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## An Experimental Study on the Effect of Ambiguity in a Coordination Game

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# An Experimental Study on the Effect of Ambiguity in a Coordination Game*†

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**Abstract**

We report an experimental test of the influence of ambiguity on behaviour in a coordination game. We study the behaviour of subjects in the presence of ambiguity and attempt to determine whether they prefer to choose an ambiguity safe option. We find that this strategy, which is not played in either Nash equilibrium or iterated dominance equilibrium, is indeed chosen quite frequently. This provides evidence that ambiguity aversion influences behaviour in games. While the behaviour of the Row Player is consistent with randomising between her strategies, the Column Player shows a marked preference for avoiding ambiguity and choosing his ambiguity-safe strategy.

**Keywords**: Ambiguity; Choquet expected utility; coordination game; Ellsberg urn, experimental economics.

**JEL Classification:** **C**72, **C**91, **D**03, D81

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# 1 Introduction

This paper reports an experimental study of the impact of ambiguity in games. There is a relatively large body of experimental literatuture on ambiguity. However few previous papers examine the effect of ambiguity in games. Previous experiments on games have established that ambiguity does affect behaviour in strategic interactions. However, most of them do not test specific theories of the impact of ambiguity (Colman and Pulford (2007), Ivanov (2011)). It is thus difficult to predict what effect ambiguity has, and in which direction it will cause behaviour to change. Our research studies experimentally the comparative statics of ambiguity in games. In particular we test whether the theory of equilibrium under ambiguity (EUA) proposed in Eichberger, Kelsey, and Schipper (2009) performs better than Nash equilibrium.[1]

We find that ambiguity does affect behaviour in games. Moreover ambiguity appears to have a larger impact in games than in single person decisions. Although there is extensive experimental literature which shows that ambiguity affects decision making, most of it studies single-person decisions. There are relatively few experiments on ambiguity which study behaviour in games.[2] A game is a stylized way of representing a situation where a group of individuals have to make a number of linked decisions and thus forms a model of many economic interactions. Games provide a useful intermediate step between single person decisions and economic applications. Since many economic problems can be represented as games we believe this research will be useful for understanding the impact of ambiguity in economics.

Table 1: Battle of Sexes Game

| | | Player 2 | | |
|---|---|---|---|---|
| | | $L$ | $M$ | $R$ |
| Player 1 | $T$ | 0, 0 | 300, 100 | $50, x$ |
| | $B$ | 100, 300 | 0, 0 | $55, x$ |

We consider a Battle of Sexes game which has an added safe strategy, $R$, available for Player

[1] The theory is based on earlier research by Dow and Werlang (1994) and Eichberger and Kelsey (2000).

[2] There are some exceptions to this rule, see for instance Colman and Pulford (2007), Eichberger, Kelsey, and Schipper (2008), Ivanov (2011) or Di Mauro and Castro (2011).

2 (See Table 1). The value of $x$, which is the safe option available to Player 2, varies every round in the range $60 - 260$. For some values of $x$, the safe strategy (in our game, option $R$) is dominated by a mixed strategy of $L$ and $M$, and thus would not be played in a Nash equilibrium. For some higher values of $x$ the game is dominance solvable.

The traditional battle of the sexes game has two pure Nash equilibria, neither of which is focal. Even if a player wishes to use a Nash equilibrium strategy they have to decide which one. Game theory does not provide guidance on how to behave in this situation. Thus we believe that it is possible that subjects will perceive considerable ambiguity. Moreover, all our games were one shot. Feedback was only provided after subjects had made all relevant choices, so no learning could take place. We expected these conditions to lead subjects to perceive the games to be ambiguous.

The effect of ambiguity-aversion is to make $R$ (the ambiguity-safe option) attractive for Player 2. Thus, even if strategy $R$ is not played in Nash equilibrium, it may be chosen in an equilibrium under ambiguity. The ambiguity-safe strategy is never chosen in Nash equilibrium for the parameter values considered by us. Moreover for some values of $x$, our games are dominance solvable and $R$ is not part of the equilibrium strategy. Despite this, we find that $R$ is chosen quite frequently by subjects. While the behaviour of the Row Player, is consistent with expected behaviour of randomising $50 : 50$ between her strategies, the Column Player shows a marked preference for avoiding ambiguity and choosing his ambiguity-safe strategy. Thus, ambiguity influences behaviour in the games. (We use the convention that female pronouns denote the row player and male pronouns denote the column player.)[3]

During the experiment, we alternated the Battle of Sexes games with decision problems based on the 3-ball Ellsberg urn. In these rounds, subjects were presented with an urn containing 90 balls, of which 30 were *Red*, and the remainder an unknown proportion of *Blue* or

[3] Of course this convention is for convenience only and bears no relation to the actual gender of subjects in our experiments.

*Yellow* and asked to pick a colour to bet on. The payoff attached to *Red* was varied in order to obtain an ambiguity threshold. Alternating experiments on urns and games had the dual aim of erasing the short term memory of subjects, and providing an independent measure of subjects' ambiguity-attitudes.

Subjects appeared to perceive a greater level of ambiguity in a two-person coordination game, than in a single person decision problem. This might be because the principle of insufficient reason implies that the probability distribution attached to the Red, Blue and Yellow balls is $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. This can be seen as creating a focal point in the urn experiment. There was no equivalent focal point in the games. Another difference is that in single person decision problems a proxy for ambiguity is introduced by the experimenter, using an artificial device such as the Ellsberg urn. However in games, ambiguity is created by the other subjects taking part in the experiment. Behaviour in the financial market is dependent on other people, and games can be used to effectively model such economic conditions. Natural disasters on the other hand, are more like single person decision problems.

If ambiguity-attitude is a fixed characteristic of the individual we would expect to see a positive correlation between choosing the ball with known probability in the urn experiment and choosing the safe strategy in games. In fact we found no statistically significant correlation, which suggests that ambiguity-attitude varies a lot from one context to another. The differences between the games and the urn experiments suggest that even for a given subject ambiguity-attitude is quite variable depending on the type of decision (s)he is making. A consequence of this is that it will be difficult to test theories by measuring ambiguity-attitude in one context and then proceeding to use the measured ambiguity-attitude to predict behaviour in another context.

**Organisation of the Paper** In Section 2, we describe the theory being tested in the experiments. Section 3 describes the experimental design employed, Section 4 consists of data analysis and results, Section 5 reviews related literature and Section 6 provides a summary of results together with future avenues of research. The appendix contains the proofs of the main results.

# 2 Preferences and Equilibrium under Ambiguity

In this section we explain neo-additive preferences which we use to model ambiguity. The model is then developed into a theory of ambiguity in games.

## 2.1 Modelling Ambiguity

The Ellsberg paradox is a violation of the Subjective Expected Utility (SEU), Savage (1954). One version of the paradox is explained below. Consider an urn filled with 90 balls, 30 of which are red ($R$) and the remaining 60 are of an unknown mix of blue ($B$) and yellow ($Y$). One ball is drawn at random, and the payoff depends on the colour of the ball drawn and the act chosen. Subjects are asked to choose between acts $f$, $g$, $f'$, $g'$ as shown in the table below (Pay-offs in Experimental Currency Units - ECU):

Table 2: The Ellsberg Options

| | 30 balls | 60 balls | |
|---|---|---|---|
| Act | Red $R$ | Blue $B$ | Yellow $Y$ |
| $f$ | 100 | 0 | 0 |
| $g$ | 0 | 100 | 0 |
| $f'$ | 100 | 0 | 100 |
| $g'$ | 0 | 100 | 100 |

Subjects are asked to choose between $f$ and $g$, generally prefer $f$ because of the definite $\frac{1}{3}$ chance of winning 100 ECU to the ambiguous act $g$, but when asked to choose between $f'$ and $g'$, the same subjects prefer $g'$ which gives a $\frac{2}{3}$ chance of winning 100 ECU, again avoiding the ambiguous act $f'$.

These choices cannot be represented as maximising expected utility with respect to a standard subjective probability distribution $\pi$. However these preferences are compatible with non-additive beliefs, introduced by Schmeidler (1989). Such beliefs may be represented by a *capacity* or non-additive set function $\nu$. In this case it is possible that $\nu(R \cup Y) \neq \nu(R) + \nu(Y)$, which could be compatible with the choices in the Ellsberg paradox. Schmeidler (1989) proposed a theory called **Choquet Expected Utility (CEU)**, where outcomes are evaluated by a weighted sum of utilities, but unlike EUT the weights used depend on the acts. The model preserves additivity in beliefs when there is conventional risk, while permitting non-additivity for ambiguous events.

The CEU model also categorises individuals' response to ambiguity. Individuals can be either optimistic or pessimistic in their outlook towards ambiguity. An optimistic outlook would over-estimate the likelihood of a good outcome - inducing one to make risky decisions such as investing in dotcom shares. On the other hand, pessimism would over-estimate the likelihood of a bad outcome - such as losing all your wealth in a bad investment. CEU uses capacities to model optimistic and pessimistic outlooks to ambiguity. A capacity $v$ is convex (resp. concave) if for all $A$ and $B \subseteq S$, $v(A \cup B)$ + $v(A \cap B) \geq v(A)$ + $v(B)$, (resp. $v(A \cup B)$ + $v(A \cap B) \leq v(A)$ + $v(B)$). In CEU, convex (resp. concave) capacities are used to model a pessimistic (resp. optimistic) outlook to ambiguity.

Neo-additive capacities were introduced by Chateauneuf, Eichberger, and Grant (2007). In this model the decision-maker has beliefs based on an additive probability distribution $\pi$. However (s)he lacks confidence in these beliefs hence they are ambiguous beliefs. The ambiguity is represented by the parameter $\delta$. The individual's attitude to ambiguity is represented by the parameter $\alpha$, with higher values of $\alpha$ corresponding to greater ambiguity-aversion.

Consider a two-player game with a finite set of pure strategies $S_i$, such that $s_i$ is the player's own strategy and $s_{-i}$ denotes the set of possible strategy profiles for $i$'s opponents. The payoff function of player $i$ is denoted $u_i(s_i, s_{-i})$. The functional form of preferences may be represented as:

$$V_i\left(s_i; \pi_i, \alpha_i, \delta_i\right) = \delta_i\left(1-\alpha_i\right) M_i\left(s_i\right) + \delta_i \alpha_i m_i\left(s_i\right) + \left(1-\delta_i\right) \int u_i(s_i, s_{-i}) d\pi_i(s_{-i}), \qquad (1)$$

where $M_i\left(s_i\right) = \max_{s_{-i} \in S_{-i}} u_i(s_i, s_{-i})$ and $m_i\left(s_i\right) = \min_{s_{-i} \in S_{-i}} u_i(s_i, s_{-i})$.[4] These preferences maximise a weighted average of the best payoff, the worst payoff and the expected payoff. They are a special case of CEU.

Intuitively, $\pi$ can be thought to be the decision-maker's belief. However, he is not sure of this belief, hence it is an ambiguous belief. His confidence about it is modelled by $(1-\delta_i)$, with $\delta_i = 1$ denoting complete ignorance and $\delta_i = 0$ denoting no ambiguity. His attitude to ambiguity is measured by $\alpha_i$, with $\alpha_i = 1$ denoting pure optimism and $\alpha_i = 0$ denoting pure pessimism. If the decision-maker has $0 < \alpha_i < 1$, he is neither purely optimistic nor purely pessimistic (i.e., ambiguity-averse), but reacts to ambiguity in a partly pessimistic way by putting a greater weight on bad outcomes and in a partly optimistic way by putting a greater weight on good outcomes.

## 2.2 Equilibrium under Ambiguity

In this section we present an equilibrium concept for strategic games with ambiguity. In a Nash equilibrium, players are believed to behave in a manner that is consistent with the actual behaviour of their opponents. They perfectly anticipate the actions of their opponent and can thus provide a best response to it in the form of their own action. However, in the case of ambiguity, represented by non-additive beliefs, the Nash idea of having consistent beliefs regarding the opponent's action and thus being able to play an optimum strategy as a response

[4] Note that Chateauneuf, Eichberger, and Grant (2007) write a neo additive capacity in the form $\mu(E) = \delta\alpha + (1-\delta)\pi(E)$. We have modified their definition to be consistent with the rest of the literature where $\alpha$ is the weight on the minimum expected utility.

to these beliefs, needs to be modified. We assume that players choose pure strategies. In equilibrium, a player's beliefs about the pure strategies of his/her opponent must be best responses for that opponent, given the opponent's beliefs.

Unlike Nash equilibrium where a player can assign an additive probability distribution to his/her opponent's actions, ambiguous beliefs are represented by capacities. The support of a capacity is a player's belief of how the opponent will act. Formally, the support of a neo-additive capacity, $\nu(A) = \delta\alpha + (1-\delta)\pi(A)$, is defined by $\operatorname{supp}(\nu) = \operatorname{supp}(\pi)$. Thus the support of a neo-additive belief is equal to the support of its additive component.[5]

**Definition 2.1 (Equilibrium under Ambiguity)** *A pair of neo-additive capacities* $(\nu_1^*, \nu_2^*)$ *is an Equilibrium Under Ambiguity (EUA) if for* $i = 1, 2$, $\operatorname{supp}(\nu_i^*) \subseteq R_{-i}(\nu_{-i}^*)$, *where* $R_i$ *denotes the best-response correspondence of player* $i$ *given that his/her beliefs are represented by* $\nu_i$ *is defined by*

$$R_i(\nu_i) = R_i(\pi_i, \alpha_i, \delta_i) := \operatorname{argmax}_{s_i \in S_i} V_i(s_i; \pi_i, \alpha_i, \delta_i).$$

This definition of equilibrium is taken from Eichberger, Kelsey, and Schipper (2009), who adapt an earlier definition in Dow and Werlang (1994). These papers show that an EUA will exist for any given ambiguity-attitudes for the players. In games, one can determine $\pi_i$ endogenously as the prediction of the players from the knowledge of the game structure and the preferences of others. In contrast, we treat the degrees of optimism, $\alpha_i$ and ambiguity, $\delta_i$, as exogenous. In equilibrium, each player assigns strictly positive likelihood to his/her opponent's best responses given the opponent's belief. However, each player lacks confidence in his/her likelihood assessment and responds in an optimistic way by over-weighting the best outcome, or in a pessimistic way by over-weighting the worst outcome.

Alternative approaches to equilibrium with ambiguity can be found in Klibanoff (1993) and

[5] This definition is justified in Eichberger and Kelsey (2014).

Lo (1996). They model players as having preferences which satisfy the axioms of maxmin expected utility (MMEU, Gilboa and Schmeidler (1989)). Players are allowed to have beliefs which are represented by sets of conventional probability distributions. As such, players can have mixed strategies that are chosen from these sets of probabilities. They model ambiguity aversion as a strict preference among players to randomise between strategies when they are indifferent to pure strategies.

# 3 Experimental Model

## 3.1 Battle of the Sexes Game

In this section, we explain the games used in our experimental sessions. These are similar to the standard battle of the sexes game, except that they have been modified by giving the column player an additional option which is secure.

### 3.1.1 Nash Equilibrium

As we argue in the introduction, we believe ambiguity will be high due to the multiplicity of equilibria in the basic Battle of the Sexes game. Ambiguity-aversion makes $R$, which is the safe option, attractive for Player 2. When $x = 60$, the secure strategy $R$ is dominated by a mixed strategy and hence is not played in Nash equilibrium or iterated dominance equilibrium.

**Theorem 3.1** *The game has the following Nash equilibria:*

*(a) When* $0 \leq x \leq 75$, *there are 3 equilibria:* $(T, M), (B, L)$ *and a mixed strategy equilibrium* $(\frac{3}{4} \cdot T + \frac{1}{4} \cdot B, \frac{3}{4} \cdot L + \frac{1}{4} \cdot M)$;[6]

*(b) When* $75 < x \leq 100$, *there are 3 equilibria:* $(T, M), (B, L)$ *and* $(\frac{x}{100} \cdot T + \frac{100-x}{100} \cdot B, \frac{1}{61} \cdot M + \frac{60}{61} \cdot R)$;

[6]The notation $\frac{3}{4} \cdot T + \frac{1}{4} \cdot B$ denotes the mixed strategy where $T$ is played with probability $\frac{3}{4}$ and $B$ is played with probability $\frac{1}{4}$.

(c) *When* $100 < x < 300$, *there is a unique equilibrium:* $(B, L)$.

### 3.1.2 Ambiguity Aversion

Ambiguity about the behaviour of Player 1 would make the secure option $R$ more attractive for Player 2. Note that the best response to $R$, is for Player 1 to play $B$. Hence of the two possible Nash equilibria $(T, M)$ and $(B, L)$, the latter may be more robust to ambiguity.[7]

We assume that the beliefs of the players may be represented by neo-additive capacities and that players are ambiguity-averse, i.e., $\alpha = 0$.

**Theorem 3.2** *The game has the following Equilibria under Ambiguity:*

(a) *when* $0 \leq x \leq (1-\delta)75$, *there are 3 equilibria,* $(T, M)$, $(B, L)$ *and* $(\frac{3}{4} \circ T + \frac{1}{4} \circ B, \frac{3}{4} \circ L + \frac{1}{4} \circ M)$; [8]

(b) *when* $(1-\delta)75 < x \leq (1-\delta)100$, *there are 3 equilibria:* $(T, M)$, $(B, L)$ *and* $(\frac{x}{(1-\delta)100} \circ T + \frac{(1-\delta)100 - x}{(1-\delta)100} \circ B, \frac{1}{61} \circ M + \frac{60}{61} \circ R)$;

(c) *when* $(1-\delta)100 < x < (1-\delta)300$, *there is a unique equilibrium:* $(B, L)$;

(d) *when* $x > (1-\delta)300$, *there is a unique equilibrium:* $(B, R)$.

In the above analysis, players are presumed to be uniformly ambiguity averse. Suppose $\delta = \frac{1}{2}$, which is in line with the findings of Kilka and Weber (2001). Then $(a)$ occurs for $0 \leqslant x \leqslant 37.5$, $(b)$ occurs for $37.5 \leqslant x \leqslant 50$, $(c)$ occurs for $50 \leqslant x \leqslant 150$ and $(d)$ occurs for $150 \leqslant x$.

The testable hypothesis that arises from the analysis, is that while Nash equilibrium predicts that $R$ cannot be chosen in the range $37.5 < x < 50$ or $150 < x < 300$, EUA predicts $R$ can be chosen in these ranges.

[7]Theorem 2 confirms that $(B, L)$ is an equilibrium for a greater parameter range than $(T, M)$.

[8]The notation $\frac{3}{4} \circ L + \frac{1}{4} \circ M$ implies that Player 1's belief is a neo-additive capacity based on the additive probability distribution which assigns probability $\frac{3}{4}$ (resp. $\frac{1}{4}$) to $L$ (resp. $M$). The interpretation is that player 1 believes that 2 will play the pure strategy $L$ with subjective probability $\frac{3}{4}$. However 1 is unsure of this belief, (i.e. perceives it to be ambiguous).

## 3.2 Ellsberg Urn Experiments

The Battle of Sexes game was alternated with single person decision problems similar to the Ellsberg Urn. Subjects were presented with an urn containing 90 balls, of which 30 were Red, and the remainder an unknown proportion of Blue or Yellow. Subjects were asked to pick a colour, and a ball was drawn from the urn. If the colour of the ball matched the colour chosen by the subject, it entitled the subject to a prize. The decisions put to the subjects took the following form:

"An urn contains 90 balls, of which 30 are Red. The remainder are either Blue or Yellow.

Which of the following options do you prefer?

a) Payoff of $y$ if a Red ball is drawn.

b) Payoff of 100 if a Blue ball is drawn.

c) Payoff of 100 if a Yellow ball is drawn."

Payoff "$y$" attached to the option *Red* was changed from round to round, with $y = 95$, 90 or 80, to measure the ambiguity threshold of subjects. In addition, we also put before subjects the classic case of Ellsberg Paradox, when $y = 100$, as described in Table 3.

# 4 Experimental Design

The Battle of Sexes game and Ellsberg Urn problem described above were used in two series of paper-based experiments, one conducted at St. Stephen's College in New Delhi, India, and the other at the Finance and Economics Experimental Laboratory in Exeter (FEELE), UK.

Sessions 1 and 2 consisted of 20 subjects each. Sessions 3 and 4 consisted of 18 and 22 subjects respectively. In total there were 80 subjects who took part in the experiment, 38 of which were females and 42 were males. We were also interested in whether or not participants had a mathematical background - of those taking part in the sessions, 45 studied a quantitative subject such as Biochemistry, Electronic Engineering or Astrophysics, while 35 studied a nonquantitative subject such as History, Philosophy, or International Relations. Each session lasted a maximum of 45 minutes.

Subjects were allowed to read through a short but comprehensive set of instructions at their own pace[9], following which the instructions were also read out to all the participants in general. The subjects were then asked to fill out some practice questions to test their understanding of the games, before the actual set of experimental questions were handed out. At the start of the experiment, subjects were randomly assigned the role of either a Row Player or a Column Player for the purpose of the Battle of Sexes game, and remained in the same role throughout the rest of the experiment.

The experiment consisted of 11 rounds, starting with a decision regarding a Battle of Sexes game, which was then alternated with an Ellsberg Urn decision, such that there were in total 6 Battle of Sexes rounds and 5 Ellsberg urn decisions to be made. Each subject had to choose one option per round: Top/Bottom if they were a Row Player or Left/Middle/Right if they were a Column Player, and in case of the Ellsberg urn rounds Red, Blue or Yellow.

The values of $x$, the ambiguity-safe payoff available to the Column Player that were used for the Battle of Sexes game rounds were: $230, 120$, 200, 170, 260, 60 (in that order). In the first three Ellsberg urn rounds, the pay-offs attached to drawing a Blue or Yellow ball were held constant at 100, while those attached to drawing a Red ball varied as 95, 90, 80. The last two Ellsberg urn rounds consisted of the classic case of the Ellsberg paradox, where subjects had to choose between a payoff of 100 for a Red or 100 for a Blue ball, followed by a choice between a payoff of 100 for drawing a Red/Yellow ball or 100 for drawing a Blue/Yellow ball.

Once subjects had made all 11 decisions, a throw of dice determined *one* Battle of Sexes round and *one* Ellsberg urn round for which payments were to be made. Row Players' decisions were matched against the Column Players' decisions according to a random and anonymous

[9]Experimental protocols available from the corresponding author on request.

matching, and pay-offs were announced.[10]

Rather than using a real urn we simulated the draw from the urn on a computer.[11] The computer randomly assigned the number of blue and yellow balls in the urn so that they summed to 60, while keeping the number of red balls fixed at 30 and the total number of balls in the urn at 90. It then simulated an independent ball draw for up to 30 subjects. If the colour of the ball drawn by the computer matched that chosen by the subject, it entitled him to the payoff specified in the round chosen for payment.

The total earnings of a subject was the sum of a show-up fee, payoff earned in the chosen Battle of Sexes round and payoff earned in the chosen Ellsberg urn round. Average payment made to Indian subjects was $Rs.420$ (£6 approximately), and to Exeter subjects was £7.40.

# 5 Data Analysis and Results

## 5.1 Behaviour of the Row Player in the Games

In the Battle of Sexes rounds of the experiment, the task of the Row Player was to choose between $T$ and $B$. In the mixed equilibrium, the Row Player randomises $\frac{3}{4} : \frac{1}{4}$ between $T$ and $B$. However we find that the subjects who played the role of Row Players in our experiments, individually and on aggregate, randomise more closely to $50 : 50$. See Figure 1, for a summary of the Row Player's behaviour.

We conducted a binomial test with the null that the Row Player randomises $50 : 50$ between $T$ and $B$, for each value of $x$. We fail to reject this hypothesis for each individual session even at a 10% level of significance. When tested for each value of $x$ on the whole (as a sum of all sessions combined), we fail to reject the null for all the values of $x$, except when $x = 200$, where

[10] For the purpose of the Indian experiments, $1ECU = Rs.1$, while for the Exeter experiments, $100ECU = £2$. In addition, a show-up fee of $Rs.250$ was paid to the Indian subjects and £5 to the Exeter subjects.

[11] The computer simulated urn can be found at the following link: http://people.exeter.ac.uk/dk210/Ellsberg-110708.xls.

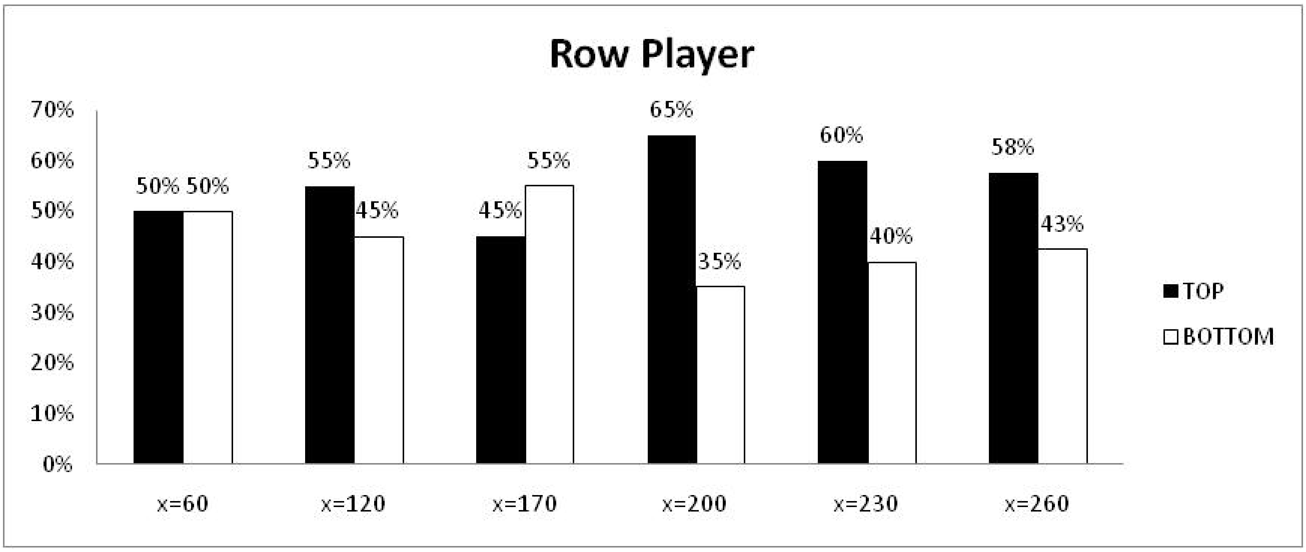


Figure 1: Row Player Behaviour

we reject the null at 5%.[12]

We conducted a chi-squared test with the null hypothesis that the Row Player chooses $T$ and $B$ with equal probability ($H_0 : prob(T) = 0.5$, $prob(B) = 0.5$) versus the alternative that this is not true ($H_1 : prob(T) \neq 0.5$). Again, we fail to reject this hypothesis even at the 10% level.

## 5.2 Behaviour of the Column Player

In the Battle of Sexes rounds of the experiment, the task of the Column Player was to choose between $L$, $M$ and the ambiguity-safe option $R$. See Figure 2, for a summary of the Column Player's behaviour.

When $x = 60$ one might expect the Column Player to pick $L$, since $L$ has a much higher maximum pay-off than $M$. As seen in Figure 2 most subjects do indeed choose $L$. However, even at this low value of $x$, where the ambiguity-safe option $R$ is dominated by randomisation between the other strategies, a significant 30% of subjects still choose it.

What is more interesting to note however, is that the number of subjects playing $R$, steadily

[12] In this case, the Row Player plays $T$ significantly more often than $B$. This is puzzling, since $B$ would be the best response to the Column Player choosing $R$.

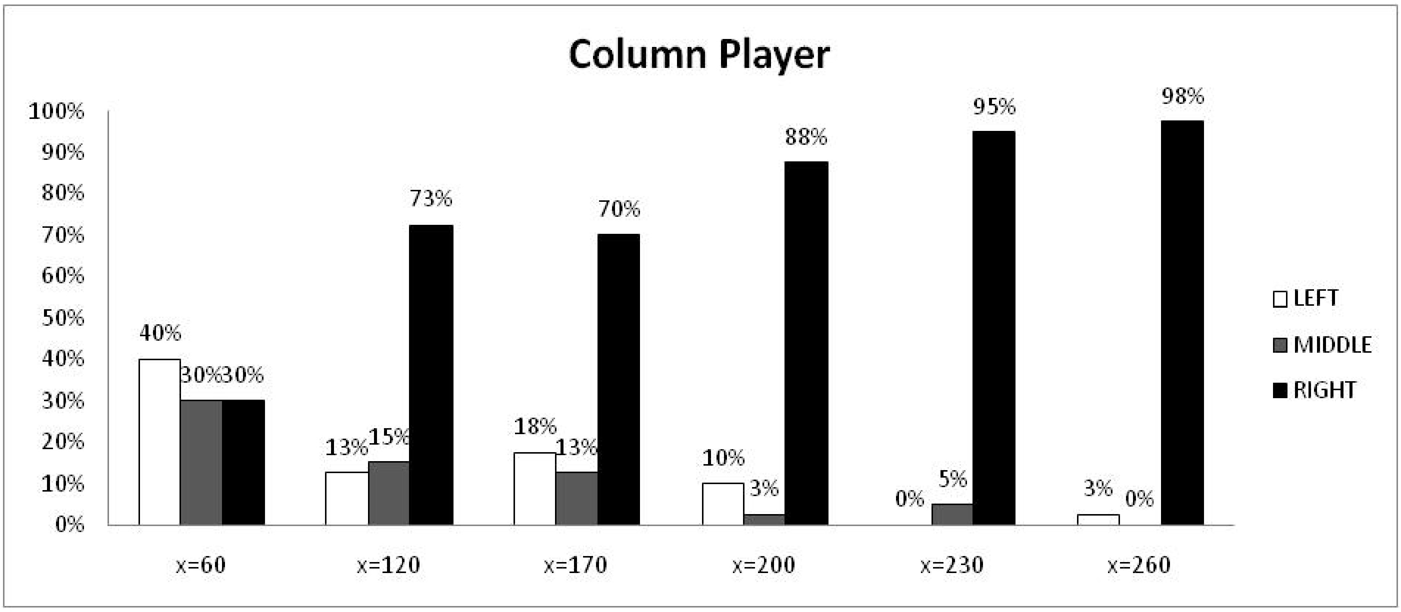


Figure 2: Column Player Behavior

increases from 73% to 98% for $120 \leq x \leq 260$. Nash equilibrium predicts that $R$ cannot be chosen for any of these values, but it is the clear choice of a majority of subjects in the presence of ambiguity, as seen in Figure 2.

We conducted a binomial test with the null that the Column Player chooses $R$ as often as he does $L+M$ ($H_0 : prob(Right) = 0.5$, $prob(Left + Middle) = 0.5$), against the alternative that he plays $R$ more often than both $L+M$ combined ($H_1 : prob(Right) > prob(Left + Middle)$), for each value of $x$.[13] We reject the null at a 1% level of significance for all the values of $x$ in the range $120 - 260$. This leads us to conclude that subjects play $R$ significantly more often than both $L$ and $M$ combined, at a 1% level of significance.

A chi-squared test with the null hypothesis that the Column Player chooses $R$ and $L + M$ with equal probability ($H_0 : prob(Right) = 0.5$, $prob(Left + Middle) = 0.5$) versus the alternative that this is not true ($H_1 : prob(Right) \neq 0.5$) is also rejected at the 1% level of significance, since $R$ is chosen significantly more often.

We ran a probit regression to ascertain what factors influenced subjects in choosing $R$ more often than $L$ or $M$. Dummy variables were defined to capture the characteristics of the data

[13] The binomial test was conducted for each value of $x$ except $x = 60$, where EUA predicts that the column player can play $L$. It may be noted that for $x = 60$, subjects play $L + M$ more than 50% of the time.

such as: $Math = 1$, if the subject was doing a quantitative degree ($Math = 0$, for degrees like English, History, Philosophy, Politics etc.); $Male = 1$, if gender is male (0, otherwise); $Delhi = 1$, if the session was run in India (0 for Exeter); $x\_60$, $x\_120$, $x\_170$, $x\_200$, $x\_230$, $x\_260 = 1$, depending on the value "$x$" took in that particular round.

A probit regression of *Right* on $Math$, $Male$, and the various $x - value$ dummies $x\_120$, $x\_170$, $x\_200$, $x\_230$, $x\_260$, has a chi-square ratio of 75.55 with a p-value of 0.0001, which shows that our model as a whole is statistically significant.[14]

All the variables in the probit regression were individually statistically significant. We see that if the subject had a quantitative degree, the z-score increases by 0.538, making him more likely to pick $R$. If the subject is male, the z-score decreases by 0.402, hence males are less likely to opt for the ambiguity-safe option $R$ than females. When $x = 120$: the z-score increases by 1.16, $x = 170$: the z-score increases by 1.08, $x = 200$: the z-score increases by 1.75, $x = 230$: the z-score increases by 2.27, $x = 260$ : the z-score increases by 2.57; more than the base which is $x = 60$. Thus, as the value of $x$ increases, the subject is more likely to pick the ambiguity-safe option.

## 5.3 Player Behaviour in the Ellsberg Urn Rounds

The Ellsberg Urn rounds were alternated with the Battles of Sexes rounds. This was designed to test whether there was a correlation between ambiguity-averse behaviour in the game and ambiguity attitude in single person decision problems.[15]

As can be seen in Figure 3, subjects chose *Blue* and *Yellow* coloured balls (the ambiguous option) more often than they chose *Red*.[16] We had expected to observe that subjects who chose

[14] An initial probit regression, showed that the dummy variable for location (*Delhi*/*Exeter*) was not significant, showing that behaviour of Indian subjects was very similar to the Exeter subjects. Thus, the location dummy variable was dropped and the model was re-run without it.

[15] We would like to thank Peter Dursch, whose suggestions helped the design of the experiment.

[16] The data for $y = 100$ is from the classic Ellsberg paradox round. It is not completely comparable as subjects were not given the option of choosing yellow. Thus it is not included in the data analysis below.

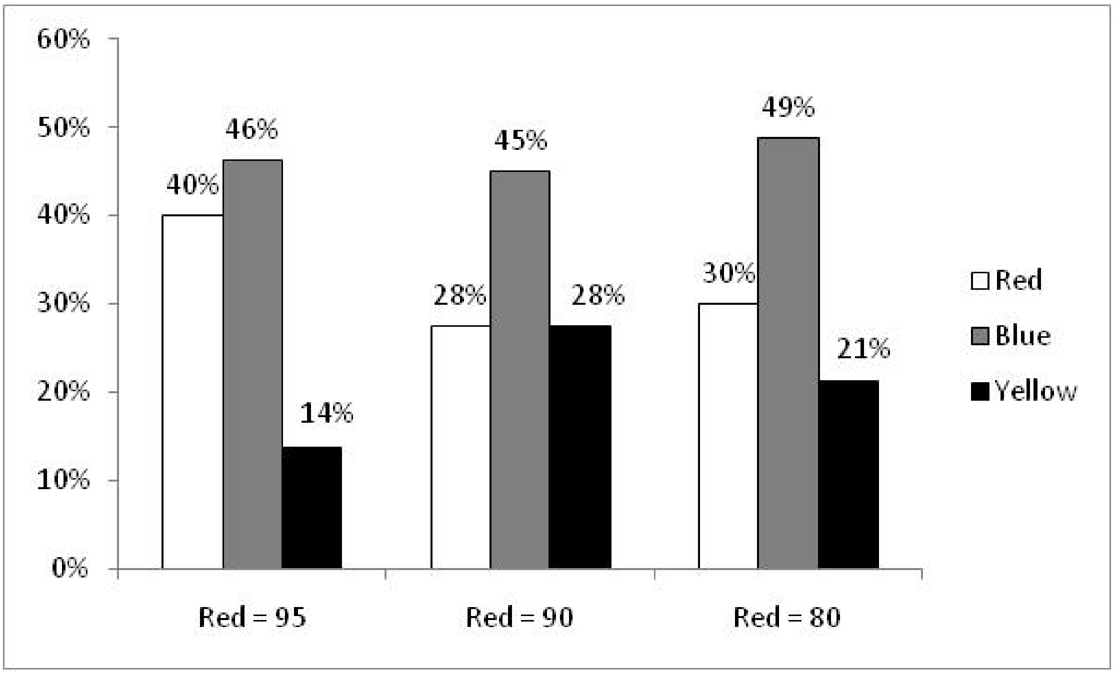


Figure 3: Subject Behaviour in Ellsberg Urn Rounds

*Right* (the ambiguity-safe option) in the Battle of Sexes rounds, would choose *Red* (the colour with the unambiguous number of balls) in the Urn rounds. However, the observed correlation was weak.

One notable feature of this data is the low level of ambiguity-aversion compared to previous studies. In the case where $y = 100$ our results are comparable to the previous literature. For lower values of $y$, subjects have to pay a monetary penalty to avoid ambiguity. Even small penalties produced a large drop in the number of subjects choosing the unambiguous option. Of the 80 subjects that took part in the experiment, only 12 subjects always chose *Red*, 11 chose *Red* twice, 20 chose *Red* once, and a significant 37 subjects never chose *Red* - always opting for either *Blue* or *Yellow*, the ambiguous options.

It is interesting to note that even in the round where the payoff attached to *Red* was 80 *ECU*, a large minority (30%) still chose the unambiguous option, despite facing a substantial monetary penalty. Of the 12 subjects who always picked *Red*, 3 are Row Players and so not relevant to our discussion. The remaining 9 are Column Players: 7 of these always chose the ambiguity-safe combination of *Right*$-$*Red* (not considering their choice when $x = 60$), while 2 chose *Left/Middle/Right* while always picking Red.

We conducted a binomial test with the null that *Red* was chosen as often as *Blue+Yellow* combined ($H_0$ : $prob(Red) = 0.5$, $prob(Blue + Yellow) = 0.5$), against the alternative that *Blue+Yellow* was chosen more often ($H_1$ : $prob(Blue + Yellow) > prob(Red)$). We reject the null at a 5% level of significance when the payoff attached to *Red* = 95, and at 1% level of significance when *Red* = 90 & 80. Looking at subject choices on the whole, over the three rounds, we can reject the null at a 1% level of significance.

Thus, the ambiguous options *Blue* and *Yellow* are chosen significantly more often than *Red*, which leads us to speculate whether the penalty for choosing *Red* was set too high or whether subjects are mildly ambiguity-seeking in the Ellsberg urn rounds, even though they appear to be ambiguity-averse in the Battle of Sexes rounds. A probit regression run to investigate whether gender, location or degree subject affected subjects' choice of *Blue* and *Yellow* was inconclusive and none of these potential explanatory variables were found to be significant.

## 5.4 Classic Ellsberg Paradox Rounds

In the last two urn rounds, the classic Ellsberg Paradox was put before the subjects. As can be seen from Table 3, a majority of them preferred *Red* to *Blue*, followed by the choice *Blue/Yellow* (rather than *Red/Yellow*). 38 of the 80 (48%) subjects chose *Red* followed by *Blue/Yellow*, thus displaying the Classic Ellsberg Paradox. The opposite preference was expressed by 7 (9%) of the subjects. They subjects chose *Blue* followed by *Red/Yellow*, which indicates ambiguity preference. Looking strictly at the Column Players who display the Ellsberg Paradox[17]: 16 (67%) subjects always chose the ambiguity-safe option *Right* − but these people do choose *Blue/Yellow* when the payoff attached to *Red* = 95, 90, 80, while 8 (33%) play a mixture of *Left/Middle/Right.*

[17] We do not consider $x = 60$, where $R$ is a dominated strategy.

Table 3: Player Behaviour in Classic Ellsberg Paradox Rounds

| Choice | Response | |
|---|---|---|
| Red followed by Red/Yellow | 19 | 24% |
| Red followed by Blue/Yellow | 38 | 48% |
| Blue followed by Blue/Yellow | 16 | 20% |
| Blue followed by Red/Yellow | 7 | 9% |

# 6 Related Literature

## 6.1 Papers on Games

One previous experimental study of the effect of ambiguity in games is Di Mauro and Castro (2011). They test a result from Eichberger and Kelsey (2002), concerning the effect of ambiguity on the voluntary provision of public goods. Eichberger and Kelsey (2002) show that when the production function for public goods is concave, ambiguity-aversion causes public good provision to be above the Nash equilibrium level. More generally they show that the deviation from the Nash equilibrium depends on the nature of strategic interactions taking place, i.e., on the basis of whether the game being played was one of strategic substitutes or complements.

Voluntary provision of public goods may be considered as an example of a game of strategic substitutes with positive externalities. If one individual contributes more, this lowers the marginal product of other people's donations. Ambiguity-aversion causes a given individual to overweigh bad outcomes. In this case, a bad outcome is when others make low donations. When others' donations are low the marginal benefit of a donation by the given individual is high. Thus the expected effect of ambiguity would be to increase the perceived marginal benefit of donations by a given individual. If all have similar perceptions of ambiguity, total donations will rise. Hence an increase in ambiguity is expected to raise both individual and total donations.

Di Mauro and Castro (2011) conduct a set of experiments designed to test this hypothesis that ambiguity rather than altruism causes an increase in contribution towards the public good. In order to negate the chance that altruism, or a feeling of reciprocation prompted the subjects' actions, they were informed that their opponent would be a virtual agent and the opponent's play was simulated by a computer. Subjects played in two scenarios, one with risk, the other with ambiguity. Contributions were significantly higher when the situation was one of ambiguity. These results are similar to our findings and showed that there was indeed evidence that ambiguity significantly affects the decisions made by individuals, in a manner that depends directly on the strategic nature of the game in consideration.

Another paper that tests whether ambiguity affects behaviour in games is Eichberger, Kelsey, and Schipper (2008). They studied games in which subjects faced either a granny, who was described as being ignorant of economic strategy, a game theorist, who was described as a successful professor of economics, or another student as an opponent. It was conjectured that subjects would view the granny as a more ambiguous opponent than the game theorist. This was confirmed by the data and ambiguity affected decision choices in the predicted way. In our paper, even though subjects are paired against one another (and not a granny), we find subjects display similar ambiguity averse behaviour.

Colman and Pulford (2007) explain the concept of ambiguity aversion as a state that arises as a result of a pessimistic response to uncertainty. This is mainly driven by a loss of decision confidence. They argue that people tend to become anxious and less confident while making decisions in the presence of ambiguity. They found that individual responses differed between ambiguous and risky versions of the game being studied. Players did not respond to ambiguity by simply equating it to riskiness, but showed a marked preference to avoid ambiguity whenever the option of doing so was provided to them. This is consistent with our findings that when an ambiguity-safe option is made available to subjects, they show a marked preference for it.

## 6.2 Papers on Ellsberg Urns

One of the earliest Ellsberg experiments was conducted by Becker and Brownson (1964). Ambiguity was implemented as the number of red balls in an urn. They found that about 50% of subjects were ambiguity-averse, on the basis of the two-colour Ellsberg problem. When given the choice between an ambiguous urn and an unambiguous one, subjects chose the unambiguous urn and were willing to pay an *ambiguity premium* of about 60% to avoid ambiguity. For instance, when given the choice between an ambiguous urn that contained between $1 - 100$ red balls and an unambiguous urn that contained exactly 50 red balls, subjects were willing to pay an average of 72% of the expected value to avoid ambiguity. When asked to choose between an ambiguous urn that contained between $40 - 60$ red balls and an unambiguous urn that contained exactly 50 red balls, subjects were willing to pay 28% of the expected value to avoid ambiguity. These results appear different to the behaviour of our subjects. In our experiments, we found that subjects were unwilling to bear even the smallest penalty in order to avoid ambiguity and choose *Red*.

Another Ellsberg experiment where subjects were found to be willing to pay a premium to avoid ambiguity was conducted by Yates and Zukowski (1976). They compared a known urn which contained five red and blue chips each, a uniform urn which contained red chips in a uniform distribution from $0 - 10$, and an ambiguous urn which contained red and blue chips in an unknown proportion. Subjects were asked to choose between urns and state minimum selling prices for bets on urns. The ambiguous urn was least preferred among the urns and priced the lowest. Moreover, subjects were willing to pay on average an ambiguity premium of 20% of the expected value of a bet, in order to bet on the known urn instead of the uniform one. As noted earlier, our subjects were unwilling to pay an ambiguity premium, and instead chose to bet on *Blue/Yellow* balls.

Bernasconi and Loomes (1992), study a two-stage lottery version of the three-colour Ellsberg problem, where drawing a red ball ($p(R) = 1/3$) was the unambiguous event, while drawing blue and yellow balls were ambiguous separately, but unambiguous together, ($p(B \cup Y) = 2/3$). Ambiguity aversion was displayed by half the subjects who chose to bet on red for £10. In addition, 90% of subjects who chose a £10 bet on red, refused to switch to a £12 bet on an ambiguous colour - thereby implying an ambiguity premium of about 20%. In our experiment, as the penalty on *Red* increased, we found more and more subjects switched to betting on *Blue/Yellow* i.e., the ambiguous balls.

An Ellsberg experiment that allows for an additional source of ambiguity is studied by Eichberger, Oechssler, and Schnedler (2011). They consider a two-colour Ellsberg experiment and insert an additional element of ambiguity in terms of the money the subject wins in the various outcomes. In the standard treatment, if the colour drawn matches the colour chosen by the subject, he receives an envelope marked with an equal sign (=) which contains €3, and if it does not match he receives an envelope with an unequal sign ($\neq$) which contains €1. This is the standard Ellsberg treatment and is referred to as $O$, or open envelope.

In the second treatment called the $S$ or sealed envelope treatment, subjects know that one envelope contains €3 and one contains €1, but do not know which envelope contains which amount. Subjects are not sure how much they would win, even if they won, and should thus, be indifferent between the ambiguous urn and the known one.[18] In the third treatment called $R$ or the random treatment, subjects are told that the amount in the envelope will be determined by the toss of a fair coin, once they have made their choice of colour for the bet on the urn. Subjects thus face equal odds of winning either amount.

In the standard Ellsberg treatment $O$, 30 of the 48 (62%) subjects preferred the known urn. This is similar to the subject behaviour in our experiment, where in the standard Ellsberg

[18] In the experiments, subjects were asked to choose an urn and the colour of the ball they would like to bet on. In addition, they could state that they were indifferent between the known urn and the unknown one, as well as being indifferent between a green ball and a blue one. In case of indifference, subjects were assigned to the unknown urn/blue ball options.

round 57 of the 80 (71%) subjects preferred *Red* to *Blue.* Moreover, significantly fewer subjects preferred the known urn to the ambiguous one in treatment $S$ (where there was additional ambiguity) when compared to treatment $O$, the standard Ellsberg case. This is analogous to our finding that fewer subjects preferred *Red* to the ambiguous choices in rounds where there was a penalty imposed on choosing the known ball.

Eliaz and Ortoleva (2011), study a three-colour Ellsberg urn in which they have increased the level of ambiguity. Subjects face ambiguity on two accounts: the unknown proportion of balls in the urn as well as the size of the prize money. In their experiment, both winning and the amount that the subject could possibly win were perfectly correlated - either positively or negatively, depending on which of the two treatments was run by them. In the experiment, most subjects preferred betting in the positively correlated treatment rather than the negative one. Moreover, subjects also showed a preference for a gamble when there was positively correlated ambiguity, as opposed to a gamble without any ambiguity. This behaviour of the subjects is compatible with our results, where we find that more subjects were willing to gamble on $Blue/Yellow$ which were the ambiguous choices rather than on *Red*.

## 6.3 Papers comparing Risk and Ambiguity Attitude

In our paper, we investigate the ambiguity attitude prevalent in subjects. Subjects who found our game "risky" would play Nash equilibria. However, on the whole, we find an absence of subjects playing Nash and hence we prefer to concentrate on equilibrium solutions in the presence of ambiguity.

Existing papers that consider the relation between risk and ambiguity aversion find mixed results. Cohen, Jaffray, and Said (1985) and Curley, Yates, and Abrams (1986) fail to find any correlation between the two. This is consistent with our beliefs, that our results are due to ambiguity and not due to risk aversion. Potamites and Zhang (2007) on the other hand find a positive but weak correlation between the two dimensions. These papers use a joint valuation task to measure risk and ambiguity attitude rather than using independent tasks. Displaying a single task that measures both risk and ambiguity attitude, causes a bias in the estimate of correlation.

Consider for instance, an individual who is averse to both risk and ambiguity. When given a choice between a risky option and an ambiguous one, the individual would find the risky alternative quite attractive. However, this is *only* in the presence of the more unattractive ambiguous option. The observed degree of risk aversion of the said individual would thus appear low, thereby reducing the correlation estimate between risk and ambiguity aversion. This phenomenon has also been documented by Fox and Tversky (1995).

Lauriola and Levin (2001) and Lauriola, Levin, and Hart (2007), find a significant positive correlation between risk and ambiguity. This is mainly due to subjects who have extreme attitudes towards risk and ambiguity. Charness and Gneezy (2010) use an investment experiment to provide further evidence of a positive relation. In their study, subject could invest $10 (or a fraction thereof) in either a risky prospect or an ambiguous one. Subjects made individual decisions without interacting with each other. They found that subjects who chose the ambiguous prospect invested significantly more than subjects investing in the risky prospect. In our paper, we find that subjects prefer the ambiguity safe option and are often willing to suffer a penalty in order to pick the secure option.[19]

## 6.4 Preference for Randomisation

As already noted, many of the subjects in our games choose the safe option even when it was dominated by a mixed strategy. This type of behaviour should not be observed in Nash equilibrium. This can be related to the debate on preference for randomisation in the context of single person decisions. Raiffa (1961) argued that in a situation, such as the classic Ellsberg

[19] When $x = 60$, subjects choose the secure option at a penalty.

paradox discussed earlier, ambiguity could be eliminated by betting on either the blue or yellow ball with objective probability one half for each. In other words ambiguity would create a strict preference for randomisation. However Eichberger and Kelsey (1996) showed that ambiguity-aversion implies indifference to randomisation in the convex capacity model, Schmeidler (1989). Experimental research has also not found evidence of a strict preference for randomisation. In particular Dominiak and Schnedler (2011) and Eichberger, Oechssler, and Schnedler (2011) found *aversion* to randomisation in the context of single person decisions.[20]

In games, the analogous question is whether a strategy which is dominated by a mixed strategy will be played. Choosing the mixed strategy is the equivalent of displaying a preference for randomisation. On the other hand choosing a dominated strategy which gives a certain pay-off is analogous to displaying aversion to randomisation. As Figure 2 indicates 30% of column players choose $R$ even when it is dominated by a mixed strategy. Thus we see a similar pattern to the experiments on single person decisions with evidence that many subjects display aversion or indifference to randomisation.

# 7 Conclusions

The Nash equilibrium prediction that $R$ cannot be chosen for $150 < x < 300$, was not observed in our experiments. The ambiguity-safe option $R$, which is selected by EUA, was the choice of a majority of subjects when $120 < x < 260$. There was also a significant minority of subjects choosing $R$ when $x = 60$. Thus, there is sufficient indication for us to conclude that ambiguity does indeed affect play in the coordination game.

One surprising feature of our results was that the links between choices in the single person decision and those in the games was not strong. Subjects appeared to perceive a greater level of ambiguity in a two-person coordination game, than a single person decision problem. More

[20] Similar results were found in unpublished research by Mafiioletti.

generally our results suggest that perceptions of ambiguity and even attitudes to ambiguity depend on context. Hence it may not be possible to measure ambiguity-attitude in one context and use it to predict behaviour in another.

It is interesting to note that there is a growing consensus that subjects find more ambiguity regarding real events as opposed to actual or simulated Ellsberg urns. It was found that when Ellsberg-type problems were put to students in a class-environment, a large proportion of PhD-level students were ambiguity-neutral, while a large proportion of MBA-level students displayed ambiguity-seeking behaviour.[21] However, when asked whether they preferred a payoff of \$100 if the US President elected in 2016 was a Democrat (or not a Democrat) or if a fair coin came up heads when tossed on the day of the election, a large proportion of the students preferred betting on the coin.

One of the reasons put forth to explain this divergence in behaviour is that it is easy to be Bayesian in an Ellsberg experiment or that the phrasing of the Ellsberg problem might lead to it being treated as a gamble. However, when asked to make a decision regarding a realistic scenario such as predicting the next President of the US, the students have no "natural" prior. A realistic scenario then is better at revealing ambiguity aversion on the part of the subject.

Parallels can be drawn between this discussion and the data we observe from our experiment, whereby subjects clearly display ambiguity-averse behaviour when put in the scenario of the coordination game while they fail to do so in the Ellsberg urn rounds. Subjects might be treating the Ellsberg urn rounds as a gamble, where they readily take a chance. However, when faced with the task of coordinating with another participant in the environment of a one-shot game with no previous learning, the subjects have no natural prior on the basis of which to make their decisions. The Column Player thus selects the strategy that gives a definite payoff of $x$ irrespective of the Row Player's decision.

[21] These observations are as recorded by Itzhak Gilboa, in a discussion on observed ambiguity in Ellsberg experiments.

One can note that our results support the Dow and Werlang (1994) model of equilibrium under ambiguity, where in the presence of ambiguity players choose their safe strategy, rather than the model of Lo (1996). Lo's equilibrium predictions coincide with those of Nash equilibrium for games with only pure equilibria. Thus for many of our game experiments Lo's predictions coincide with Nash equilibrium. Hence for these experiments EUA appears to predict the implications of ambiguity better.

# 8 Appendix

**Theorem 3.1** *The game has the following Nash equilibria:*

1. *When* $0 \leq x \leq 75$, *there are 3 equilibria:* $(T, M), (B, L)$ *and a mixed strategy equilibrium* $(\frac{3}{4} \cdot T + \frac{1}{4} \cdot B, \frac{3}{4} \cdot L + \frac{1}{4} \cdot M)$;

2. *When* $75 < x \leq 100$, *there are 3 equilibria:* $(T, M), (B, L)$ *and* $(\frac{x}{100} \cdot T + \frac{100-x}{100} \cdot B, \frac{1}{61} \cdot M + \frac{60}{61} \cdot R)$;

3. *When* $100 < x < 300$, *there is a unique equilibrium:* $(B, L)$.

**Proof of Theorem 3.1**

**Part (1)** $0 \leq x < 75$ **:** By inspection $(T, M)$ and $(B, L)$ are pure strategy Nash equilibria. For $x$ in this range, $R$ is dominated by $\frac{3}{4} \cdot L + \frac{1}{4} \cdot M$, which yields an expected pay-off of 75 no matter what player 1 chooses. Thus $R$ cannot be played in Nash equilibrium. Player 1 is indifferent between $T$ and $B$ when $300(1-p) = 100p$, $p = 3/4$. There are 3 equilibria: $(T, M)$, $(B, L)$ and $(\frac{3}{4} \cdot T + \frac{1}{4} \cdot B, \frac{3}{4} \cdot L + \frac{1}{4} \cdot M)$.

**Part (2)** $75 < x \leq 100$ : For $x$ in this range, $(T, M)$ and $(B, L)$ remain pure strategy Nash equilibria. Player 2 is indifferent between $M$ and $R$ when: $100q = x$ or $q = x/100$. Player 1 is indifferent between $T$ and $B$ when: $300p + 50(1-p) = 55(1-p)$, or $p = 1/61$. There are

3 equilibria: $(T, M)$, $(B, L)$ and $(\frac{x}{100} \cdot T + \frac{100-x}{100} \cdot B, \frac{1}{61} \cdot M + \frac{60}{61} \cdot R)$.[22]

**Part (3)** $100 < x < 300$ : For this range, $M$ is dominated for Player 2 by $R$. Once $M$ is eliminated, Player 1 will never play $T$, which is now a dominated strategy. He thus plays $B$. The best response for Player 2 is to play $L$. In this case there is a unique Nash equilibrium: $(B, L)$, which is also an iterated dominance equilibrium. □

**Theorem 3.2** *The game has the following Equilibria under Ambiguity:*

1. *when* $0 \leq x \leq (1-\delta)75$, *there are 3 equilibria,* $(T, M)$, $(B, L)$ *and* $(\frac{3}{4} \circ T + \frac{1}{4} \circ B, \frac{3}{4} \circ L + \frac{1}{4} \circ M)$*;*

2. *when* $(1-\delta)75 < x \leq (1-\delta)100$, *there are 3 equilibria:* $(T, M)$, $(B, L)$ *and* $(\frac{x}{(1-\delta)100} T + \frac{(1-\delta)100 - x}{(1-\delta)100} B, \frac{1}{61} \circ M + \frac{60}{61} \circ R)$*;*

3. *when* $(1-\delta)100 < x < (1-\delta)300$, *there is a unique equilibrium:* $(B, L)$;

4. *when* $x > (1-\delta)300$, *there is a unique equilibrium:* $(B, R)$.

**Proof.**

**Part 1.** $0 \leqslant x \leqslant (1-\delta)75$ : In this range there are two EUA in pure strategies and one in mixed strategies. In the pure equilibria, the supports of the equilibrium beliefs are given by $(T, M)$ and $(B, L)$. Consider the first of these. Define $\nu^1$ by $\nu^1 = (1-\delta)\,\pi_M(A)$, where $\pi_M$ is the additive probability on $S^2$ defined by $\pi_M(A) = 1$ if $M \in A$, $\pi_M(A) = 0$ otherwise. Similarly define Player 2's beliefs $\nu^2$ by $\nu^2 = (1-\delta)\,\pi_T(A)$. By definition $\operatorname{supp}\nu^1 = M$ and $\operatorname{supp}\nu^2 = T$. Denote this equilibrium by $\langle T, M\rangle$. By similar reasoning we may show that there exists a pure equilibrium where $\operatorname{supp}\nu^1 = L$ and $\operatorname{supp}\nu^2 = B$, which we denote by $\langle B, L\rangle$.

Now consider the mixed equilibria. Denote the equilibrium beliefs of Players 1 and 2 respectively by $\tilde{\nu}^1 = (1-\delta)\,\tilde{\pi}^1$ and $\tilde{\nu}^2 = (1-\delta)\,\tilde{\pi}^2$. Player 2's Choquet expected pay-offs

[22] Consider what would happen if Player 2 mixes between $L$ and $R$. Player 2 will be indifferent between $L$ and $R$ when: $300(1-q) = x$ or $q = \frac{300-x}{300}$. Player 1 is indifferent between $T$ and $B$ when: $100p + 55(1-p) = 50(1-p)$, or $p = -\frac{5}{95}$. It is impossible for a probability to be negative, hence there can be no such equilibria.

are given by, $V^2(L) = 300(1-\delta)\tilde{\pi}^2(B)$, $V^2(M) = 100(1-\delta)\tilde{\pi}^2(T)$ and $V^2(R) = x$. If $V^2(L) < x \leqslant (1-\delta)75$ then $\tilde{\pi}^2(B) < \frac{1}{4}$, which implies $\tilde{\pi}^2(T) > \frac{3}{4}$. Hence $V^2(M) = 100(1-\delta)\tilde{\pi}^2(T) > (1-\delta)75 \geqslant x$. Thus $R$ cannot be a best response for Player 2, hence $\tilde{\pi}^1(R) = 0$. Consequently in any mixed equilibrium 2's strategies are $L$ and $M$.

In a mixed equilibrium Player 2 must be indifferent between $L$ and $M$, hence,

$$\begin{aligned} V^2(L) &= V^2(M) \Leftrightarrow 300(1-\delta)\tilde{\pi}^2(B) = 100(1-\delta)\tilde{\pi}^2(T) \\ &\Leftrightarrow \tilde{\pi}^2(T) = \frac{3}{4}. \end{aligned}$$

In this equilibrium $V^2(L) = V^2(M) = 75(1-\delta)$. Similarly we may show that for Player 1 to be indifferent between $T$ and $B$, we must have $\tilde{\pi}^1(L) = \frac{3}{4}$ and $\tilde{\pi}^1(M) = \frac{1}{4}$.

Thus in the mixed equilibrium $\tilde{\nu}^1 = (1-\delta)\tilde{\pi}^1$ with $\tilde{\pi}^1(L) = \frac{3}{4}$ and $\tilde{\pi}^1(M) = \frac{1}{4}$ and $\operatorname{supp}\tilde{\nu}^1 = \{L, M\}$ while $\tilde{\nu}^2 = (1-\delta)\tilde{\pi}^2$ with $\tilde{\pi}^2(T) = \frac{3}{4}$ and $\tilde{\pi}^2(B) = \frac{1}{4}$, with support $\{T, B\}$. In this equilibrium $V^2(L) = V^2(M) = 75(1-\delta)$. We shall denote this equilibrium by $\left\langle \frac{3}{4} \circ T + \frac{1}{4} \circ B, \frac{1}{4} \circ L + \frac{3}{4} \circ M)\right\rangle$.

**Part 2.** $(1-\delta)75 < x < (1-\delta)100$ : In this range, there are two EUA in pure strategies: $(T, M)$ and $(B, L)$. The reasoning is similar to that used in Part (3.2) above.

In addition, there is a mixed strategy equilibrium. Denote the equilibrium beliefs of Players 1 and 2 respectively by $\tilde{\nu}^1 = (1-\delta)\tilde{\pi}^1$ and $\tilde{\nu}^2 = (1-\delta)\tilde{\pi}^2$. Player 2's Choquet expected payoffs are given by, $V^2(L) = 300(1-\delta)\tilde{\pi}^2(B)$, $V^2(M) = 100(1-\delta)\tilde{\pi}^2(T)$ and $V^2(R) = x$. Thus $L$ cannot be a best response for Player 2, hence $\tilde{\pi}^1(L) = 0$.[23] Consequently in any mixed

[23] Consider what would happen if Player 2 mixes between $L$ and $R$. For Player 2 to be indifferent between $L$ and $R$ :

$$V^2(L) = V^2(R) \Leftrightarrow 300(1-\delta)\tilde{\pi}^2(B) = x \Leftrightarrow \tilde{\pi}^2(B) = \frac{x}{300(1-\delta)}.$$

Player 1 is then indifferent between playing $T$ and $B$ when,

$$\begin{aligned} V^1(T) &= V^1(B) \Leftrightarrow 50(1-\delta)\tilde{\pi}^1(R) = 100(1-\delta)\tilde{\pi}^1(L) + 55(1-\delta)\tilde{\pi}^1(R) \\ &\Leftrightarrow 100\tilde{\pi}^1(L) = -5(1-\tilde{\pi}^1(L) \Leftrightarrow \tilde{\pi}^1(L) = -\frac{5}{95}. \end{aligned}$$

It is impossible for a belief to be negative, hence there can be no such equilibria.

equilibrium 2's strategies are $M$ and $R$.

Player 2 is indifferent between $M$ and $R$ when:

$$\begin{aligned} V^2(M) &= V^2(R) \Leftrightarrow 100(1-\delta)\tilde{\pi}^2(T) = x \\ &\Leftrightarrow \tilde{\pi}^2(T) = \frac{x}{(1-\delta)100}. \end{aligned}$$

Similarly, Player 1's Choquet expected payoff is given by: $V^1(T) = 300(1-\delta)\tilde{\pi}^1(M) + 50(1-\delta)\tilde{\pi}^1(R)$ and $V^1(B) = 55(1-\delta)\tilde{\pi}^1(R)$. Player 1 is indifferent between $T$ and $B$ when:

$$\begin{aligned} V^1(T) &= V^1(B) \\ &\Leftrightarrow 300(1-\delta)\tilde{\pi}^1(M) + 50(1-\delta)\tilde{\pi}^1(R) = 55(1-\delta)\tilde{\pi}^1(R) \\ &\Leftrightarrow \tilde{\pi}^1(M) = \frac{1}{61}. \end{aligned}$$

Thus in the mixed equilibrium $\tilde{\nu}^1 = (1-\delta)\tilde{\pi}^1$, with $\tilde{\pi}^1(M) = \frac{1}{61}$ and $\tilde{\pi}^1(R) = \frac{60}{61}$ and $\operatorname{supp}\tilde{\nu}^1 = \{M, R\}$, while $\tilde{\nu}^2 = (1-\delta)\tilde{\pi}^2$ with $\tilde{\pi}^2(T) = \frac{x}{(1-\delta)100}$ and $\tilde{\pi}^2(B) = \frac{((1-\delta)100)\,-\,x}{(1-\delta)100}$, with support $\{T, B\}$. In this equilibrium $V^2(M) = V^2(R) = x$. The mixed strategy equilibrium is $\left\langle \frac{x}{(1-\delta)100}T + \frac{((1-\delta)100)\,-\,x}{(1-\delta)100}B, \frac{1}{61}M + \frac{60}{61}R \right\rangle$.

**Part 3.** $(1-\delta)100 < x < (1-\delta)300:$ Denote the equilibrium beliefs of Players 1 and 2 respectively by $\tilde{\nu}^1 = (1-\delta)\tilde{\pi}^1$ and $\tilde{\nu}^2 = (1-\delta)\tilde{\pi}^2$. Player 2's Choquet expected pay-offs are given by, $V^2(L) = 300(1-\delta)\tilde{\pi}^2(B)$, $V^2(M) = 100(1-\delta)\tilde{\pi}^2(T)$ and $V^2(R) = x$, where $(1-\delta)100 < x < (1-\delta)300$.

For $x$ in this range, $V^2(R) > V^2(M)$ for any beliefs of Player 2, hence $\tilde{\pi}^1(M) = 0$. Player 1's Choquet expected pay-offs are given by, $V^1(T) = 50(1-\delta)\tilde{\pi}^1(R)$ and $V^1(B) = 100(1-\delta)\tilde{\pi}^1(L) + 55(1-\delta)\tilde{\pi}^1(R)$. Strategy $B$ yields a higher Choquet expected payoff than $T$ for any beliefs of Player 1, with support contained in $\{L, R\}$. For Player 2, $L$ is the best response to $B$. In this case there is a unique EUA: $\langle B, L\rangle$.

**Part 4.** $x > (1-\delta)300$ : Denote the equilibrium beliefs of Players 1 and 2 respectively by $\tilde{\nu}^1 = (1-\delta)\,\tilde{\pi}^1$ and $\tilde{\nu}^2 = (1-\delta)\,\tilde{\pi}^2$. Player 2's Choquet expected pay-offs are given by, $V^2(L) = 300(1-\delta)\,\tilde{\pi}^2(B)$, $V^2(M) = 100(1-\delta)\,\tilde{\pi}^2(T)$ and $V^2(R) = x$, where $x > (1-\delta)300$.

For $x$ in this range, $R$ strictly dominates both $L$ and $M$ for any beliefs of Player 2, hence $\tilde{\pi}^1(L) = \tilde{\pi}^1(M) = 0$. Player 1's best response is to play $B$, with $\operatorname{supp}\nu^1 = R$. There is a unique EUA: $\langle B, R\rangle$. $\square$

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