z_0} = \frac{-s a_0}{z_0}$ is the elasticity of emission with respect to afforestation, evaluated at the status-quo. Thus, $\Psi \cdot q$ measures, starting from the status-quo, the proportional change in emission due to the direction of proportional changes q in active policies. It can be shown that

$$\dot{\hat{z}} = \frac{\dot{z}}{z_0} = \psi_{zc}\frac{\dot{c}}{c_0} + \psi_{zg}\frac{\dot{g}}{g_0} + \psi_{zo}\frac{\dot{o}}{o_0} + \psi_{za}\frac{\dot{a}}{a_0} = \Psi \cdot q. \tag{5.10}$$

Furthermore, (5.9) implies that, evaluated at the status-quo, the *derivative of emission due to the direction of proportional changes q in active policies* is given by

$$\dot{z} = z_0 \Psi \cdot q. \tag{5.11}$$

Defined in this manner, the derivative of the emission function is also independent of the units of measurement of all inputs as elements of vectors Ψ and q are elasticities of emission with respect to inputs and proportionate changes in inputs, respectively, and hence are independent of units.

The problem, below, is the elasticity analogue of problem (4.5) that identifies, at the tight status-quo, the direction vector of proportional changes in active policies, which results in the maximal proportional increase in profit, given a change $\dot{z}$ (and hence a given proportional increase $\frac{\dot{z}}{z_0}$) in the sporadic (emission) policy.

$$\mathcal{V}(\nu_0, z_0, \Theta_0, \dot{z}) = \max_q \left\{ \varepsilon(\nu_0, z_0, \Theta_0) \cdot q \;\middle|\; \Psi(\nu_0, z_0, \Theta_0) \cdot q \leq \frac{\dot{z}}{z_0} \;\wedge\; q \cdot q \leq 1 \right\}. \tag{5.12}$$

5.2. A measure of marginal abatement cost that is independent of the units of measurement of all inputs.

The Lagrangian of problem (5.12) evaluated at $\dot{z} = 0$ is

$$\tilde{L}\left(\nu_0, z_0, \Theta_0, \dot{z};\; \tilde{\mu}, \tilde{\lambda}\right)\bigg|_{\dot{z}=0} = \left[\varepsilon(\nu_0, z_0, \Theta_0) \cdot q - \tilde{\mu}\left[\Psi(\nu_0, z_0, \Theta_0) \cdot q - \frac{\dot{z}}{z_0}\right] - \tilde{\lambda}[q \cdot q - 1]\right]\bigg|_{\dot{z}=0}. \tag{5.13}$$

Let the choice variables and the Lagrange multipliers evaluated at the optimum of problem (5.12) be

$$\left\langle q\left(\nu_0, z_0, \Theta_0, \dot{z}\right), \tilde{\mu}\left(\nu_0, z_0, \Theta_0, \dot{z}\right), \tilde{\lambda}\left(\nu_0, z_0, \Theta_0, \dot{z}\right)\right\rangle\Bigg|_{\dot{z}=0} = \left\langle \overset{*}{q}, \overset{*}{\tilde{\mu}}, \overset{*}{\tilde{\lambda}}\right\rangle. \tag{5.14}$$

The maximum proportionate increase in profit at the status-quo with no change in emission policy is given by

$$\frac{\overset{*}{\pi}}{\pi_0} = \mathcal{V}\left(\nu_0, z_0, \Theta_0, \dot{z}\right)\Bigg|_{\dot{z}=0} = \varepsilon(\nu_0, z_0, \Theta_0) \cdot \overset{*}{q}.$$

This, combined with (5.8), implies that the increase in profit (derivative of profit function) at the status-quo due to the direction of proportionate changes $\overset{*}{q}$ in active policies is given by

$$\overset{*}{\pi} = \pi_0\ \mathcal{V}\left(\nu_0, z_0, \Theta_0, \dot{z}\right)\Bigg|_{\dot{z}=0} = \pi_0\varepsilon(\nu_0, z_0, \Theta_0) \cdot \overset{*}{q}.$$

Hence, if $\pi_0 > 0$, then a measure of MAC, evaluated at the status-quo, can be obtained by employing the envelope theorem as

$$\begin{aligned} MAC_{\Psi}(\nu_0, z_0, \Theta_0) = \frac{\partial\overset{*}{\pi}}{\partial\dot{z}}\Bigg|_{\dot{z}=0} &= \pi_0 \frac{\partial\mathcal{V}\left(\nu_0, z_0, \Theta_0, \dot{z}\right)}{\partial\dot{z}}\Bigg|_{\dot{z}=0} \\ &= \pi_0 \frac{\partial\tilde{L}\left(\nu_0, z_0, \Theta_0, \dot{z};\ \overset{*}{\tilde{\mu}}, \overset{*}{\tilde{\lambda}}\right)}{\partial\dot{z}}\Bigg|_{\dot{z}=0} \\ &= \frac{\pi_0}{z_0}\overset{*}{\tilde{\mu}}. \end{aligned} \tag{5.15}$$

In the case when $\pi_0 \leq 0$, we define the MAC as

$$MAC_{\Psi}(\nu_0, z_0, \Theta_0) = \frac{\|\pi_0\|}{z_0}\overset{*}{\tilde{\mu}}. \tag{5.16}$$

Intuitively, when π_0 is positive (respectively, negative), then the MAC defined in (5.15) (respectively, (5.16)) measures the reduction in local profit (the increase in local loss) per unit proportionate reduction in the emission cap, when the optimal directions of proportionate changes in active policies are adopted.

The following theorem, proof of which is exactly analogous to the proof of Theorem 1, computes the Lagrange multiplier $\overset{*}{\tilde{\mu}}$.[26]

[26] With an abuse of notation, we denote $\varepsilon\left(\nu_0, z_0, \Theta_0\right)$ by ε and $\Psi\left(\nu_0, z_0, \Theta_0\right)$ by Ψ.

Theorem 4: *Let $\langle \nu_0, z_0, \Theta_0 \rangle$ be a tight status-quo with $z_0 > 0$, $\nu_0 \gg 0_6$, and $\pi_0 \neq 0$. Then $\varepsilon \cdot \overset{*}{\tilde{q}} \geq 0$.*

(1) *Suppose $\varepsilon \cdot \overset{*}{\tilde{q}} > 0$. Then the following are true.*

(i) if $\Psi \cdot \overset{*}{\tilde{q}} = 0$ then

$$\overset{*}{\tilde{\mu}} = (\Psi \cdot \Psi)^{-1} (\varepsilon \cdot \Psi) \quad \wedge \quad \overset{*}{\tilde{\lambda}} = \frac{1}{2} \sqrt{\left[\varepsilon - \overset{*}{\tilde{\mu}} \Psi\right] \cdot \left[\varepsilon - \overset{*}{\tilde{\mu}} \Psi\right]} \quad \wedge \quad \overset{*}{\tilde{q}} = \frac{\varepsilon - \overset{*}{\tilde{\mu}} \Psi}{\sqrt{\left[\varepsilon - \overset{*}{\tilde{\mu}} \Psi\right] \cdot \left[\varepsilon - \overset{*}{\tilde{\mu}} \Psi\right]}}.$$

(ii) if $\Psi \cdot \overset{*}{\tilde{q}} < 0$ then

$$\overset{*}{\tilde{\mu}} = 0 \quad \wedge \quad \overset{*}{\tilde{\lambda}} = \frac{1}{2} \sqrt{\varepsilon \cdot \varepsilon} \quad \wedge \quad \overset{*}{\tilde{q}} = \frac{\varepsilon}{\sqrt{\varepsilon \cdot \varepsilon}}.$$

(2) *Suppose $\varepsilon \cdot \overset{*}{\tilde{q}} = 0$. Then*

$$0_6 \in q(\nu_0, z_0, \Theta_0, 0) \quad \wedge \quad \overset{*}{\tilde{\lambda}} = 0 \quad \wedge \quad \varepsilon = \overset{*}{\tilde{\mu}} \Psi.$$

Corollary to Theorem 4: *Let $\langle \nu_0, z_0, \Theta_0 \rangle$ be a tight status-quo with $z_0 > 0$, $\nu_0 \gg 0_6$, and $\pi_0 \neq 0$. Then $MAC_\Psi(\nu_0, z_0, \Theta_0)$ is independent of the units of measurement of all inputs.*[27]

The corollary follows because, from Theorem 4 and (5.15), we can rewrite the marginal abatement cost when $\varepsilon \cdot \overset{*}{\tilde{q}} > 0$ as

$$\begin{aligned} MAC_\Psi(\nu_0, z_0, \Theta_0) &= \frac{\|\pi_0\|}{z_0} \frac{\varepsilon \cdot \ \Psi}{\Psi \cdot \ \Psi} \quad \text{if} \quad \Psi \cdot \overset{*}{\tilde{q}} = 0 \\ &= 0 \qquad\qquad\quad \text{if} \quad \Psi \cdot \overset{*}{\tilde{q}} < 0, \end{aligned} \tag{5.17}$$

where vectors ε and Ψ are both vectors of elasticities with respect to all inputs and $\overset{*}{\tilde{q}}$ is the vector of proportional changes in active policy instruments. Hence, all these vectors are independent of the units of measurement of inputs.

[27] In the case when $\epsilon \cdot \overset{*}{\tilde{q}} = 0$, $MAC_\Psi(\nu_0, z_0, \Theta_0)$ is an estimate of the true MAC.

5.3. Decomposition of marginal abatement cost into the ability to abate and the reduction in profit.

We will now decompose the measure of MAC defined in (5.17) into a measure of the ability to abate (ATA) of the producing unit and a measure of reduction in profit (RIP) of the producing unit due to the adoption of the abatement strategy that results in the maximum proportionate reduction of emission of the producing unit at the status-quo.

Starting from the tight status-quo $\langle \nu_0, z_0, \Theta_0 \rangle$ with $z_0 > 0$, $\nu_0 \gg 0_6$, and $\pi_0 \neq 0$, from (5.10) it follows that the direction of proportionate changes in active policies that results in the maximum proportionate decrease in emission is $q_\Psi := -\frac{\Psi}{\|\Psi\|}$.[28]

Employing (5.8) and (5.11), the derivatives of profit and emission functions, evaluated at the tight status-quo, and corresponding to the direction of proportionate changes in active policies q_Ψ, are

$$\begin{aligned} \dot{\pi}_\Psi &= \pi_0 \varepsilon \cdot q_\Psi = & -\pi_0 \; \varepsilon \cdot \frac{\Psi}{\|\Psi\|} \\ \dot{z}_\Psi &= z_0 \Psi \cdot q_\Psi = & -z_0 \; \Psi \cdot \frac{\Psi}{\|\Psi\|}. \end{aligned} \tag{5.18}$$

Below, we define two quantities, which are evaluated at the states-quo:

$$RIP_\Psi = \|\pi_0\| \; \varepsilon \cdot \frac{\Psi}{\|\Psi\|} = -\|\pi_0\| \; \varepsilon \cdot \; q_\Psi \qquad \text{and} \qquad ATA_\Psi = z_0 \Psi \cdot \frac{\Psi}{\|\Psi\|} = -z_0 \Psi \cdot \; q_\Psi.$$

(5.18) implies that, intuitively, $\pi_0 \; \varepsilon \cdot q_\Psi$ and $z_0 \Psi \cdot q_\Psi$ measure, respectively, the change in profit and the change in emission at the status-quo due to the adoption of the direction of proportionate changes in active policies that results in the maximum proportionate decrease in emission. Hence, RIP_Ψ and ATA_Ψ measure, respectively, the *reduction in profit* and the reduction in emission (the *ability to abate*) at the status-quo due to the adoption of the direction of proportionate changes in active policies that results in the maximum proportionate decrease in emission.[29] It follows from (5.17) and (5.18) that, evaluated at the status-quo, the MAC is

$$\begin{aligned} MAC_\Psi(\nu_0, z_0, \Theta_0) &= \frac{RIP_\Psi}{ATA_\Psi} \quad && \text{if} \quad \Psi \cdot \overset{*}{q} = 0 \\ &= 0 && \text{if} \quad \Psi \cdot \overset{*}{q} < 0. \end{aligned} \tag{5.19}$$

Theorem 5, below, follows in a manner analogous to Theorem 2 and from employing (5.15) and the definition of RIP_Ψ. The theorem states that the marginal abatement cost, MAC_Ψ, is positive if and only if the reduction in profit, RIP_Ψ, is greater than zero.

[28] This follows because, from (5.10), the proportionate reduction in emission is a dot product of the elasticity vector Ψ and the vector of proportionate changes in active policies.

[29] Note, the reduction in profit or emission will be positive (respectively, negative) if the change in profit or emission (*i.e.*, $\dot{\pi}_\Psi$ or $\dot{z}_\Psi$) is negative (respectively, positive).

Theorem 5: *Let $\langle \nu_0, z_0, \Theta_0 \rangle$ be a tight status-quo with $z_0 > 0$ and $\pi_0 \neq 0$. Then $\overset{*}{\mu} = 0$ if and only if $\varepsilon \cdot \Psi \leq 0$ and*

$$MAC_\Psi(\nu_0, z_0, \Theta_0) = 0 \quad \Longleftrightarrow \quad \varepsilon \cdot \Psi \leq 0 \quad \Longleftrightarrow \quad RIP_\Psi \leq 0.$$

Together, Theorem 5 and (5.19) imply

$$\begin{aligned} MAC_\Psi(\nu_0, z_0, \Theta_0) &= \frac{RIP_\Psi}{ATA_\Psi} \quad \text{if} \quad RIP_\Psi > 0 \\ &= 0 \qquad\quad\ \ \text{if} \quad RIP_\Psi \leq 0. \end{aligned} \tag{5.20}$$

5.4. Factors affecting the ability to abate and the reduction in profit.

Remarks 4, 5, and 6 in this section describe the factors that affect the ATA and the RIP of the producing unit due to adoption of the abatement strategy that results in the maximum proportionate reduction in its emission. Given the definition of Ψ in (5.9), the vector of proportionate changes in active policies q_Ψ is obtained as

$$\begin{aligned} q_\Psi = \left\langle q_{l_\Psi},\ q_{c_\Psi},\ q_{g_\Psi},\ q_{r_\Psi},\ q_{o_\Psi},\ q_{a_\Psi} \right\rangle &= -\frac{\Psi}{\|\Psi\|} = \frac{-1}{\|\Psi\|} \langle 0, \psi_{zc}, \psi_{zg}, 0, \psi_{zo}, \psi_{za} \rangle \\ &= \frac{-1}{\sqrt{\alpha_c^2 c_0^2 + \alpha_g^2 g_0^2 + \alpha_o^2 o_0^2 + s^2 a_0^2}} \langle 0,\ \alpha_c c_0,\ \alpha_g g_0,\ 0,\ \alpha_o o_0,\ -s a_0 \rangle . \end{aligned} \tag{5.21}$$

Recalling (5.5), (5.21) implies that the change in (derivatives of) active policy variables corresponding to the direction of proportionate changes in active policies q_Ψ are:

$$\begin{aligned} \dot{\nu}_\Psi := \left\langle \dot{l}_\Psi,\ \dot{c}_\Psi,\ \dot{g}_\Psi,\ \dot{r}_\Psi,\ \dot{o}_\Psi, \dot{a}_\Psi \right\rangle &= \langle q_{l_\Psi} l_0,\ q_{c_\Psi} c_0,\ q_{g_\Psi} g_0,\ q_{r_\Psi} r_0,\ q_{o_\Psi} o_0,\ q_{a_\Psi} a_0 \rangle \\ &= \frac{1}{\sqrt{\alpha_c^2 c_0^2 + \alpha_g^2 g_0^2 + \alpha_o^2 o_0^2 + s^2 a_0^2}} \langle 0,\ -\alpha_c c_0^2,\ -\alpha_g g_0^2,\ 0,\ -\alpha_o o_0^2,\ s a_0^2 \rangle . \end{aligned} \tag{5.22}$$

The vector $\dot{\nu}_\Psi$ implies that the changes in fossil-fuel usage implied by q_Ψ are negative and the change in afforestation is positive. Thus, q_Ψ discourages the use of fossil fuels and encourages afforestation, with the extent of *reductions* in fossil fuels and *increase* in afforestation given, respectively, by

$$\begin{aligned} \langle -\dot{c}_\Psi,\ -\dot{g}_\Psi,\ -\dot{o}_\Psi \rangle &= \frac{1}{\sqrt{\alpha_c^2 c_0^2 + \alpha_g^2 g_0^2 + \alpha_o^2 o_0^2 + s^2 a_0^2}} \langle \alpha_c c_0^2,\ \alpha_g g_0^2,\ \alpha_o o_0^2 \rangle \\ \dot{a}_\Psi &= \frac{s a_0^2}{\sqrt{\alpha_c^2 c_0^2 + \alpha_g^2 g_0^2 + \alpha_o^2 o_0^2 + s^2 a_0^2}}. \end{aligned} \tag{5.23}$$

Remark 4. *From (5.23) it follows that, for $i = c, g, o$, the derivative of $-\dot{i}_\Psi$ with respect to i_0 is positive.*[30] *Hence, the reduction in any fossil-fuel implied by the direction of proportionate changes in active policies q_Ψ is higher, the higher is the consumption of the fossil-fuel at the status-quo. (5.23) also implies that the the derivative of $\dot{a}_\Psi$ with respect to a_0 is positive. Hence, the increase in afforestation implied by q_ψ is higher, the higher is the status-quo level of afforestation.*

It can be shown that the ATA at the status-quo is given by

$$\begin{aligned} ATA_\Psi &\equiv -z_0 \Psi \cdot q_\Psi = -\nabla_\nu \mathcal{Z} \cdot \dot{\nu}_\Psi \\ &= [\alpha_c(-\dot{c}_\Psi) + \alpha_g(-\dot{g}_\Psi) + \alpha_o(-\dot{o}_\Psi) + s\dot{a}_\Psi] \\ &= \sqrt{\alpha_c^2 c_0^2 + \alpha_g^2 g_0^2 + \alpha_o^2 o_0^2 + s^2 a_0^2}. \end{aligned} \tag{5.24}$$

Remark 5: *(5.24) and (5.23) imply that the ATA implied by q_Ψ will be higher,*

(i) the higher are the reductions of fossil fuels and the greater is the increase in afforestation implied by q_Ψ at the status-quo. Remark 4 implies that this will be true the higher are the consumptions of the fossil fuels or the higher is the afforestation at the status-quo.

(ii) the higher are the reductions in usage of those fossil fuels whose emission factors (α_c, α_g, or α_o) are high or the greater is the increase in afforestation whenever the sequestration factor (s) of forest is high.

From the definition of ε in (5.6), we have

$$\begin{aligned} \pi_0 \varepsilon &= \langle l_0 \Pi_l,\ c_0 \Pi_c,\ g_0 \Pi_g,\ r_0 \Pi_r,\ o_0 \Pi_o,\ a_0 \Pi_a \rangle \\ &= \big\langle l_0[F_l - w],\ c_0[F_e - p_c],\ g_0[F_e - p_g],\ r_0[F_e - p_r],\ o_0[F_o - p_o],\ -a_0[G_f + G_a] \big\rangle . \end{aligned} \tag{5.25}$$

For all $i = l,\ c,\ g,\ r,\ o$, we will call the element $i_0 \Pi_i = i_0[F_i - p_i]$ of $\pi_0 \varepsilon$ as the *profitability of one-per cent increase in input i* and $-a_0 \Pi_a = a_0[G_f + G_a]$ as the *cost of one-percent increase in afforestation.*

From (5.21) it follows that the proportionate changes in fossil fuels implied by q_Ψ are negative and the proportionate change in afforestation implied by q_Ψ is positive. It follows that the proportionate *reductions* in fossil fuels and the proportionate *increase* in afforestation implied by q_Ψ are

$$\begin{aligned} \langle -q_{c_\Psi}, -q_{g_\Psi}, -q_{o_\Psi} \rangle &= \frac{1}{\sqrt{\alpha_c^2 c_0^2 + \alpha_g^2 g_0^2 + \alpha_o^2 o_0^2 + s^2 a_0^2}} \langle \alpha_c c_0,\ \alpha_g g_0,\ \alpha_o o_0 \rangle \\ q_{a_\Psi} &= \frac{s a_0}{\sqrt{\alpha_c^2 c_0^2 + \alpha_g^2 g_0^2 + \alpha_o^2 o_0^2 + s^2 a_0^2}}. \end{aligned} \tag{5.26}$$

[30] For example, the derivative $-\frac{\partial \dot{c}_\Psi}{\partial c_0} > 0$.

Employing (5.21), (5.25), and (5.26) we can show that the RIP implied by q_Ψ is

$$\begin{aligned} RIP_\Psi &= -\pi_0\varepsilon \cdot q_\Psi \\ &= c_0[F_e - p_c](-q_{c_\Psi}) + g_0[F_e - p_g](-q_{g_\Psi}) + o_0[F_o - p_o](-q_{o_\Psi}) + a_0[G_f + G_a]q_{a_\Psi} \\ &= \frac{\big[[F_e - p_c]\alpha_c c_0^2 + [F_e - p_g]\alpha_g g_0^2 + [F_o - p_o]\alpha_o o_0^2 + [G_f + G_a]s a_0^2\big]}{\sqrt{\alpha_c^2 c_0^2 + \alpha_g^2 g_0^2 + \alpha_o^2 o_0^2 + s^2 a_0^2}} \end{aligned} \tag{5.27}$$

Remark 6. *From (5.27) and (5.26) it follows that the reduction in profit due to adoption of the proportionate direction of change in active policies q_Ψ is higher,*

(i) the higher are the profitabilities of one-percent increases in the fossil fuels (namely, $c_0\Pi_c = c_0[F_e - p_c]$, $g_0\Pi_g = g_0[F_e - p_g]$, or $o_0\Pi_o = o_0[F_o - p_o]$) and the lower is the cost of one-percent increase in afforestation (i.e., $a_o\Pi_a = a_0[G_f + g_a]$) at the status-quo.

(ii) the higher are the proportionate reductions in the usage of those fossil fuels whose profitabilities of one-percent increases are the greatest at the status-quo or the greater is the proportionate increase in afforestation whenever the marginal (per unit) cost of one-percent increase in afforestation is high at the status-quo.

6. Data and estimation procedures.

Our sample consists of 118 countries – 26 countries from Africa, 31 from Asia, 39 from Europe, 13 from North America, 7 from South America, and 2 from Oceania.

6.1. Data on GDP, capital, labour, and energy.

The data on GDP, capital, and labour for these countries was derived from Version 4 of the Extended Penn Tables (EPWT) for four years, 1990, 2000, 2005, and 2010. Real GDP and capital are measured in 2005 purchasing power parity in USD, while labour is measured as number of workers.

Data on total energy use and its split into fossil-fuel and alternative (including nuclear and renewable) energy sources was obtained from the World Development Indicators (WDI), published by the World Bank and International Energy Agency (IEA), for four years 1990, 2000, 2005, and 2010. Energy is measured in kilotonnes of oil equivalents (ktoe). The split of fossil fuel energy into coal, gas, and oil was obtained from the country tables of the US Energy Information Agency (US-EIA). A similar country-level detailed split of energy from alternative sources into independent sources such as nuclear, hydro, wind, solar, etc., was not available for an extensive coverage of countries.

6.2. Emission data and emission factors.

Country-level data on CO_2 emissions from fossil-fuel combustion was obtained from IEA for 2010. For parity with carbon sequestration by forests, the emissions of CO_2 are converted into million tonnes of carbon (mtc) by multiplying them by a factor, 12/44.[31] Tables D.1 and D.2 provide lists of countries with highest and lowest carbon emissions in our sample. China, USA, India, and Russia are the top emitters, while Bhutan has the lowest emission level in our sample. The emission factors α_c, α_g, and α_o, for coal, gas, and oil, respectively, are derived from IPCC (2006) guidelines. Table 2.3 of these guidelines was employed to derive an emission factors representative of these three fossil fuels. The emission factors for coal, gas, and oil, were taken to be 100,000, 57,000, and 75,000 kilograms of CO_2 per tera joules of energy, respectively. Employing appropriate conversion factors, these amount to $\alpha_c = 0.001141854$, $\alpha_g = 0.000650857$, and $\alpha_o = 0.000856391$ mtc per ktoe of coal, gas, and oil, respectively.

6.3. Forest data and computation of carbon sequestration factors.

Country-level forest data was obtained from the Global Forest Resources Assessment (FRA) 2010 of the Food and Agricultural Organisation. In its guidelines for country reporting (2008), FRA asks countries to estimate and report the carbon content of their forests in 2010 in accordance with the guidelines given in IPCC (2006). Measures of carbon sequestered in forests are based on the estimates of forest biomass, which in turn are derived from data on the growing stock (volume) of forests measured in million meter-cube (mm3). The growing stock of forests is converted into forest biomass, based on conversion factors given in IPCC (2006). Forest biomass, measured in millions of tonnes (mt), is a sum of the above ground biomass (AGB), the below ground biomass (BGB), and dead wood.[32] A biomass expansion and contraction factor (BECF) converts the forest stock into AGB. The BGB is obtained by multiplying the AGB by the root-shoot ratio (RSR). The forest biomass is converted into carbon stock in forest, measured in mtc, by multiplying the former by a carbon fraction (CF). In general, the BCEF, the RSR, and the CF depend on the type of trees and other vegetation in forests, and hence will be forest-type and more generally country-specific. IPCC (2006) provides a global default value of CF equal to 0.47, though many countries prefer to measure this independently.

Since carbon sequestration depends ultimately on the volume (stock) of forests, we measure the extent of forests of any country, and its afforestation and deforestation in mm3. Thus, the carbon sequestration factor s in our theoretical model is defined as the amount of carbon sequestered per unit volume of forest and is measured in mtc per mm3.

31 See *e.g.,* EPA brochure: http://www.epa.gov/cpd/pdf/brochure.pdf.

32 In our analysis we omit the dead wood component as it is usually small for countries.

From the discussion above, it is derived as $s = BECF \times (1 + RSR) \times CF$. Individual country reports of FRA were employed to compute, for each country, its BECF as the ratio of the reported volume of forests and the amount of AGB. RSR was computed as the ratio of reported amounts of BGB and AGB. Finally, country statements on CF, in combination with the derived BCEF and RSR, were employed to derive the value of s for each country. Carbon sequestration factors by continent and climate zone-types are listed in Tables D.3 and D.4. Table D.4 shows that the average carbon sequestration is highest in tropical region followed by the regions with warm deserts. The continent with the highest average carbon sequestration factor is Africa. Countries in Africa lie mainly in tropical or warm desert climate zones, the zones with highest carbon sequestration factors. Table D.5 provides a list of countries with highest carbon sequestration factors. Most of these countries are in Africa and lie in the tropical or the warm desert zones.

Given time series of data on forest stock from FRA, the change in the forest stock over time is computed. The change in the forest stock could be attributed to afforestation, deforestation, and natural regeneration. FRA does not provide independent information on these three components.[33] As a result, we make the following assumptions: We assume that there is some positive afforestation in every country.[34] Change in the forest stock during the period 2009-2010 is computed. If the change in the forest stock is positive, it is attributed completely to afforestation. If it is found to be negative then a country is assigned a small amount of afforestation, which is assumed to be 0.0001 mm3 by default. Table D.6 shows the average afforestation levels across various continents. Afforestation levels are highest in North America and Asia. There are only two countries in our sample in Oceania. These are Australia and New Zealand. Afforestation levels are close to zero in Australia, while they are high in New Zealand (27.4 mm3). Ten countries with the highest afforestation levels in our sample are listed in Table D.7. USA, China, Russia, and India top this list.

6.4. Price data and imputations of energy costs and expenditures on afforestation.

6.4.1. Cost of energy.

International data on the price of labour, the wage rate, is obtained from EPWT [2014] in 2005 USD purchasing power parity. Prices of various sources of energy (excluding oil) are derived from the Projected Costs of Generating Electricity (PCGE) [2010] published by

[33] FRA provides information on the increase in area of planted forests, but the conversion of forest area into forest volume is very much dependent on the forest type and the tree species involved. Factors that convert a country's forest area into its growing stock are not readily available. As discussed above, carbon sequestration depends on the extent of the growing stock of forest.

[34] We do not study reforms in the deforestation policy.

IEA and the Nuclear Energy Agency (NEA). This report includes 21 countries (16 OECD; 3 non-member countries including Brazil, Russia, and South Africa; industry participants from Australia, France, and European Union; and some plants from China). It is based on cost data for 190 power plants from these countries. The study computes levelised costs of producing electricity (LCOE) from various sources – coal, gas, nuclear, and renewables. LCOE includes investment costs, fuel costs, operations and maintenance costs, etc. For fossil fuels, an additional carbon (externality) cost is added for some (mainly OECD) countries. This is like a fixed Pigouvian tax levied per unit of emission. However, in our model, the emission-policy instrument is a carbon cap, which makes the Pigouvian tax redundant. Hence, in our analysis, we consider the LCOE net of the carbon cost. In this report, LCOE are computed in USD per mega-watt-hours of electricity generated. We convert this into USD per ktoe of electricity generated employing appropriate conversion factors. In our analysis we employ the LCOE computed at a 5% discount rate.[35] The study demonstrates great variations in the LCOE across regions, depending upon regional differences in factors such as availability of low cost fuels (some regions may be richly endowed with fuels, while other have to import them at higher prices), natural endowments of renewable resources, capital and other input costs, etc. In general, LCOE from most sources are lowest in Asia, especially for China, and are high in Europe.

In the absence of other data sources that estimate LCOE for an extensive coverage of countries, we extrapolate the findings of the PCGE report to countries outside its sample of study. *E.g.,* with respect to coal-fired power plants, the extrapolation scheme is as follows: We impute the LCOE in China to all countries in Asia, except the Middle Eastern countries, Japan, South Korea, and countries in the former Soviet Block. Estimates for Japan and South Korea are provided by PCGE. To the countries in the former Soviet Block in both Asia and Europe we impute Russia's LCOE. The average LCOE for plants in Europe in the sample considered by PCGE is imputed to all countries in Europe except for those for which we have data from PCGE. Coal does not seem to be an important source of electrical energy in most African and Gulf countries due to their poor endowment of coal. These countries rely on gas-powered or other renewable sources of energy (such as hydroelectricity). At the same time, we assume that other input costs such as labour and capital costs in these countries are closer to those in Asia. Hence, we impute Japan's LCOE (which is the highest in Asia) to all countries in Africa that produce some coal-fired electricity, except South Africa, for which PCGE provides estimates. South Africa seems to be an exception in Africa in this regards, as it produces and imports coal-fired electricity. To all countries in North America, we impute the LCOE of either USA or Mexico, which are provided by PCGE. To all countries in South America, we impute

[35] In the PCGE report these are computed for 5% and 10% discount rates.

the LCOE of Brazil, and to countries in Oceania in our sample, namely, Australia and New Zealand, we impute the LCOE in Australia. Similar extrapolations are done also for imputing LCOE to countries in the case of gas-powered and renewable electricity.

Several internet sources and repositories of the International Energy Agency (IEA) and US-IEIA were employed to obtain country-level energy profiles for nuclear and renewable sources of energy. The profiles noted, for each country, the different types of alternative sources of energy in place as well as its plans and intentions for extensions to newer renewable sources in the future.[36] Note, the geography and the natural endowments of any country define or place constraints on the types of renewable energy sources available to it currently and in the future, *e.g.,* hydropower may not be a viable source of renewable energy in a country comprising chiefly deserts. It can only be a small-scale provider of power for countries with no big rivers. For each country, the LCOE for renewable energy is computed as the average levelised cost computed over renewables that were noted in the profile of the country. A more disaggregated approach could not be followed due to lack of country-level data on the actual amounts of energy produced by each renewable source.[37]

Table D.8 shows the wage rate and average LCOE for coal-fired, gas-fired, and renewable energy plants for some countries in the sample studied by PCGE. The table shows that labour and energy costs are lowest in China.

Oil is a significant source of non-electrical energy. Oil-powered electricity generation is relatively more costly in most countries. Moreover, the split of data on oil energy between electrical and non-electrical usage is not available at the level of all countries. At the same time, consumption of oil is highly positive for all countries. Thus, we assume that it is an essential input in production for all countries.[38] Hence, we assume that all the data on oil energy pertains to non-electrical energy. Hence, the price of oil energy is taken to be the crude oil price, which is obtained from the BP Statistical Review as 83.7 USD per barrel in 2010. This is equivalent to 572591.7 USD per ktoe.

6.4.2. Cost of afforestation.

There are several case studies that have been conducted for selected countries, which study costs of afforestation and maintaining current stocks of forests. These estimates vary widely. Even within a single country, estimates vary widely depending on the location of forests and the methodologies used. Given the wide variation in estimates in recent studies and due to incomplete coverage of countries by these studies, we rely on two

[36] Note, the information on the actual amount of energy produced from each source was not available consistently for all countries in the sample.

[37] Note, this information is needed to compute profit elasticity of renewables in our model.

[38] This is unlike coal and gas. There are countries in our sample which consume no coal or gas. *E.g.,* some African countries consume no coal.

earlier sources: Chp. 24 of the Second Assessment Report (SAR) of IPCC (1996) and Sathaye and Ravindranathan (SR) (1998), which compile estimates from different case-studies in a manner that greatly facilitates attribution of comprehensive costs of forests (including afforestation costs) to all countries in our sample. We consider seven climate zones of the world – polar, boreal, cold desert, temperate, Mediterranean (sub-tropical), warm deserts, and tropical. For each country, we identify the major zones that it lies in. SAR and SR tabulate ranges of costs of forests across different latitudinal belts of the world in USD per tonn of carbon (tc) sequestered. In our analysis, the comprehensive cost of afforestation is captured by the term $G_f + G_a$, and is hence measured in USD per meter-cube (m3) of forest. Hence, we multiply the cost estimates in the SAR and SR by the sequestration factor s, which is measured in tc per m3. Parts of different continents that lie in each latitudinal belt are identified by the SAR and SR. We relate the latitudinal belts to the climate zones. Employing their tables, for each continent, we attribute costs of forest to the different types of climate zones within which it lies. See Table D.9. For every country, depending on the continent that it lies in, an estimate of the cost of forest is obtained as the average of costs across different climate zones that it lies in. There seem no studies which explicitly report costs of forests in the desert regions of the world. It should be noted, however, that the sequestration factors are very high for deserts (see Table D.4), and many desert countries derive huge profits from export of fossil-fuel resources, which could be used in part to finance afforestation reforms. Hence, it is not clear at the outset that afforestation cannot form a part of profit increasing and emission-non increasing set of reforms for desert countries. Since forest cost estimates are not available for these countries, we impute the highest forest costs in the SAR and SR to these countries, namely, 29 and 27 USD per tc sequestered for warm and cold desert regions, respectively.

The average afforestation costs across continents are presented in Table D.10. Afforestation costs are the highest in Africa and Oceania (Australia and New Zealand). Significant parts of Africa and Australia are deserts, and this explains the high afforestation costs in these regions. Asia has a wide variation in its climatic zones. Although, significant parts of Asia lie in tropical, subtropical, and temperate zones with lower afforestation costs, regions of several countries in Asia are also deserts. Hence, the average afforestation cost in Asia is higher than in Europe and the Americas. The countries with the highest afforestation costs in our sample are listed in Table D.11. The table shows that most of these countries lie in major warm or cold deserts of the world.

6.5. Estimation and computation procedures.

Energy is an intermediate input, requiring resources for its production. Data on how capital and labour resources are divided between the energy and non-energy sectors are not readily available for all countries. Rather, we have data on aggregate capital and labour inputs. Hence, we proceed in a manner similar to many works in the literature, *e.g.,* Van der Werf (2008), where output is the sum of total value added and the value of energy. In our case, the output variable y, is the sum of GDP and the value of energy.

The production function in (2.1) was assumed to have a Cobb Douglas form:[39]

$$y = Ak^{\beta_k} l^{\beta_l} e^{\beta_e} o^{\beta_o} \iff \ln y = \ln A + \beta_k \ln k + \beta_l \ln l + \beta_e \ln e + \beta_o \ln o.$$

An unbalanced panel of 118 countries across four years, 1990, 2000, 2005, and 2010 was employed, and a stochastic production frontier with the above functional form was estimated with country-specific dummies. Estimates of coefficients of all inputs can be found in Table D.12. These coefficients are all positive and significant. The sum of input coefficients is $\beta_k + \beta_l + \beta_e + \beta_o = 0.8306 < 1$. Hence, the the production function exhibits decreasing returns to scale. We also obtain country-specific estimate of the total factor productivity term due to the use of country-specific dummies. The descriptive statistics of this can be found in Table D.13.

The MAC is computed employing (5.17), where elasticity vectors ε and Ψ are defined as in (5.6) and (5.9), and computed using the status-quo data. The ATA and RIP are computed employing (5.24) and (5.27), respectively.[40]

7. Results.

The empirical analysis computes the measure of MAC, derived in Section 5, that is independent of the units of measurements of inputs. None of the 118 countries in our sample maximises its profit at the status-quo subject to its existing emission level. Hence, profit increasing and emission non-increasing reforms exist at the status-quo for every country. In Section 5, the MAC was shown to be the ratio of the ATA and the RIP. Hence, we first compute and study international differences in abilities to abate (AsTA) and the reductions in profits (RIPs). Based on these trends, we next study the international differences in the marginal abatement costs (MACs). The role of afforestation in lowering MACs is also studied.

[39] We adopt this simple production structure in this paper for convenience. This assumption can be relaxed and functional forms which are nested to capture different degrees of substitutability's between energy, capital, and labour can also be adopted as in Van der Werf (2008) and in references therein.

[40] MatLab codes were created to compute the ATA, RIP, and MAC for all countries in our sample.

7.1. *International differences in abilities to abate (AsTA).*

The measure ATA_{Ψ} is computed employing (5.24) in tonnes of carbon (tc). Table R.1 shows that the ATA ranges in our sample of countries from 0.12 to 1905.89. The average ATA is 55.61. For 50% of the countries in our sample, the AsTA are lesser than 8.67. Nearly 90% of the countries have low AsTA (AsTA less than 100). 6% of the countries in our sample (*i.e.,* seven countries) have moderately high AsTA (AsTA lying between 100 and 200). Table R.2 shows that the countries with the highest AsTA (AsTA greater than 200) are China, USA, India, Russia, and Japan. They comprise around 4% of our sample.

As explained in (i) of Remark 5, the ATA is related to fossil-fuel usage by countries. Both the magnitude of fossil-fuel usage (measured by the average usage across the three fossil fuels) and the composition of fossil fuels used influence the ATA. The rank correlation between average fossil-fuel usage and ATA is very high, nearly 0.986. The countries with the highest AsTA are also the largest users of fossil fuels. As explained in Remark 4, a high fossil-fuel usage implies that the direction of proportionate changes, q_{Ψ}, in active policies that leads to the maximum proportionate reductions in emission discourages greatly the usage of fossil fuels. This leads to greater reduction in emission and, hence, higher ATA. The lists of eighteen countries with the highest AsTA and with the highest average fossil fuel consumption are provided in Tables R.2 and R.3, respectively. Any discrepancy in the two rankings is attributed to the differences in the compositions of fossil fuels used. For example, although India has lower average fossil-fuel consumption than Russia, it has a higher ATA than Russia. Table R.4 shows that this is because India consumes disproportionately more than Russia those fossil fuels that have the highest emission factors, namely, coal and oil (see Section 6.2). Russia consumes more gas than India, the emission factor of which is lower than that of coal and oil. This is consistent with (ii) of Remark 5, which explains how the composition of fossil-fuel usage affects the ATA. In our sample of countries, the country with the least fossil-fuel usage is Bhutan, but it is not the country with the least ATA. (Bhutan is the 37^{th} lowest ATA country.) The discrepancy is again explained by the composition of fossil-fuel usage in Bhutan. Bhutan uses only two fossil fuels, coal followed by oil. As seen above, these are associated with the highest emission factors. A moderately high sequestration factor s coupled with its good afforestation effort also contribute towards increasing Bhutan's ATA.

To study further international differences in the AsTA, we divide our sample into 34 OECD and 84 non-OECD countries. Table R.6 and frequency and cumulative frequency polygons in panels (a) and (b) of Figure 6 indicate that 88% of the non-OECD countries have AsTA less than 50, while less than 60% of the OECD countries have AsTA less than 50. Table R.2 shows that there are five OECD countries with AsTA between 90 to 200, namely, Germany, South Korea, Canada, UK, and Mexico. There are two OECD countries with AsTA greater than 200. These are USA and Japan. In the group of non-OECD

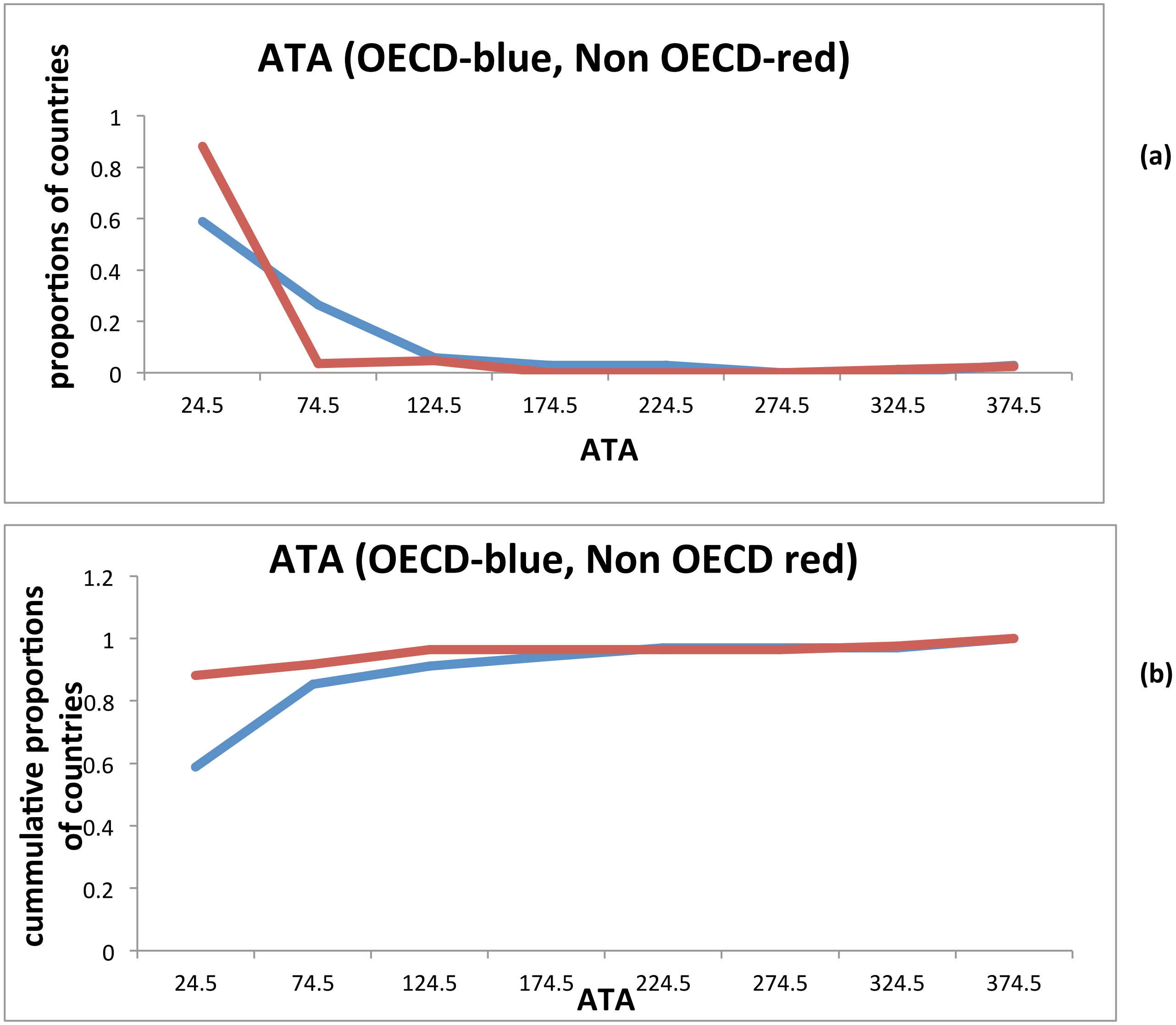

ATA (OECD-blue, Non OECD-red)
proportions of countries
1
0.8
0.6
0.4
0.2
0
24.5
74.5
124.5
174.5
224.5
274.5
324.5
374.5
ATA
(a)
ATA (OECD-blue, Non OECD red)
cummulative proportions of countries
1.2
1
0.8
0.6
0.4
0.2
0
24.5
74.5
124.5
174.5
224.5
274.5
324.5
374.5
ATA
(b)

Figure 6

countries, China, India, Russia, South Africa, Brazil, Iran, Saudi Arabia, and Indonesia have discretely high AsTA (greater than 80) as compared to the other countries in this group, which have AsTA less than 55. The non-OECD countries with AsTA greater than 200 are China, India, and Russia. As stated earlier, all the high AsTA countries (whether OECD or non-OECD) are associated with the highest usage of fossil fuels.

Consider now the groups of OECD and non-OECD countries after excluding the countries listed above that have high and very high AsTA. Table R.5 shows that the AsTA in these groups of OECD and non-OECD countries vary from 0.95 to 86.54 and 0.12 to 53.98, respectively. The average and median AsTA in the OECD group of such countries is considerably higher than in the corresponding group of non-OECD countries. As seen in this table, this is also associated with higher average and median fossil-fuel usage in the OECD set as opposed to the corresponding set of non-OECD countries.

7.2. International differences in reductions in profits.

RIP in USD is computed by employing (5.27). Table R.7 shows that the RIP, due to adoption of the abatement strategy q_Ψ that results in the greatest proportional decrease in emission at the status-quo, ranges in our sample from $-20,079.3$ to $1,152,288.08$. The average RIP is $40,700.79$, while 50% of the countries in the sample have RIPs less than $6,502.41$. Almost 90% of the countries in our sample have RIP less than $100,000$. Lists of countries with the lowest and highest RIPs can be found in Tables R.8 and R.10, respectively. Twelve countries have RIPs greater than $100,000$, with five countries (namely, Germany, France, China, UK, and India) having RIPs between $200,000$ to $400,000$ and two countries (USA and Japan) having RIPs greater than $400,000$. USA has the highest RIP in our sample, which is nearly three times that of Japan.

As explained in Remark 6, the magnitude of RIP depends, on the one hand, on the profitabilities of one-percent increases in fossil fuels and the cost of one-percent increase in afforestation and, on the other, the extent to which abatement strategy q_Ψ discourages usage of fossil fuels and encourages afforestation. This is consistent with our empirical results:

Firstly, we find that the rank correlation coefficient between the average of profitabilities of fossil fuels and afforestation and RIP is 0.897, which is high. The top twelve countries with respect to RIP are also the top twelve countries with respect to average profitability (see Tables R.8 and R.9).

Secondly, RIP is often negative in the set of countries with the lowest RIPs (see Table R.10), indicating that, in these countries, there exist inputs for which the profitabilities per percentage increases are negative (see Table R.11). Thus, reductions in such fossil fuels actually increase profits in these economies. It is also interesting to note that some countries with the lowest RIPs such as Russia, Ukraine, Kazakhstan, and South Africa are also countries whose average profitabilities of inputs are significantly positive. As seen in Table R.11, this is because, in these countries, the profitabilities of one percent increases in coal and gas are negative and the abatement strategy q_{Ψ} discourages disproportionately more the usage of one or both of these inputs. At the same time it discourages less the usage of oil, whose profitability of one percent increase is positive.

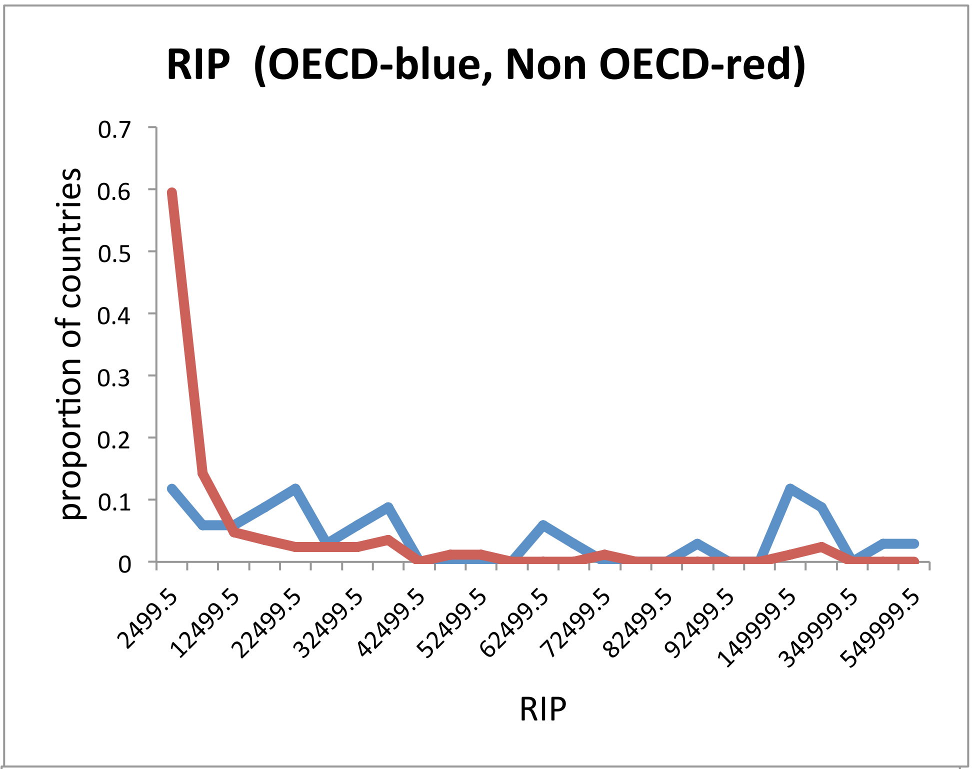
In most of the top twelve countries with respect to RIP, profitability of coal and oil is higher as compared to gas, and the abatement strategy q_{Ψ} discourages disproportionately more the use of coal and oil (see Table R.12). This accounts for the high RIPs in these countries.

We now study the differences in RIPs between the OECD and non-OECD countries based on Table R.13 and frequency and cumulative frequency polygons in panels (a) and (b) of Figures 7. These indicate that RIP in the OECD countries varies between $1,162$ and $1,152,288$, while it varies between $-20,079$ and $227,456$ in the non-OECD countries. Further, while only 36% of the non-OECD countries have RIPs greater than $1,000$, almost 82% of the OECD countries have RIPs greater than $1,000$. The mean and median RIPs in OECD countries are hence higher (respectively, $103,970.7$ and $32,137.8$) than in the non-OECD countries (respectively, $15,091.5$ and $3,172.1$). The OECD countries with RIPs greater than $100,000$ are US, Japan, Germany, France, UK, Italy, France, Mexico, Spain, and Canada (see Table R.8). The top five RIP countries in the non-OECD block are China, India, Brazil, Indonesia, and Iran, of which RIPs of only the first three countries are greater than $100,000$ (see Table R.8). The RIPs of the latter two countries are $71,630.18$ and $53,232.02$, respectively.

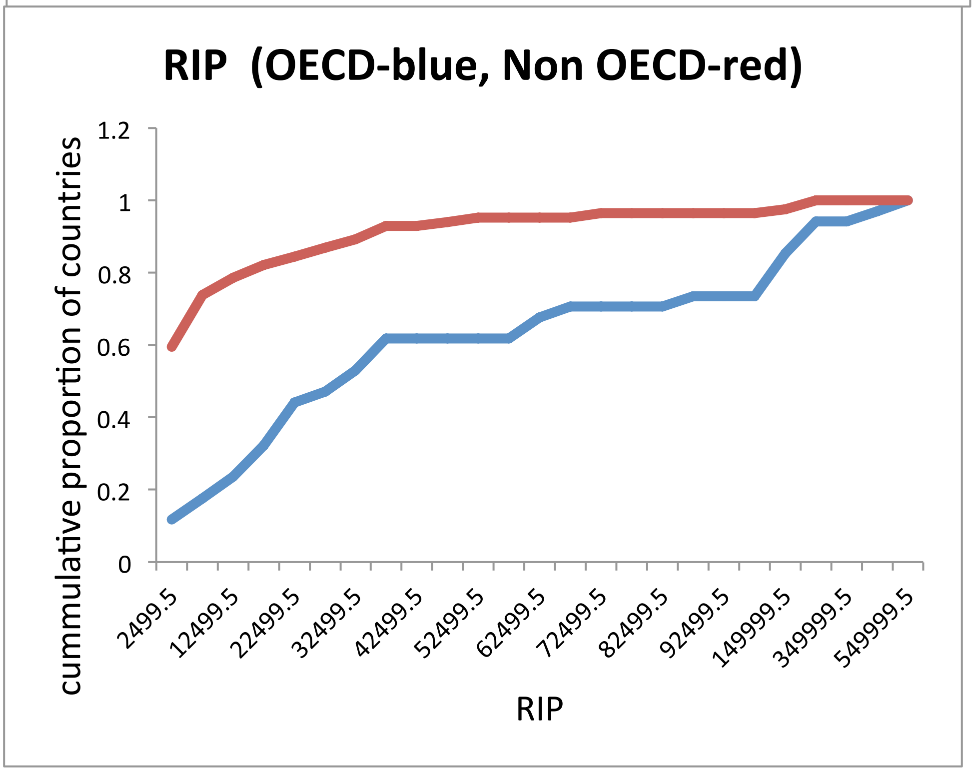
If we exclude the high RIP countries from the OECD and non-OECD groups of countries and study the remaining groups of countries, then Table R.14 shows that the mean and median RIPs and average profitability of inputs in these non-OECD countries are significantly smaller than in the corresponding OECD countries.

7.3. International differences in marginal abatement costs and relation to abilities to abate and reductions in profits.

The measure of MAC derived in (5.20) is computed in dollars per ton of carbon (dptc). The MACs of 118 countries in our sample vary from zero to $6,700.24$. Table R.15 shows that the mean and median MACs are $1,345.7$ and $1,082.5$, respectively. Majority of the countries (more than 94% of the countries) have MAC less that $3,000$. There are seven countries with exceptionally high MACs (MACs greater than $3,000$). These are listed in Table R.16. The countries with lowest MACs (MACs lying between zero and 10) and MACs of countries with the highest AsTA in our sample are also listed in Tables R.17 and R.18, respectively.

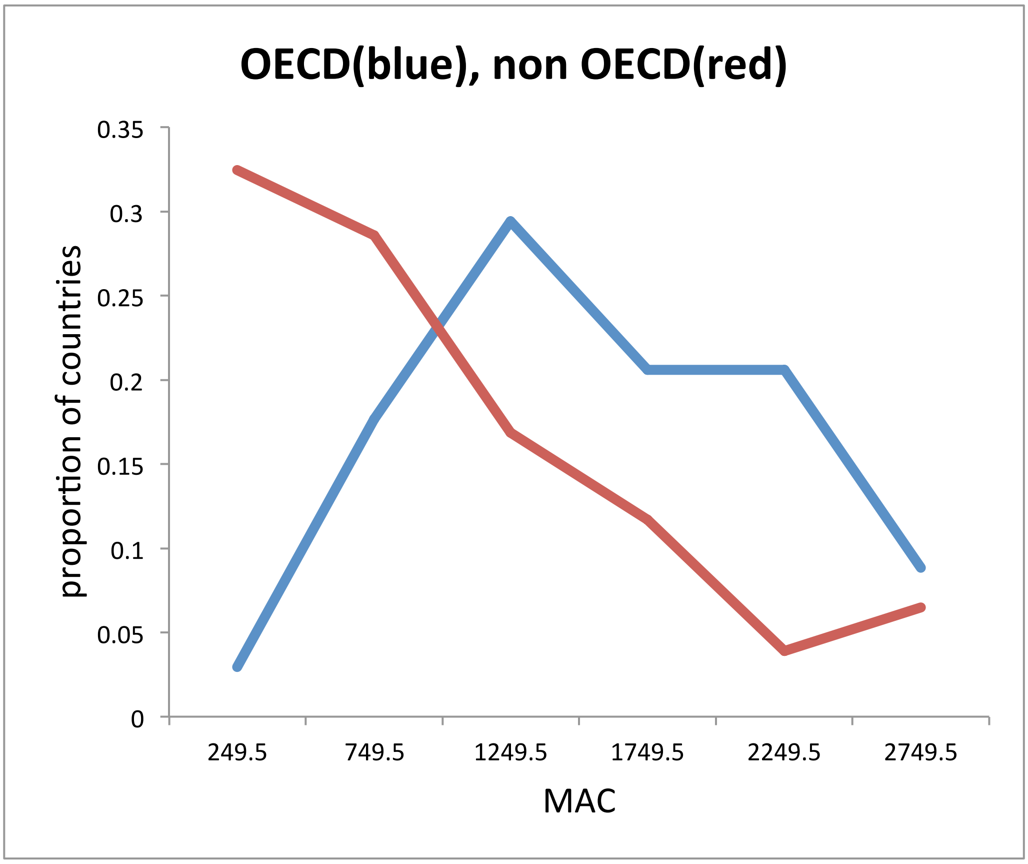

RIP (OECD-blue, Non OECD-red)
proportion of countries
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
2499.5
12499.5
22499.5
32499.5
42499.5
52499.5
62499.5
72499.5
82499.5
92499.5
149999.5
349999.5
549999.5
RIP

(a)

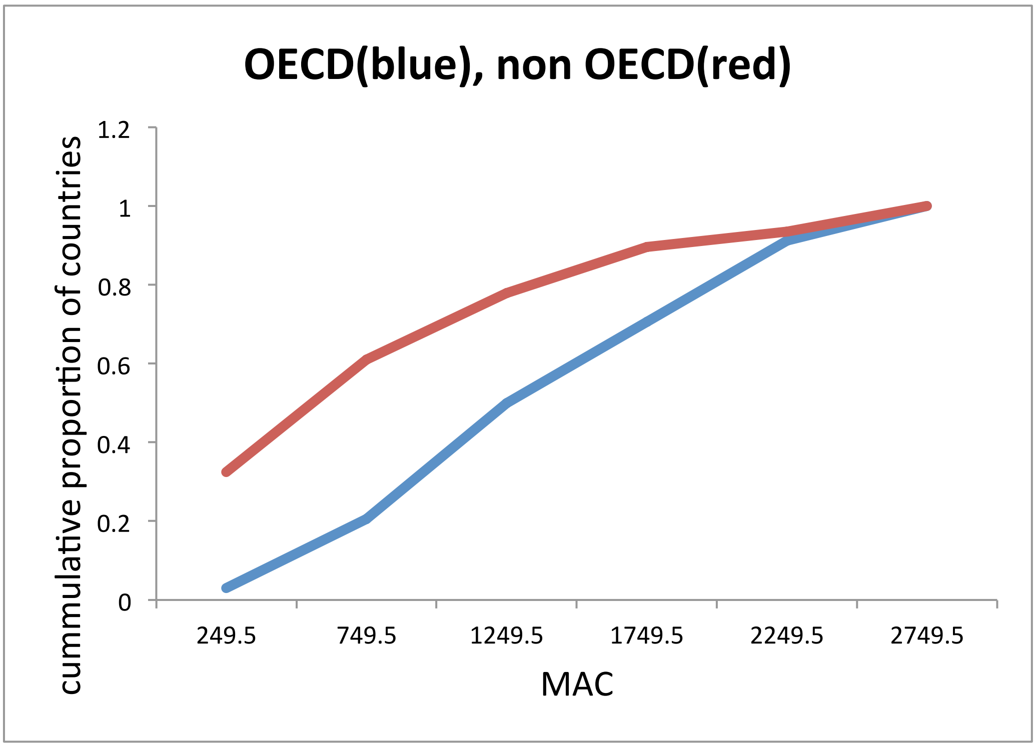

RIP (OECD-blue, Non OECD-red)
cummulative proportion of countries
1.2
1
0.8
0.6
0.4
0.2
0
2499.5
12499.5
22499.5
32499.5
42499.5
52499.5
62499.5
72499.5
82499.5
92499.5
149999.5
349999.5
549999.5
RIP

(b)

Figure 7


OECD(blue), non OECD(red)
proportion of countries
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
249.5
749.5
1249.5
1749.5
2249.5
2749.5
MAC

(a)


OECD(blue), non OECD(red)
cummulative proportion of countries
1.2
1
0.8
0.6
0.4
0.2
0
249.5
749.5
1249.5
1749.5
2249.5
2749.5
MAC

(b)

Figure 8

As derived in (5.20), MAC is the ratio of the RIP and ATA whenever $RIP > 0$ and is zero whenever $RIP \leq 0$.

Table R.17 shows that nine countries in our sample have zero MACs. This is because the RIPs are negative for all these countries. From Theorems 4 and 5 it follows that these are the countries for which the emission constraint in problem (5.12) is non-binding (corresponding to the case in Theorem 4 where $\overset{*}{\tilde{\mu}} = 0$).

For positive values of RIP, the MAC depends on both the RIP and the ATA. Since it is the ratio of RIP and ATA, MAC is higher the higher is RIP and the lower is ATA. However, a low (respectively, high) ATA can result in a low (respectively high) MAC if the RIP is disproportionately low (respectively, high) relative to it. Our sample exhibits all these cases.

For example, all countries in Table R.16 with the highest MACs, have very low AsTA. The RIPs for these countries are all positive and vary considerably in magnitude. However, in each of these countries, the ATA is disproportionately smaller than the RIP resulting in very high MAC.

Table R.18 shows that there is a considerable variation of MACs across countries with the highest AsTA in our sample. China and USA are the two countries with the highest AsTA (more than $1,000$), with China leading the USA. Nevertheless, the MAC of USA is nearly nine times that of China. This is because the RIP for USA is more than five times that of China. China and India have similar RIPs, nevertheless India's MAC is four to five times that of China. This is because India has a lower ATA than China (China's ATA is more than five times that of India). UK, Germany, and India have similar RIPs, but because Germany's ATA is bigger than UK's and much smaller than India's, we find that the MAC of Germany is higher than India's but lower than UK's. RIP for France is bigger than UK's, while its ATA is lower than UK's. Hence it has a higher MAC than UK. Among the countries with the highest AsTA, UK, Italy, and France have the highest MACs (greater than $2,000$). Brazil, Mexico, USA, and Canada have the next highest MACs (between 1000 and 2000). All the other countries have MAC less than 1000, with Russia and South Africa having the lowest MACs, the MAC of Russia being zero.

We now study the differences in the MACs across OECD and non-OECD countries. Excluding the set of seven countries listed in Table R.16 with exceptionally high MACs, Table R.19 shows that the MACs in the OECD countries ranges from 383 to $2,942$, while in the set of non-OECD countries, it ranges ranges from zero to $2,930$. Poland is the country with the lowest MAC in the OECD countries, while there are twenty-four countries in the non-OECD block with MACs less than that of Poland. The mean and median MACs are $1,543$ and 1553, respectively, in the OECD countries, which are higher than those in the non-OECD countries (937 and 772, respectively). Panels (a) and (b) of Figure 8 and Table R.19 reveal that 61% of (*i.e.,* forty-seven) of non-OECD countries as opposed to only 20% of (*i.e.,* seven) OECD countries have MACs less than $1,000$. About 50% of (*i.e.,* seventeen) OECD countries have MACs in the range $1,000$ and $2,000$, while only 29% of (*i.e.,* twenty-two) non-OECD countries have MAC in this range. 29% of (*i.e.,* ten) OECD countries as opposed to 10% of (*i.e.,* eight) non-OECD countries have MACs greater than $2,000$.

Thus, for low MACs (MACs less than $1,000$), the proportion of the non-OECD countries is much higher than that of the OECD countries. While for high MACs (MACs lying between $1,000$ and $2,000$ and MACs higher than $2,000$), the reverse is true.

7.4. Importance of afforestation as an active policy variable.

The effect of afforestation efforts on MACs can be studied by comparing the MACs computed when afforestation is not an active policy of reform at the status-quo with the marginal abatement costs obtained (in the above section) when it is. When afforestation reforms are not undertaken at the status-quo, then it implies that the afforestation effort is held fixed at the status-quo level, *i.e.,* $\dot{a} = 0$. The vector Θ of exogenous environmental variables is extended to include status-quo level of afforestation and is denoted by $\Theta^{(a)} = \langle w, p, k, f, d, a\rangle$. The active policy variables are labour, coal, gas, renewables, and oil, so that an active policy vector is $\nu^{(-a)} = \langle l, c, g, r, o\rangle \in \mathbf{R}^5_+$. Define the vectors

$$
\begin{aligned}
\Psi^{(-a)} &= \left\langle 0, \frac{\alpha_c c_0}{z_0}, \ \frac{\alpha_g g_0}{z_0}, \ 0, \ \frac{\alpha_o o_0}{z_0} \right\rangle \\
q_{\Psi^{(-a)}} &= \left\langle 0, \ q_{c_{\Psi^{(-a)}}}, \ q_{g_{\Psi^{(-a)}}}, \ 0, \ q_{o_{\Psi^{(-a)}}} \right\rangle = \frac{-\Psi^{(-a)}}{\|\Psi^{(-a)}\|} \\
&= \frac{-1}{\sqrt{\alpha_c^2 c_0^2 + \alpha_g^2 g_0^2 + \alpha_o^2 o_0^2}} \left\langle 0, \alpha_c c_0, \ \alpha_g g_0, \ 0, \ \alpha_o o_0 \right\rangle,
\end{aligned} \tag{7.1}
$$

where $\Psi^{(-a)}$ is the vector of elasticities of emission with respect to all active policy variables other than afforestation and $q_\Psi^{(-a)}$ is the direction of proportionate changes in all active policies other than afforestation that results in the greatest possible decline in emission.

Following steps similar to those in Sections 5.2 and 5.3, it is easy to derive the MAC when afforestation is not an active policy variable as

$$
\begin{aligned}
MAC_{\Psi^{(-a)}}\left(\nu_0^{(-a)}, z_0, \Theta_0^{(a)}\right) &= \frac{RIP_{\Psi^{(-a)}}}{ATA_{\Psi^{(-a)}}} \qquad \text{if} \qquad RIP_{\Psi^{(-a)}} > 0 \\
&= 0 \qquad\qquad\quad \text{if} \qquad RIP_{\Psi^{(-a)}} \leq 0,
\end{aligned} \tag{7.2}
$$

where

$$
\begin{aligned}
ATA_{\Psi(-a)} &= -z_0 \Psi^{(-a)} \cdot q_{\Psi(-a)} \\
&= \sqrt{\alpha_c^2 c_0^2 + \alpha_g^2 g_0^2 + \alpha_o^2 o_0^2} \\
RIP_{\Psi(-a)} &= \pi_0 \left[c_0[F_e - p_c](-q_{c_{\Psi(-a)}}) + g_0[F_e - p_g](-q_{g_{\Psi(-a)}}) + o_0[F_o - p_o](-q_{o_{\Psi(-a)}}) \right]
\end{aligned}
\tag{7.3}
$$

are the ATA (the reduction in emission) and the RIP, respectively, evaluated at the status-quo, due to the direction of proportionate changes $q_{\Psi(-a)}$ in all active policy variables other than afforestation.

A comparison of ATA_Ψ derived in (5.24) with $ATA_{\Psi(-a)}$ derived in (7.3) clearly indicates that the ATA will decrease if afforestation is not an available active policy variable at the status-quo, *i.e.,* $ATA_\Psi > ATA_{\Psi(-a)}$.

Moreover, a comparison of (5.27) with $RIP_{\Psi(-a)}$ derived in (7.3) clearly implies that, if afforestation is not an active policy variable, then no cost need to be incurred on additional afforestation and this will tend to reduce the RIP. On the other hand, a comparison of (5.26) with (7.1) implies that the direction of proportionate changes $q_{\Psi(-a)}$ discourages usage of coal, gas, and oil more than the direction of proportionate changes q_Ψ, *i.e.,* proportionate reductions in usage of fossil fuels will be higher when afforestation is not an active policy variable; precisely, $-q_{c_\Psi} < -q_{c_{\Psi(-a)}}$, $-q_{g_\Psi} < -q_{g_{\Psi(-a)}}$, and $-q_{o_\Psi} < -q_{o_{\Psi(-a)}}$. This will tend to increase the RIP. Hence, the relationship between RIP_Ψ and $RIP_{\Psi(-a)}$ is ambiguous, *i.e.,* RIP_Ψ can be greater than, equal to, or less than $RIP_{\Psi(-a)}$.

A comparison of (5.20) and (7.2) combined with the arguments made above imply that, theoretically, the relationship between $MAC_{\Psi(-a)}$ and MAC_Ψ is ambiguous when afforestation levels are positive at the status-quo. When a country does no afforestation, then there is no difference in the MACs in the situations with and without afforestation as an active policy variable at the status-quo.[41]

Fifty-eight out of 118 countries in our sample engage in positive levels of afforestation, *i.e.,* their afforestation levels are greater than 0.0001 mm3. The twenty countries with highest afforestation levels in our sample are listed in Table D.7. Among the OECD countries, Australia, Canada, Mexico, Finland, Estonia, Luxembourg, Japan, and Austria are assumed to have afforestation levels close to zero.[42]

In the set of fifty-eight countries with positive levels of afforestation, Russia, Ukraine, and Uzbekistan continue to have zero marginal abatement costs even when afforestation is not allowed to be an active policy variable.

[41] This is because, when afforestation is an available policy for reform and $a_0 = 0$, then the proportionate change in afforestation implied by $q_{a_\Psi} = \frac{a_0 s}{\|\Psi\|z} = 0$ (see (5.26)), *i.e.,* q_Ψ does not encourage afforestation.

[42] Their actual afforestation levels are zero. As discussed in Section 6, for such countries, we assume a very low and positive afforestation level of 0.0001 mm3.

As theory predicts, the ATA reduces in all the fifty-eight countries with positive afforestation. The top twenty countries in which reductions are the most are listed in Table R.20. The list shows that in three countries, namely, Bhutan, Latvia, and Kyrgyztan, the ATA reduces by more than 50% when afforestation is not allowed to be an active policy instrument. This shows the importance of increasing afforestation relative to decreasing usage of fossil fuels for emission abatement in these countries.

Theory implies that the sign of the change in reduction in profit when afforestation is not an active policy variable is ambiguous. In our sample, the RIPs of all countries with positive afforestation, excluding Russia, Ukraine, and Uzbekistan, increase. Further, comparing Tables R.20 with R.21 reveals that countries (excluding Russia) for which reductions in AsTA are the most are also the countries in which RIPs increase the most.

Since AsTA reduce and RIPs increase for all countries in our sample with positive afforestation levels (except Russia, Ukraine, and Uzbekistan), the MACs are higher in these countries when afforestation is not allowed to be an active policy instrument. Hence, in general, afforestation is a policy that helps in lowering MACs of countries. Countries with highest increases in MACs are listed in Table R.22.

In the case of Russia, Ukraine, and Uzbekistan, though AsTA reduce when afforestation is not allowed to be an active policy variable, the RIPs continue to be negative. Hence, their MACs continue to be zero even when afforestation is not an active policy variable.

8. Conclusions.

Climate change is a public good externality. However, standard price-based or quantity-based solutions (such as a carbon tax or Kyoto-type cap and trade scheme) are not perfectly effective in optimally regulating it because of the sovereign status of the involved economic units (namely, countries), which creates incentives for non-compliance in order to avoid abatement costs. Recently, countries have been advised to submit their Intended Nationally Determined Contributions (INDCs) to the UNFCCC, which will form the basis of new climate negotiations in December 2015. An INDC is a comprehensive package, which outlines a country's voluntary emission targets for future years as well as its proposed plans and actions to achieve these targets. In general, there is broad spectrum of national views and perspectives with regards to control of climate change. While acknowledging the need for controlling climate change, many sovereign nations are cautious, if not reluctant, to commit to undertaking large-scale abatement. This could be because climate change is not considered by them as an imminent threat or because they consider that its control is unaffordable at their existing levels of economic development. On the other hand, there are also nations that treat tackling climate change as both top national and global priorities, are more willing to incur the costs involved, and take pride in acting as global leaders in controlling it and encouraging other countries to do the same. A great diversity can hence be expected in the INDCs that will be submitted by nations.

It is obvious that magnitudes of costs of abatement will play an important role in determination of national emission targets under the INDC scheme. At the same time, international differences in MACs can help also in determining the manner in which these targets can be achieved. This will be especially true in scenarios where countries that are keen to abate have high MACs, while countries that are cautious or reluctant to abate have lower MACs. Our empirical results, based on the policy reform approach, support this argument. Several OECD countries in Europe have submitted intentions to cut emissions by $40-50\%$ by 2020. Our results show that these are also the countries whose current MACs are among the highest in our sample.[43] Several non-OECD countries, which have shown caution in submitting their emission-reduction targets, are countries with low current MACs. USA, whose MAC lies between these two sets, has submitted an emission-reduction target in the range of $26-28\%$.

We show that MAC of a country computed using the policy reform approach is a ratio of its RIP and its ATA. We show that the ATA depends mainly on the fossil-fuel consumption levels, the volume of carbon sequestration activities of the country, the emission factors of fossil fuels it uses, and carbon sequestration factor of its forest stock. For example, intuitively, when fossil fuel consumption levels are very high, then a one-percent decrease in the fossil fuel implies a high absolute decrease in fossil fuel usage and, hence, a big reduction in emission. The RIP depends on the profitability of its inputs. The higher the profitability of inputs such as fossil fuels, the more the reduction in profits when their usage is reduced to cut down emission. ATA varies widely in the non-OECD countries. It is very low for a majority of these countries, due to their low fossil fuel consumption levels. But this is often offset by very low profitabilities of inputs. But there are also high fossil fuel consuming non-OECD countries such as China, India, Russia, Brazil, Iran, Saudi Arabia, and Indonesia, whose AsTA are among the highest in our sample. In many of these countries, the effect of RIP is dominated by their high AsTA leading to low MACs. In fact, there are countries like Russia in the non-OECD group, whose RIPs are negative (so that their MACs are zero), indicating negative profitability of some fossil fuels in these countries. Thus, our results imply that reducing fossil fuel usage actually increases allocative efficiency and increases profits of these countries. The AsTA of many OECD countries in Europe are lower than that of China, India, and Russia. At the same time their RIPs are comparable or higher. Hence, the MACs in these countries turn out to be among the highest in our sample. Countries like USA and Japan have both

[43] Barring seven non-OECD countries in Table R.16.

high AsTA and high RIPs. Their MACs, hence, lie between the high MAC countries in OECD-Europe and non OECD countries such as China, India, and Russia.

The formulae for MAC derived in this paper are useful where long-run abatement plans, based on future projections of the economy, are implemented in a piecemeal (stage-by-stage) manner to minimise uncertainties associated with future projections. At every stage, our formulae, which are based on data summarising the existing states of the economy and its technology, can be employed to compute both the optimal profit increasing and emission-non increasing direction of reform and the short-run MAC. These reflect existing inefficiencies in production and indicate potential gains from international emission trading that exist at that point in time. A country with high MAC at a given stage and with long-run abatement targets can gainfully minimise pressure on its own energy sector by adequately compensating a country having low MAC at the same point in time for abating on its behalf.

The accuracy of the estimates of MACs derived by employing the proposed policy reform approach and their soundness for international comparability depend very much on the extent to which the model employed captures all pertinent factors and the availability of reliable data on these factors that depicts international differences in an accurate manner. In this regards, the model presented in this paper is only a prototype of the types of models required to compute MACs employing the reform approach. Depending on the availability of data, the model can be extended further by incorporating a more bottom-up approach that captures in greater detail the energy-economy links and incorporates a richer set of carbon sequestration and abatement options. There is considerable room for improving the accuracy of the estimates of MACs derived in this paper by extending the model presented along these line and as richer and more detailed data sets from individual countries become available.

APPENDIX

Proof of Theorem 1: Since 0_6 is an emission non-increasing reform with magnitude less that one, it satisfies constraints of problem (4.7). Hence, $\dot{\mathbf{\Pi}}(\nu_0, z_0, \Theta_0, 0) = \nabla_\nu \Pi(\nu_0, z_0, \Theta_0) \cdot \overset{*}{\dot{\nu}} \geq 0 = \nabla_\nu \Pi(\nu_0, z_0, \Theta_0) \cdot 0_6$.

Since $\overset{*}{\dot{\nu}}$ solves problem (4.7) and $\overset{*}{\mu}$ and $\overset{*}{\lambda}$ are the values of the Lagrange multipliers at the optimum of problem (4.7), the vector $\langle \overset{*}{\dot{\nu}}, \overset{*}{\mu}, \overset{*}{\lambda} \rangle$ solves the Kuhn-Tucker first-order conditions (4.14).

(1) In this case, Remark 1 implies that

$$\overset{*}{\dot{\nu}} \cdot \overset{*}{\dot{\nu}} = 1. \tag{A.1}$$

(i) Suppose $\nabla_\nu \mathcal{Z} \cdot \overset{*}{\nu} = 0$.
Premultiplying both sides of the first equation of (4.14) by $\nabla_\nu \mathcal{Z}$, we obtain

$$\nabla_\nu \mathcal{Z} \cdot \nabla_\nu \Pi - \overset{*}{\mu} \nabla_\nu \mathcal{Z} \cdot \nabla_\nu \mathcal{Z} - 2\overset{*}{\lambda} \nabla_\nu \mathcal{Z} \cdot \overset{*}{\nu} = 0$$

Since $\nabla_\nu \mathcal{Z} \cdot \overset{*}{\nu} = 0$, we have

$$\nabla_\nu \mathcal{Z} \cdot \nabla_\nu \Pi - \overset{*}{\mu} \nabla_\nu \mathcal{Z} \cdot \nabla_\nu \mathcal{Z} = 0 \quad \Longrightarrow \quad \overset{*}{\mu} = (\nabla_\nu \mathcal{Z} \cdot \nabla_\nu \mathcal{Z})^{-1} (\nabla_\nu \mathcal{Z} \cdot \nabla_\nu \Pi). \tag{A.2}$$

The first equation in (4.14) also implies

$$\overset{*}{\nu} = \frac{1}{2\overset{*}{\lambda}} \left[\nabla_\nu \Pi - \overset{*}{\mu} \nabla_\nu \mathcal{Z} \right].$$

This, combined with (A.1), gives us

$$\overset{*}{\lambda} = \frac{1}{2} \sqrt{\left[\nabla_\nu \Pi - \overset{*}{\mu} \nabla_\nu \mathcal{Z} \right] \cdot \left[\nabla_\nu \Pi - \overset{*}{\mu} \nabla_\nu \mathcal{Z} \right]} \quad \Longrightarrow \quad \overset{*}{\nu} = \frac{\nabla_\nu \Pi - \overset{*}{\mu} \nabla_\nu \mathcal{Z}}{\sqrt{\left[\nabla_\nu \Pi - \overset{*}{\mu} \nabla_\nu \mathcal{Z} \right] \cdot \left[\nabla_\nu \Pi - \overset{*}{\mu} \nabla_\nu \mathcal{Z} \right]}}. \tag{A.3}$$

(ii) Suppose $\nabla_\nu \mathcal{Z} \cdot \overset{*}{\nu} < 0$.
The second condition in (4.14) immediately implies $\overset{*}{\mu} = 0$. The first condition in (4.14) implies

$$\overset{*}{\nu} = \frac{1}{2\overset{*}{\lambda}} \nabla_\nu \Pi.$$

Combined with (A.1), this implies

$$\overset{*}{\lambda} = \frac{1}{2} \sqrt{\nabla_\nu \Pi \cdot \nabla_\nu \Pi} \quad \Longrightarrow \quad \overset{*}{\nu} = \frac{\nabla_\nu \Pi}{\sqrt{\nabla_\nu \Pi \cdot \nabla_\nu \Pi}}. \tag{A.4}$$

(2) In this case, $\dot{\mathbf{\Pi}}(\nu_0, \dot{z}, \Theta_0, 0) = 0$. Since 0_6 is an emission non-increasing reform with magnitude less than one, it satisfies all constraints of problem (4.7). Hence, since $\nabla_\nu \Pi \cdot 0_6 = 0 = \dot{\mathbf{\Pi}}(\nu_0, \dot{z}, \Theta_0, 0)$, we have $0_6 \in \dot{\nu}(\nu_0, z_0, \Theta_0, 0)$. Since $0_6 \cdot 0_6 < 1$, the third set of conditions in (4.14) imply $\overset{*}{\lambda} = 0$. The first condition of (4.14) hence implies $\nabla_\nu \Pi = \overset{*}{\mu} \nabla_\nu \mathcal{Z}$.
■

Proof of Theorem 2:
(1) $\nabla_\nu \mathcal{Z} \cdot \nabla_\nu \Pi \leq 0 \implies \overset{*}{\mu} = 0$.
(i) Suppose $\nabla_\nu \mathcal{Z} \cdot \nabla_\nu \Pi < 0$. Define $\overset{*}{\delta} := \frac{\nabla_\nu \Pi}{\|\nabla_\nu \Pi\|} \equiv \frac{\nabla_\nu \Pi}{\|\nabla_\nu \Pi\|}$. Then $\overset{*}{\delta} \cdot \overset{*}{\delta} = 1$. Hence $\overset{*}{\delta}$ solves the problem

$$\max_\delta \left\{ \nabla_\nu \Pi \cdot \delta \;\middle|\; \delta \cdot \delta = 1 \right\}. \tag{A.5}$$

The definition of $\overset{*}{\delta}$ implies $\nabla_\nu \mathcal{Z} \cdot \overset{*}{\delta} \equiv \nabla_\nu \mathcal{Z} \cdot \frac{\nabla_\nu \Pi}{\|\nabla_\nu \Pi\|} < 0$. Hence, (A.5) implies that $\overset{*}{\delta}$ also solves

$$\max_{\delta} \left\{ \nabla_\nu \Pi \cdot \delta \;\middle|\; \nabla_\nu \mathcal{Z} \cdot \delta \leq 0 \ \wedge \ \delta \cdot \delta = 1 \right\}.$$

Hence, $\overset{*}{\nu} = \overset{*}{\delta}$. But this implies $\nabla_\nu \mathcal{Z} \cdot \overset{*}{\nu} < 0$. Hence, (ii) of part (1) of Theorem 1 implies that $\overset{*}{\mu} = 0$.

(ii) Suppose $\nabla_\nu \mathcal{Z} \cdot \nabla_\nu \Pi = 0$. Then both (i) and (ii) of part (1) of Theorem 1 imply that $\overset{*}{\mu} = 0$.

(2) $\overset{*}{\mu} = 0 \implies \nabla_\nu \mathcal{Z} \cdot \nabla_\nu \Pi \leq 0$.

Suppose $\nabla_\nu \mathcal{Z} \cdot \nabla_\nu \Pi > 0$. Suppose $\nabla_\nu \mathcal{Z} \cdot \overset{*}{\nu} < 0$. Since we are maintaining the assumption that $\nabla_\nu \Pi \cdot \overset{*}{\nu} > 0$, Remark 1 implies that $\overset{*}{\nu} \cdot \overset{*}{\nu} = 1$. Define a function $\delta : [0,1] \longrightarrow \mathbf{R}^6$ with image $\delta(\lambda) = \lambda \overset{*}{\nu} + (1-\lambda)\frac{\nabla_\nu \Pi}{\|\nabla_\nu \Pi\|}$, where $\lambda \in [0,1]$. Then $\nabla_\nu \mathcal{Z} \cdot \delta(\lambda) = \lambda \nabla_\nu \mathcal{Z} \cdot \overset{*}{\nu} + (1-\lambda) \nabla_\nu \mathcal{Z} \cdot \frac{\nabla_\nu \Pi}{\|\nabla_\nu \Pi\|}$. Since, $\nabla_\nu \mathcal{Z} \cdot \overset{*}{\nu} < 0$ and $\nabla_\nu \mathcal{Z} \cdot \nabla_\nu \Pi > 0$, continuity of δ implies that there exists $\hat{\lambda} \in (0,1)$ such that $\nabla_\nu \mathcal{Z} \cdot \delta(\hat{\lambda}) = 0$. The definition of δ implies that $\nabla_\nu \Pi \cdot \delta(\hat{\lambda}) = \hat{\lambda} \nabla_\nu \Pi \cdot \overset{*}{\nu} + (1-\hat{\lambda}) \nabla_\nu \Pi \cdot \frac{\nabla_\nu \Pi}{\|\nabla_\nu \Pi\|} = \hat{\lambda} \|\nabla_\nu \Pi\| \cos\theta + (1-\hat{\lambda})\|\nabla_\nu \Pi\|$, where θ is the angle between $\nabla_\nu \Pi$ and $\overset{*}{\nu}$. Since, $\nabla_\nu \Pi \cdot \overset{*}{\nu} > 0$, we have $0 < \cos\theta < 1$. Hence, $\|\nabla_\nu \Pi\| \cos\theta < \|\nabla_\nu \Pi\|$. Hence, $\hat{\lambda}\|\nabla_\nu \Pi\| \cos\theta + (1-\hat{\lambda})\|\nabla_\nu \Pi\| > \|\nabla_\nu \Pi\| \cos\theta = \nabla_\nu \Pi \cdot \overset{*}{\nu}$. Hence, $\nabla_\nu \Pi \cdot \delta(\hat{\lambda}) > \nabla_\nu \Pi \cdot \overset{*}{\nu}$. Further, $\|\delta(\hat{\lambda})\| = \sqrt{\left[\hat{\lambda}\overset{*}{\nu} + (1-\hat{\lambda})\frac{\nabla_\nu \Pi}{\|\nabla_\nu \Pi\|}\right] \cdot \left[\hat{\lambda}\overset{*}{\nu} + (1-\hat{\lambda})\frac{\nabla_\nu \Pi}{\|\nabla_\nu \Pi\|}\right]} = \sqrt{\hat{\lambda}^2 + (1-\hat{\lambda})^2 + 2\hat{\lambda}(1-\hat{\lambda})\cos\theta} < \sqrt{\hat{\lambda}^2 + (1-\hat{\lambda})^2 + 2\hat{\lambda}(1-\hat{\lambda})} = 1$. Hence, $\hat{\delta} := \frac{\delta(\hat{\lambda})}{\|\delta(\hat{\lambda})\|} > \delta(\hat{\lambda})$. Hence, $\nabla_\nu \Pi \cdot \hat{\delta} > \nabla_\nu \Pi \cdot \delta(\hat{\lambda}) > \nabla_\nu \Pi \cdot \overset{*}{\nu}$, $\hat{\delta} \cdot \hat{\delta} = 1$, and $\nabla_\nu \mathcal{Z} \cdot \hat{\delta} = 0$. This contradicts the optimality of $\overset{*}{\nu}$ in solving problem (4.7). Hence, $\nabla_\nu \mathcal{Z} \cdot \nabla_\nu \Pi > 0$ implies $\nabla_\nu \mathcal{Z} \cdot \overset{*}{\nu} = 0$. Case (i) of Theorem 1, hence, implies $\overset{*}{\mu} > 0$. This is a contradiction to our maintained hypothesis that $\overset{*}{\mu} = 0$. ■

Proof of Theorem 3: Since $\nabla_\nu \Pi \cdot \overset{*}{\nu} > 0$, Remark 1 implies that $\overset{*}{\nu} \cdot \overset{*}{\nu} = 1$. Hence, (3.11) implies that the directional derivative of the profit function in the direction of reform $\overset{*}{\nu}$ is

$$\lim_{t \longrightarrow 0} \frac{\Pi\left(\nu_0 + \overset{*}{\nu} t, \Theta_0\right) - \Pi\left(\nu_0, \Theta_0\right)}{t} = \nabla_\nu \Pi\left(\nu_0, \Theta_0\right) \cdot \overset{*}{\nu} =: \pi > 0. \tag{A.6}$$

Choose $\epsilon > 0$ such that $N_\epsilon(\pi) \subset \mathbf{R}_{++}$ and a sequence $\{t^n\} \longrightarrow 0$ such that $t^n > 0$ for all all positive integers n. Then (A.6) implies that there exists a positive integer $n' > 0$ such that for all positive integers $n > n'$, we have

$$\frac{\Pi\left(\nu_0 + \overset{*}{\nu} t^n, \Theta_0\right) - \Pi\left(\nu_0, \Theta_0\right)}{t^n} \in N_\epsilon(\pi) \implies \Pi\left(\nu_0 + \overset{*}{\nu} t^n, \Theta_0\right) > \Pi\left(\nu_0, \Theta_0\right).$$

Pick any $\overset{*}{n} > n'$ and define $\overset{*}{t} := t^{\overset{*}{n}}$, $\overset{*}{\delta} = \overset{*}{\nu}\overset{*}{t}$, and $\overset{*}{\nu} := \nu_0 + \overset{*}{\delta}$. Then

$$\Pi\left(\nu_0 + \overset{*}{\delta}, \Theta_0\right) \equiv \Pi\left(\overset{*}{\nu}_0, \Theta_0\right) > \Pi\left(\nu_0, \Theta_0\right).$$

Further, $\nabla_\nu \mathcal{Z}(\nu, z, \Theta) \cdot \overset{*}{\nu} \leq 0$ implies $\nabla_\nu \mathcal{Z}(\nu, z, \Theta) \cdot \overset{*}{\delta} \leq 0$. Hence, we have[44]

$$\begin{aligned} \nabla_\nu \mathcal{Z}(\nu_0, z_0, \Theta_0) \cdot \left[\overset{*}{\nu} - \nu_0\right] \leq 0 \implies & \alpha_c \overset{*}{c} + \alpha_g \overset{*}{g} + \alpha_o \overset{*}{o} - s\overset{*}{a} \leq \alpha_c c_0 + \alpha_g g_0 + \alpha_o o_0 - s a_0 \leq z_0 \\ \implies & \mathcal{Z}(\nu_0 + \overset{*}{\delta}, z_0, \Theta_0) \equiv \mathcal{Z}(\overset{*}{\nu}, z_0, \Theta_0) \leq 0. \blacksquare \end{aligned}$$

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[44] Note $\nu_0 = \langle l_0, c_0, g_0, r_0, o_0, a_0\rangle$ and $\overset{*}{\nu} = \langle \overset{*}{l}, \overset{*}{c}, \overset{*}{g}, \overset{*}{r}, \overset{*}{\delta}, \overset{*}{a}\rangle$

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Table D.1

	Countries with highest carbon emissions	
country	**carbon**	**co2**
China	2030.86905	7446.51986
United States	1535.45869	5630.01518
India	467.653219	1714.72847
Russia	453.138701	1661.50857
Japan	321.975341	1180.57625
Germany	218.553625	801.36329
S. Korea	158.461184	581.02434
Iran	153.900706	564.30259
Canada	147.670748	541.45941
United Kingdom	144.243679	528.89349
South Africa	129.210344	473.77126
Saudi Arabia	127.834797	468.72759
Brazil	122.470587	449.05882

Table D.2

	Countries with lowest carbon emissions	
country	**carbon**	**co2**
Bhutan	0.09538636	0.34975
Cape Verde	0.10343455	0.37926
Gambia	0.13126909	0.48132
Haiti	0.57810818	2.11973
Zambia	0.59820545	2.19342
Mozambique	0.69639	2.55343
Tajikistan	0.72165	2.64605
Namibia	0.85258364	3.12614
Congo Dem Rep	0.86507182	3.17193
Iceland	0.89158364	3.26914

Table D.3

Average carbon sequestration by continents

	world	Africa	Asia	Europe	N. America	S. America	Oceania	
min	0.287	0.342	0.347	0.287	0.385	0.450	*Australia*	0.839
max	1.935	1.935	1.560	0.705	1.104	1.049	*New Zealand*	0.339
mean	0.694	0.994	0.744	0.427	0.716	0.831		
median	0.553	0.874	0.729	0.402	0.758	1.013		
std dev	0.357	0.429	0.341	0.097	0.215	0.268		

Table D.4

Average carbon sequestration by climate zones

	CZ 1	CZ 2	CZ 3	CZ 4	CZ 5	CZ 6	CZ 7
min	0.484	0.374	0.353	0.287	0.342	0.338	0.371
max	0.704	0.704	0.783	1.244	1.432	1.331	1.935
mean	0.594	0.478	0.539	0.450	0.896	0.620	0.905
median	0.594	0.400	0.474	0.409	0.845	0.481	0.841
std dev	0.155	0.130	0.182	0.178	0.286	0.291	0.377

Climate zones

CZ1	Polar
CZ2	Boreal Tundra and Taiga
CZ3	Cold deserts
CZ4	Temperate
CZ5	Warm deserts
CZ6	Sub tropical (Mediterranean)
CZ7	Tropical

Table D.5

Countries with highest carbon sequestration factors (CSF)

country	CSF	CZ
Angola	1.935	7
Gambia	1.753	7
Tanzania	1.632	7
Benin	1.631	7
Sri Lanka	1.560	7
Sudan	1.432	5,7

Table D.6

	Afforestation volume						
	Africa	**Asia**	**Europe**	**N. America**	**S. America**	**Oceania**	
min	0.0001	0.0001	0.0001	0.0001	0.0001	Australia	0.0001
max	0.4000	219.6600	29.0400	609.6000	4.5600	New Zealand	27.4000
mean	0.0229	17.7676	6.5500	47.3238	0.7658		
median	0.0001	0.0200	1.6000	0.0001	0.0001		
std dev	0.0797	54.1283	8.7773	168.9463	1.6994		

Table D.7

	Countries with highest afforestation
country	**afforestation**
United States	609.6
China	219.66
Russia	208.78
India	72
Belarus	29.04
Poland	28
New Zealand	27.4
Sweden	24.8
Ukraine	23
Italy	22.92

Table D.8

	Average costs of energy and labour costs			
country	**coal-fired**	**gas-fired**	**renewable**	**labour**
China	347233.03	420133.75	940358.19	4727.90
Russia	588536.15	671632.50	1034955.32	11264.10
Korea	504334.95	929353.30	943876.26	19542.70
Japan	746646.00	935052.00	1364960.77	33626.70
Europe avera	724095.43	971209.67	1671712.89	23371.43
USA	549866.40	806889.40	1177392.13	47459.70
Brazil	744087.40	975175.50	1021477.44	7396.40
Mexico	593013.70	979943.80	1109918.74	8446.00
Australia	379021.70	714702.27	1083034.06	37502.40
South Africa	374369.70	420133.75	989013.08	9025.60

Table D.9

CZ	Asia	Africa	Europe	Americas	Oceania
Cost of forestation: USD per tonn of carbon					
7	3	7		3	
6	4	4	6	6	
5	29	29		29	29
4	6		7	7	7
3	27		27	27	
2	8		8	8	
1	27		27	27	

Table D.10

	Cost of forestation by continent (USD per mm3)
Africa	14167265.86
Asia	9246360.48
Europe	3353281.91
North Amer	2675001.77
South Amer	3414453.77
Oceania	13355569.23

Table D.11

Countries with highest costs of afforestation

country	CZ	afforestation costs (USD per mm3)
Sudan	5,7	41521018.52
Morocco	6, 5	34548770.05
Namibia	5	32400457.14
Yemen	5	29931480.00
UAE	5	28725225.00
Egypt	5	25246167.50
Botswana	5,7	24641605.26
Australia	5	24341347.61
Ethiopia	5	24059015.15
Jordan	5,6	22807533.33
Pakistan	5, 6	21956343.75
Libya	5	21910225.00
Saudi Arabia	5	21484287.50
Turkmenistan	3	21150000.00
Israel	5	21126500.00
Uzbekistan	3	20035557.69
South Africa	5,6	19873634.33
Algeria	5,6	17814649.12
Iran	5, 6	13960578.36
Angola	7	13545856.31
Gambia	7	12273156.74
Tanzania	7	11423241.71
Benin	7	11414869.57
Iceland	2	10636111.11

Table D.12

coefficients	**estimates**	**p-values**
constant	9.791503	0
capital	0.3722823	0
Electrical energy	0.0819302	0
Oil	0.130042	0
labour	0.2463787	0
total	0.8306332	**implies drs**

Table D.13

	Country specific total factor productivity
min	9.0617487
max	11.323674
mean	10.1856496
median	10.2365058
std dev	0.48802418

Table R.1

ATA-class	World AsTA frequency	proportion	cumm. proportion
0-49	94	0.7966	0.7966
50-99	12	0.1017	0.8983
100-149	6	0.0508	0.9492
150-199	1	0.0085	0.9576
200-249	1	0.0085	0.9661
250-299	0	0.0000	0.9661
300-349	1	0.0085	0.9746
350-	3	0.0254	1.0000
count	118		
min	0.1202		
max	1905.8898		
mean	55.6128		
median	8.6717		
stand dev	205.9294		

Table R.2

Countries with highest ATA

country	ATA
China	1905.89
United States	1087.51
India	380.02
Russia	329.39
Japan	247.74
Germany	153.45
S. Korea	135.84
South Africa	122.94
Brazil	117.95
Iran	117.51
Saudi Arabia	113.81
Canada	112.01
United Kingdo	98.73
Mexico	95.98
France	88.52
Italy	86.54
Australia	83.84
Indonesia	81.83

Table R.3

Countries with highest fossil-fuel usage

country	Avg. fossil fuel (ktoe)
China	735935.94
United States	684429.62
Russia	221857.28
India	177565.62
Japan	150306.88
Germany	98156.89
S. Korea	78685.53
Iran	75882.27
Canada	71405.39
United Kingdo	68049.66
Saudi Arabia	65980.86
Brazil	57283.66
Mexico	56971.62
Italy	56693.37
France	51237.65
Indonesia	50635.76
Australia	47275.66
South Africa	45866.93

Table R.4

Countries with highest to AsTA: composition of fossil-fuels and afforestation

country	coal	gas	oil	s	afforestation
China	1626556.54	99703.15	481548.14	0.4224	219.6600
USA	525412.34	619751.15	908125.37	0.3854	609.6000
Russia	305387.47	59732.77	167576.62	0.3707	72.0000
India	113472.48	400298.76	151800.61	0.3987	208.7800
Japan	121738.02	102836.36	226346.27	0.3585	0.0001
Germany	79570.30	86292.18	128608.18	0.4023	6.8000
S. Korea	75967.26	42902.60	117186.74	0.4436	19.8000
Iran	105509.83	3642.89	28448.07	1.2045	0.0001
Canada	12604.47	23744.97	135501.55	0.4854	0.0001
UK	1432.69	135975.16	90238.96	0.4814	1.8000
Saudi Arabia	0.00	79632.34	118310.25	0.7408	0.0001
Brazil	29022.55	73394.92	111798.70	0.4840	0.0001
Mexico	31523.78	89866.72	82758.49	0.3588	7.8000
Italy	9316.41	59965.29	101633.16	0.7119	0.0001
France	11923.05	46202.26	95587.66	0.4675	14.4000
Indonesia	14132.74	75709.27	80238.10	0.3859	22.9200
Australia	59877.02	30102.43	51847.53	0.8394	0.0001
South Africa	37384.93	38403.53	76118.83	1.1477	0.0001

Country with least fossil-fuel usage

Bhutan (ATA=3)	18.66	0	102.14	0.5177	5.8000

Table R.5

OECD and non-OECD countries excluding high ATA countries

	OECD	non-OECD
ATA		
count	27	76
min	0.95	0.12
max	86.54	53.98
mean	27.54	9.45008816
median	14.45	3.77809247
Avg fossil-fuel usage		
min	399.8056	40.2663713
max	56693.37	35481.3982
mean	15784.2276	5358.9691
median	7203.72948	1979.6932

Table R.6

ATA		**OECD**				**Non-OECD**			
Bin	**Avg bin**	**freq**	**cumm freq**	**proportion**	**cumm prop.**	**freq**	**cumm freq**	**proportion**	**cumm prop**
0-49	24.5	20	20	0.58823529	0.58823529	74	74	0.88095238	0.88095238
50-99	74.5	9	29	0.26470588	0.85294118	3	77	0.03571429	0.91666667
100-149	124.5	2	31	0.05882353	0.91176471	4	81	0.04761905	0.96428571
150-199	174.5	1	32	0.02941176	0.94117647	0	81	0	0.96428571
200-249	224.5	1	33	0.02941176	0.97058824	0	81	0	0.96428571
250-299	274.5	0	33	0	0.97058824	0	81	0	0.96428571
300-349	324.5	0	33	0	0.97058824	1	82	0.01190476	0.97619048
350-	374.5	1	34	0.02941176	1	2	84	0.02380952	1
count		34				**count**	84		
min	0.95218721					**min**	0.12021607		
max	1087.5114					**max**	1905.88977		
mean	78.6699498					**mean**	46.2802022		
median	18.3558194					**median**	4.62620925		
std dev	186.578585					**std dev**	213.615855		

Table R.7

RIP class	**World RIP frequency**	**proportion**	**cumm proportion**
0-4999	54	0.4576	0.4576
5000-9999	14	0.1186	0.5763
10000-14999	6	0.0508	0.6271
15000-19999	6	0.0508	0.6780
20000-24999	6	0.0508	0.7288
25000-29999	3	0.0254	0.7542
30000-34999	4	0.0339	0.7881
35000-39999	6	0.0508	0.8390
40000-44999	0	0.0000	0.8390
45000-49999	1	0.0085	0.8475
50000-54999	1	0.0085	0.8559
55000-59999	0	0.0000	0.8559
60000-64999	2	0.0169	0.8729
65000-69999	1	0.0085	0.8814
70000-74999	1	0.0085	0.8898
75000-79999	0	0.0000	0.8898
80000-84999	0	0.0000	0.8898
85000-89999	1	0.0085	0.8983
90000-94999	0	0.0000	0.8983
95000-99999	0	0.0000	0.8983
100000-199999	5	0.0424	0.9407
200000-299999	5	0.0424	0.9831
300000-399999	0	0.0000	0.9831
400000-499999	1	0.0085	0.9915
500000-	1	0.0085	1.0000
count	118		
min	-20079.29456		
max	1152288.075		
mean	40700.78972		
median	6502.414269		
std dev	121844.3889		

Table R.8

Countries with highest RIP

United State:	1152288.08
Japan	407911.41
Germany	256809.02
France	230312.77
China	227456.39
United Kingd	219375.16
India	212303.93
Italy	188077.06
Brazil	179706.85
Spain	142722.18
Mexico	142682.99
Canada	116133.65

Table R.9

Countries with highest avg profitability

United State:	413835.85
China	233457.32
Japan	138380.46
India	119034.95
Germany	91339.51
United Kingd	80415.31
Italy	63424.03
France	63226.66
Brazil	46674.43
Mexico	44908.00
Spain	43355.10
Canada	34452.77

Table R.10

Countries with lowest RIP and their avg profitability

country	RIP	Avg profitability
Uzbekistan	-20079.29	-3635.95
Trinidad & Tobago	-15747.08	-2558.30
Russia	-10133.09	24000.75
Turkmenistan	-6848.12	-1308.93
Ukraine	-5577.35	5949.80
Kazakhstan	-3380.18	2709.37
Zimbabwe	-723.03	83.91
Bosnia and Herzego	-444.08	517.34
Malta	-7.02	-1.75
Bhutan	24.94	124.93
South Africa	52.08	8410.66

Table R.11

country	profit-coal	qψc	profit-gas	qψg	profit-oil	qψo	profit-affor	qψa
Russia	-25600.31	-0.3933645	-123573.99	-0.7909769	246592.25	-0.394675	-1414.9498	0.2526888
Ukraine	-9059.70	-0.7435692	-17246.75	-0.610124	50160.01	-0.2327353	-54.350986	0.1438326
Kazakhstan	-7757.20	-0.9685511	-2564.34	-0.1314402	21159.01	-0.2112632	-0.0005829	0.0000008
Zimbabwe	-1154.65	-0.9649428	0.00	0	1490.28	-0.2624602	-0.0005769	0.0000366
Bosnia and Herzegovina	-1222.50	-0.9749918	-103.47	-0.023664	3395.35	-0.2209773	-0.0002748	0.0000074
Bhutan	19.12	-0.0070933	0.00	0	491.11	-0.0291162	-10.509778	0.9995509
South Africa	-8566.83	-0.9799779	-462.50	-0.0192861	42671.96	-0.1981699	-0.0019874	0.0000010

Table R.12

country	profit-coal	qψc	profit-gas	qψg	profit-oil	qψo	profit-affor	qψa
United States	154275.15	-0.551667	22685.25	-0.3709105	1480027.73	-0.7151285	-1644.7424	0.2160559
Japan	53668.80	-0.5610913	25960.93	-0.270165	473892.10	-0.7824241	-0.0002868	0.0000001
Germany	37397.52	-0.5920913	2099.65	-0.3660025	325880.01	-0.717741	-19.151775	0.0178295
France	3386.99	-0.1537974	1707.49	-0.3397034	247859.27	-0.9247515	-47.120294	0.0760433
China	31819.27	-0.9745003	-5318.00	-0.0340484	908472.40	-0.2163785	-1144.4054	0.0486857
United Kingdom	20158.60	-0.3645839	35259.99	-0.5924247	266262.26	-0.7178494	-19.592612	0.0283493
India	64306.84	-0.9176003	8223.64	-0.1023034	403689.40	-0.3776394	-80.0656	0.0702290
Italy	10096.34	-0.1864824	32435.62	-0.5694236	211217.23	-0.7940604	-53.075367	0.1022216
Brazil	3505.69	-0.1220235	1117.03	-0.1310284	182074.98	-0.9838403	-0.0001456	0.0000004
Spain	7252.32	-0.1336569	22189.86	-0.3231289	144005.82	-0.9345515	-27.602401	0.0658559
Mexico	7924.29	-0.1108365	27802.46	-0.4066389	143905.24	-0.9068407	-0.0002136	0.0000007
Canada	5931.69	-0.2958537	-3863.58	-0.4264639	135742.96	-0.8547509	-0.0006776	0.0000004

Table R.13	OECD RIP			Non-OECD RIP		
Bin	**frequency**	**proportion**	**cumm prop**	**frequency**	**proportion**	**cumm prop**
0-4999	4	0.118	0.118	50	0.595	0.595
5000-9999	2	0.059	0.176	12	0.143	0.738
10000-14999	2	0.059	0.235	4	0.048	0.786
15000-19999	3	0.088	0.324	3	0.036	0.821
20000-24999	4	0.118	0.441	2	0.024	0.845
25000-29999	1	0.029	0.471	2	0.024	0.869
30000-34999	2	0.059	0.529	2	0.024	0.893
35000-39999	3	0.088	0.618	3	0.036	0.929
40000-44999	0	0.000	0.618	0	0.000	0.929
45000-49999	0	0.000	0.618	1	0.012	0.940
50000-54999	0	0.000	0.618	1	0.012	0.952
55000-59999	0	0.000	0.618	0	0.000	0.952
60000-64999	2	0.059	0.676	0	0.000	0.952
65000-69999	1	0.029	0.706	0	0.000	0.952
70000-74999	0	0.000	0.706	1	0.012	0.964
75000-79999	0	0.000	0.706	0	0.000	0.964
80000-84999	0	0.000	0.706	0	0.000	0.964
85000-89999	1	0.029	0.735	0	0.000	0.964
90000-94999	0	0.000	0.735	0	0.000	0.964
95000-99999	0	0.000	0.735	0	0.000	0.964
100000-199999	4	0.118	0.853	1	0.012	0.976
200000-299999	3	0.088	0.941	2	0.024	1.000
300000-399999	0	0.000	0.941	0	0.000	1.000
400000-499999	1	0.029	0.971	0	0.000	1.000
500000-	1	0.029	1.000	0	0.000	1.000
count	34			**count**	84	
min	1662.164			**min**	-20079.29	
max	1152288			**max**	227456.39	
mean	103970.7			**mean**	15091.538	
median	32137.83			**median**	3172.0971	
stand dev	206994.1			**stand dev**	39857.836	

Table R.14

RIP and avg profitability (excluding high RIP countries)

		OECD	**Non-OECD**
RIP	count	25	79
	min	1662.164	-20079.295
	max	89897.085	47631.468
	mean	27147.669	6624.807
	median	21192.803	3045.214
avg profitability	min	409.785	-3635.948
	max	29105.435	12321.332
	mean	9849.593	2990.927
	median	7170.166	1094.536

Table R.15

MAC class	World MAC frequency	proportion	cumm prop
0-499	26	0.220	0.220
500-999	28	0.237	0.458
1000-1499	23	0.195	0.653
1500-1999	16	0.136	0.788
2000-2499	10	0.085	0.873
2500-2999	8	0.068	0.941
3000-3499	1	0.008	0.949
3500-3999	0	0.000	0.949
4000-4499	1	0.008	0.958
4500-4999	1	0.008	0.966
5000-5499	3	0.025	0.992
5500-5999	0	0.000	0.992
6000-6499	0	0.000	0.992
6500-6999	1	0.008	1.000
count	118		
min	0		
max	6700.24364		
mean	1345.726		
median	1082.47351		
stand dev	1213.29537		

Table R.16

Countries with highest MACs

country	MAC	ATA	RIP
Haiti	3104.71	0.63	1961.98
Nigeria	4361.38	10.92	47631.47
Mozambique	4526.80	0.72	3252.10
Zambia	5078.40	0.65	3306.85
Tanzania	5090.42	1.46	7456.10
Ethiopia	5290.28	1.92	10165.35
Nepal	6700.24	0.95	6393.21

Table R.17

Countries with lowest MACs

country	MAC	ATA	RIP
Uzbekistan	0.00	27.34	-20079.29
Trinidad &Tobago	0.00	14.22	-15747.08
Russia	0.00	329.39	-10133.09
Turkmenistan	0.00	12.74	-6848.12
Ukraine	0.00	53.98	-5577.35
Kazakhstan	0.00	42.75	-3380.18
Zimbabwe	0.00	2.25	-723.03
Bosnia and Herzegovin	0.00	5.72	-444.08
Malta	0.00	2.25	-7.02
South Africa	0.42	122.94	52.08
Bhutan	8.30	3.00	24.94

Table R.18

	MACs in countries with highest ATA		
country	**MAC**	**ATA**	**RIP**
China	119.343938	1905.89	227456.39
United State:	1059.56414	1087.51	1152288.08
India	558.662857	380.02	212303.93
Russia	0	329.39	-10133.09
Japan	1646.50332	247.74	407911.41
Germany	1673.54481	153.45	256809.02
S. Korea	661.778384	135.84	89897.08
South Africa	0.42360351	122.94	52.08
Brazil	1523.60679	117.95	179706.85
Iran	453.011115	117.51	53232.02
Saudi Arabia	180.840466	113.81	20580.87
Canada	1036.78529	112.01	116133.65
United Kingd	2221.95954	98.73	219375.16
Mexico	1486.60533	95.98	142682.99
France	2601.77156	88.52	230312.77
Italy	2173.38479	86.54	188077.06
Australia	719.462737	83.84	60323.19
Indonesia	875.3343	81.83	71630.18

Table R.19

	OECD MACs			Non-OECD excluding very high MAC countries		
Bin	**frequency**	**prop**	**cumm prop**	**frequency**	**prop**	**cumm prop**
0-499	1	0.029	0.029	25	0.325	0.325
500-999	6	0.176	0.206	22	0.286	0.610
1000-1499	10	0.294	0.500	13	0.169	0.779
1500-1999	7	0.206	0.706	9	0.117	0.896
2000-2499	7	0.206	0.912	3	0.039	0.935
2500-2999	3	0.088	1.000	5	0.065	1.000
count	34			**count**	77	
min	383.382449			**min**	0	
max	2942.34346			**max**	2930	
mean	1543.31157			**mean**	937.3	
median	1552.81327			**median**	772.4	
stand dev	649.003591			**stand dev**	808.4	

Table R.20

Countries with greatest decreases in AsTA in the absence of afforestation reforms

country	ATA with afforest	ATA with no afforest
United States	1087.511398	1061.825472
Russia	329.386677	318.697287
New Zealand	11.906682	7.464863
Latvia	5.741546	1.677056
Belarus	18.568529	14.792315
Bhutan	3.004143	0.090028
Sweden	17.564192	14.918426
Kyrgyz Republic	4.044316	1.716165
Norway	12.322722	10.061416
China	1905.889770	1903.629665
Slovenia	4.767473	3.153421
Poland	70.647630	69.592477
Croatia	5.128992	4.106700
India	380.021551	379.083239
Costa Rica	2.981353	2.117813
Bulgaria	10.248519	9.433550
Ukraine	53.982396	53.421089
Cuba	8.169513	7.675373
Turkey	51.814083	51.347866
Italy	86.536475	86.083167

Table R.21

Countries with greatest increases in RIPs in the absence of afforestation reforms

country	RIP with afforest	RIP with no afforest
United States	1152288.075296	1179798.370511
Sweden	37779.846853	44379.246687
Norway	23239.398872	28374.420748
New Zealand	7716.000827	12226.567439
Latvia	1050.936287	3472.114003
Slovenia	3577.014375	5379.499497
Costa Rica	3896.791192	5479.472811
Croatia	5704.807019	7108.827747
Italy	188077.059264	189062.005829
Belarus	3079.080778	3805.500162
Cuba	10480.534788	11152.210046
France	230312.774350	230977.984555
Turkey	67408.511692	68016.806638
Kyrgyz Republic	479.305324	1067.013883
India	212303.925490	212823.786719
Romania	16171.097333	16659.038604
Bhutan	24.939780	481.674514
Poland	27085.061425	27480.837960
Bulgaria	3853.515084	4174.519586
Spain	142722.181604	143030.863498

Table R.22

Countries with highest increases in MACs in the absence of afforestation reforms

country	MAC with afforest	Mac with no afforest
Bhutan	8.301794781	5350.30216
Latvia	183.040648	2070.36277
Costa Rica	1307.054735	2587.32602
New Zealand	648.0395598	1637.88234
Slovenia	750.295737	1705.92511
Norway	1885.898205	2820.1221
Sweden	2150.95846	2974.7942
Gambia	1310.069804	1984.61129
Croatia	1112.266647	1731.03184
Kyrgyz Republic	118.5133291	621.743162
Georgia	892.8902832	1282.5467
Cuba	1282.883734	1452.98607
Uruguay	1168.795255	1289.28023
Belarus	165.8225455	257.261973
Cape Verde	1660.49754	1731.78486
Romania	1141.389917	1211.7429
Bulgaria	376.0070245	442.518404
United States	1059.564136	1111.10385
Slovak Republic	778.4568145	821.305718
Turkey	1300.968918	1324.62773