Economics Department Discussion Papers Series

ISSN 1473 – 3307

Necessary and sufficient conditions for an environmental Kuznets curve with some illustrative examples

Sushama Murty

Paper number 14/07

URL: http://business-school.exeter.ac.uk/economics/papers/
URL Repec page: http://ideas.repec.org/s/exe/wpaper.html

Necessary and sufficient conditions for an environmental Kuznets curve with some illustrative examples.

Sushama Murty*

April 2014

*Department of Economics, University of Exeter Business School, Streatham Court, Rennes Drive, University of Exeter, Exeter EX4 4PU, Devon, UK.

Tel: + 44 (0) 1392 726265 Fax: + 44 (0) 1392 263242 email: s.murty@exeter.ac.uk

ABSTRACT

Necessary and sufficient conditions for an environmental Kuznets curve with some illustrative examples.

We propose a set of necessary and sufficient conditions for the environmental Kuznets curve (EKC) phenomenon in a general model that permits non-smooth preferences and feasible sets and corner solutions for welfare maximisation. These conditions pertain to the relationship between the sets of preference and technology-based shadow prices (defined using the Clarke's normal cone), at an outcome reached by an emission-held-fixed-effect (EHFE), where the emission policy does not adjust to an increase in the economic resource base and only consumption adjusts. In particular, an EKC arises if and only if there exists a threshold level of resource such that, at any level of resource below (respectively, above) the threshold, the outcome reached by the EHFE is one where the set of preference-based shadow prices lies completely below (respectively, above) the set of technological shadow prices. This characterisation could be helpful in empirical testing for an EKC based on smooth-parametric or non-smooth non-parametric specifications of preferences and technology. We illustrate the characterisation with some examples that study homothetic economies, increasing returns to abatement, and emission as a normal good.

Keywords: environmental Kuznets curve, Clarke's tangent and normal cones, shadow prices, marginal willingness to pay, marginal abatement cost, welfare maximisation, homotheticity, normal good, increasing returns to abatement.

JEL classification codes: Q56, D62, C60

Necessary and sufficient conditions for an environmental Kuznets curve with some illustrative examples.

by

Sushama Murty

1. Introduction.

Environmental Kuznets curve (EKC) is a central concept in the literature on growth and environment. It hypothesises an inverted-U shaped relationship between environmental degradation and economic well-being. Such a relationship is often seen to suggest an optimism that the intuitive negative trade-off between environmental quality and economic growth is a short-lived phenomenon, which will eventually be overcome as economies grow and become richer. The hypothesis is based on a reasoning that encompasses social preferences over environment and material well-being and what is essentially made feasible by technological, scarcity, and institutional constraints. It is hypothesised that, on the one hand, there is a marked change in the valuation of environment relative to material well-being as we move from a subsistence to a developed economy and, on the other, the feasible sets of the richer economies may permit lower costs of abating environmental degradation.

A large volume of empirical work, beginning with Grossman and Krueger [1993, 1995], Shafik and Bandyopadhyay [1992], Panayotou [1993], Holtz-Eakin and Seldon [1992], and Seldon and Song [1994] have tested this hypothesis in different contexts employing different measures of environmental degradation; typically these are various forms of noxious emissions. The evidence is often mixed. EKC appears to hold for pollutants with more localised impacts such as sulphur dioxide and nitrous oxide emissions. For emissions whose impacts are more diffused such as several green house gasses, *e.g.,* carbon dioxide, emissions have continued to rise with growth.[1]

There is also a significant theoretical literature that seeks to explore the channels through which preferences, technological, scarcity, and institutional constraints could interact to generate an EKC. While papers such as Seldon and Song [1995] and Stokey [1998] emphasise on the role of the scarcity constraints that lead to corner choices for poorly endowed economies, Andreoni and Levinson [2001] bring to light a special technological phenomenon – increasing returns to abatement and the scale of emission as measured by the scale of consumption. Lopez [1994] and McConnell [1997] highlight the importance of some properties of preferences such as non-homotheticity and the magnitude of the income elasticity of demand for environmental quality.

[1] See, *e.g.,* the World Bank Report [1992]. See also Mason and Swanson [2001] for evidence regarding chloroflurocarbons. See Dasgupta et al [2002], Stern [2004], Dinda [2004], and Kijima et al [2010] for more recent reviews.

Papers such as John and Pecchenino [1994], Jones and Manuelli [2001], and Lopez and Mitra [2000] consider different institutional structures such as taxation of the young and voting over tax rates versus voting over environmental standards by the young for improvement of future environmental quality in an overlapping generations setting and bargaining between the government and the private sector in the presence of political constraints such as corruption and rent seeking behaviour by governments.

This work views EKC as a possible outcome of the comparative statics of a welfare maximisation problem subject to a feasibility constraint, where the locus of optimal emission level is traced when the productive capacity of an economy (as measured by the level of its economic resource-base) changes. It has two main objectives.

Firstly, it provides a set of necessary and sufficient conditions that characterise an EKC in a general setting that allows for non-smooth economies and permits both corner and interior choices during welfare maximisation. They are very intuitive and pertain to the relative responses of preference and feasibility-based shadow prices of emission to increases in the economic resource. Intuitively, in the case of smooth economies, the former shadow price reflects the marginal willingness to pay (MWTP) for a reduction in emission, while the latter reflects the marginal abatement cost (MAC) of emission reduction. Starting from a welfare optimum corresponding to an initial (say the status-quo) level of resource, the effect of a change in the level of the resource on the optimal emission level can be decomposed into two effects: an emission-held-fixed-effect (EHFE), where the emission policy does not initially adjust and only consumption adjusts to the change in the resource, and a difference-in-the shadow-prices-effect (DISPE), where the emission policy also adjusts optimally. We show that an increase in the economic resource will lead to an increase (respectively, decrease) in the optimal level of emission if and only if the set of preference-based shadow prices of emission is completely below (respectively, above) the set of feasibility-based shadow prices of emission at the outcome reached by the EHFE. The necessary and sufficient conditions for an EKC follow easily from this: *e.g.,* in the smooth case, there must exist a threshold level of resource such that, at the social optima corresponding to all levels of the resource lying below (respectively, above) the threshold, the EHFE due to an increase in the economic resource implies that the MAC increases more (respectively, less) than the MWTP.

Intuitive as this characterisation of an EKC may seem, it does not seem that these conditions hold ubiquitously. Finding preference and feasible set structures for which these conditions hold is very challenging, as is demonstrated by the efforts in the theoretical literature mentioned above. The *second* objective of this work is to illustrate this characterisation of the EKC with the help of examples of preference and feasible set structures that can potentially result in this phenomenon. We show how some main results in the existing literature can be explained in this light.[2]

The literature predicts that EKC is precluded when both preference and feasible set structures are homothetic. Here we show that the validity of this conjecture depends on the way we define homothetic preferences. If preferences are homothetic in the space of emission and consumption then, as in Lopez [1994], an EKC cannot happen. However, if preferences are defined over the space of environmental quality and consumption (as in Plassman and Khanna [2006]) and are homothetic in this space, then an EKC could be possible. In the latter case, homotheticity implies that both the MWTP and the MAC are increasing in the level of the economic resource when the emission level is held fixed. However, the relative increases vary across regions of the commodity space and may permit an EKC for some parametric specifications of preferences and the technology.

We next answer two questions – to what extent can the EKC phenomenon be attributed to pure technological and pure preference-based factors? Andreoni and Levinson's [2001] work provides one answer to the first question. In the context of our characterisation of the EKC, if the abatement technology exhibits increasing returns to scale then we show that there is a subset of the feasible set, where the MAC is increasing in the level of resource when emission is held fixed and there is also a feasible subset of such combinations where it is decreasing. Our necessary and sufficient characterisation implies that an EKC can arise if, in the former (respectively, latter) subset, the responsiveness of the MWTP is non-positive (respectively, non-negative). In particular, in their numerical examples, Andreoni and Levinson assume linear preferences so that the MWTP is a constant.

We show that an EKC can be attributed to pure preference-based factors if there is a region in the commodity space where the MWTP is decreasing in consumption and there is also a region where it is increasing in consumption. We show that emission is a normal good in the former region, while it is an inferior good in the latter region. Much of the literature assumes that emission is an inferior good (or environmental quality is a normal good). Here we try to demonstrate that normality of emission (or inferiority of environmental quality) is also a plausible property, which is not in contradiction with emission yielding disutility. In fact, if preferences are homothetic in emission and consumption, then there is a region in the commodity space where emission is a normal good (*i.e.,* the MWTP is decreasing in consumption). Moreover, it is in

[2] In a companion working paper, Murty [2014], we offer further examples.

this region alone that the social optimum lies.[3]

Lastly, we employ our main results to identify conditions under which the optimal trajectory of emission has an inverted-U shape in a growth model.

Lieb [2002] also provides conditions for an EKC in smooth economies, where emission is a consumption externality. We feel that the scope of this characterisation to study preference and technology combinations that can result in an EKC is not fully explored in that paper, *e.g.,* the possibility of preference structures where emission can exhibit normality or technological structures where the MAC can be decreasing in the level of the economic resource are, in general, excluded in Lieb as being non-standard.

Since our characterisation of the EKC is in terms of shadow prices, we adopt tools such as the tangent cone and its dual, the normal cone, that facilitate defining them in the most general contexts that permit non-smoothness and non-convexities. In empirical works, both smooth parametric and non-smooth non-parametric methods such as data envelopment analysis (DEA) are commonly used to estimate preferences and technologies.[4]. In the former case, derivatives of the utility and production functions define the normal cone and, hence, the concepts of the MWTP and the MAC. In the non-smooth case, derivatives are not defined in a conventional sense and MWTP and the MAC are not unique. Normal cones help to define preference and technology-based shadow prices in both cases. This could be useful for testing for an EKC based on our necessary and sufficient conditions and data obtained from revealed choices of countries.

In Section 2, we present the general model and the welfare maximisation problem. In Section 3, we derive the necessary conditions for welfare maximisation. We also specialise this result to the differentiable case. In Section 4, we define the EHFE and the DISPE and employ these concepts to derive the necessary and sufficient conditions for an EKC in terms of the relationship between preference and feasibility-based shadow prices at the outcome reached by EHFE. Section 5 assumes smooth economies, where the MWTP and the MAC are also differentiable. It presents an equivalent characterisation of the conditions derived in Section 4 in terms of the relative responses of the MWTP and the MAC to changes in the level of the resource, when the emission level is held fixed. Theorem 4.1 and its implication summarised in Theorem 5.1 are the two main results of this paper on the necessary and sufficient conditions for an EKC. Sections 6, 7, and 8 apply these results to study the cases of homothetic economies, increasing returns to abatement, and preference-based EKCs, respectively. Section 9 employs results in the previous sections to study EKC as an outcome of economic growth. We conclude in Section 10.

[3] See Section 6 of this paper.

[4] See, for instance, Varian [1982, 1983, 1984], Färe et al [1993], and Färe et al [1994].

2. The basic framework of analysis and the welfare maximisation problem.

There are two goods: emission and consumption. The amounts of emission and consumption are denoted by $z \in \mathbf{R}_+$ and $c \in \mathbf{R}_+$, respectively. As seen below, the representation of preferences is standard. The feasible set of emission-consumption combinations, given any level of the resource-base, is assumed to contain all the emission and consumption combinations that are feasible. In general, it can be a set that is defined by technological or institutional constraints, in addition to the scarcity constraint. In this paper, we will interpret it as a set defined by technological and scarcity constraints. Such a set could have a functional representation. We provide some examples to illustrate this set. The welfare maximisation problem that is relevant for our analysis is also posed and an analytically more tractable equivalent of this problem is derived.

2.1. Preferences, technology, and strictly efficient production frontier.

Preferences over the two goods are represented by a utility function $u : \mathbf{R}^2_+ \longrightarrow \mathbf{R}$ with image $u(z, c)$. Emission yields disutility. Assumption 2.1 requires u to be decreasing in z and increasing in c.

Assumption 2.1: u is a continuous function. For all $\langle z, c\rangle$ and $\langle z', c'\rangle$ in $\mathbf{R}^2_+$ such that $\langle z, c\rangle \neq \langle z', c'\rangle$, if $z \leq z'$ and $c \geq c'$, then $u(z, c) > u(z', c')$.

The endowment level is denoted by $y \in \mathbf{R}_+$. The set of emission and consumption levels that are feasible given an endowment level is represented by the set-valued mapping $\hat{Y} : \mathbf{R}_+ \mapsto \mathbf{R}^2_+$ with image $\hat{Y}(y)$.

Assumption 2.2: The set-valued mapping $\hat{Y}$ is continuous; $0^2 \in \hat{Y}(y)$; and $y' > y \geq 0$ implies $\hat{Y}(y) \subset \hat{Y}(y')$.[5]

Assumption 2.2 implies that, at every level of endowment $y \geq 0$, the feasible set $\hat{Y}(y)$ is closed and permits shut-down, *i.e.,* a point with no emission and no consumption is feasible. Assumption 2.2 also implies that the set of feasible emission and consumption levels expands with increase in the endowment level.

It is intuitive that, if the endowment level is fixed, then efficiency requires the maximal production of the consumption good and the minimal production of emission. Hence, we define the strictly efficient (production) frontier of $\hat{Y}(y)$ for $y \geq 0$ as the set

$$\left\{\langle z, c\rangle \in \hat{Y}(y) \;\middle|\; \nexists\ \langle z', c'\rangle \in \hat{Y}(y) \text{ such that } \langle z', c'\rangle \neq \langle z, c\rangle,\ z' \leq z, \text{ and } y' \geq y\right\}.$$

[5] 0^n denotes a zero vector in $\mathbf{R}^n$.

Thus, given $y \geq 0$, $\langle z, c\rangle \in \hat{Y}(y)$ is a *strictly efficient point of* $\hat{Y}(y)$ if there is no other emission-consumption combination in $\hat{Y}(y)$ with consumption level greater than or equal to c and emission level less than or equal to z.

2.2. The welfare maximisation problem and a disposal hull of the technology.

The planner maximises welfare subject to a feasibility constraint. The solution mapping of our concerned welfare maximisation problem is $\varphi : \mathbf{R}_+ \mapsto \mathbf{R}^2_+$ with image defined as

$$\varphi(y) := \arg\max_{z,c}\{u(z,c)\big|\ \langle z, c\rangle \in \hat{Y}(y)\}. \tag{2.1}$$

The following remark is straightforward to infer given the monotonicity properties of u under Assumption 2.1,

Remark 2.1 *Under Assumption 2.1, for any $y \geq 0$, $\varphi(y)$ lies on the strictly efficient frontier of $\hat{Y}(y)$.*

The tractability of our analysis can be enhanced by focussing on a particular disposal hull of every image of mapping $\hat{Y}$. Define a set-valued mapping $Y : \mathbf{R}_+ \mapsto \mathbf{R}^2_+$ with image

$$Y(y) = \Big\{\langle z, c\rangle \in \mathbf{R}^2_+ \ \big|\exists\ \langle z', c'\rangle \in \hat{Y}(y) \text{ such that } z \geq z' \text{ and } c \leq c'\Big\}. \tag{2.2}$$

The image $Y(y)$ is a disposal hull of $\hat{Y}(y)$ with the following disposability properties: $\hat{Y}(y) \subseteq Y(y)$. If an emission-consumption combination belongs to $Y(y)$, then so does any emission-consumption combination with a greater amount of emission and a lesser amount of consumption. Note, $Y(y)$ satisfies standard output-free disposability with respect to the consumption good. Further, it can be verified that the strictly efficient frontiers of $Y(y)$ and $\hat{Y}(y)$ coincide for every $y \geq 0$.[6] This and Remark 2.1 lead to the following remark.

Remark 2.2: *If Assumption 2.1 is true, then $\varphi(y)$ also solves the problem*

$$\max_{z,c}\{u(z,c)\big|\ \langle z, c\rangle \in Y(y)\}. \tag{2.3}$$

Under Assumption 2.2, the set-valued mapping Y is continuous, $0^2 \in Y(y)$, and $y' > y \geq 0$ implies $Y(y) \subset Y(y')$.

The focus of our work will be the study of the comparative statics of problem (2.1). Since the outcomes of Problems (2.1) and (2.3) are equivalent, without loss of generality, we can work with mapping Y instead of mapping $\hat{Y}$ and focus on Problem (2.3). This is what we do in the rest of the analysis.

[6] The strictly efficient frontier of $Y(y)$ can be defined analogously to the strictly efficient frontier of $\hat{Y}(y)$.

2.3. A functional representation specialising to a convex technology.

Assumption 2.3, below, permits a representation of the image of mapping Y with smooth functions that results in a convex technology set. Some examples are presented in Section 2.4.

Assumption 2.3: Assumption 2.2 holds and there exist functions $F : \mathbf{R}^3_+ \longrightarrow \mathbf{R}$ and $B : \mathbf{R}_+ \longrightarrow \mathbf{R}_+$ such that the image of the set-valued mapping Y is

$$Y(y) = \left\{ \langle z, c \rangle \in \mathbf{R}^2_+ \mid F(z, c, y) \leq 0,\ c \leq B(y) \right\}, \tag{2.4}$$

where B and F are continuously differentiable and quasi-convex with $F_z < 0,\ F_c > 0,\ F_y < 0,$ and $B' > 0$. For all $\bar{y} > y \geq 0$, if $c = B(y)$, $\bar{c} = B(\bar{y})$, $F(\bar{z}, \bar{c}, \bar{y}) = 0$, and $F(z, c, y) = 0$ then $\bar{z} > z$.

It is intuitive that, given a finite amount of resource, consumption is bounded. In Assumption 2.3, function $B : \mathbf{R}_+ \longrightarrow \mathbf{R}_+$ with image $B(y)$ defines the upper bound on consumption for every level of endowment. Remark 2.3, which can be inferred readily from this assumption, characterises the strictly efficient points of $Y(y)$.

Remark 2.3: *Under Assumption 2.3, for every $y \geq 0$, $\langle z, c \rangle \in \mathbf{R}^2_+$ is a strictly efficient point of $Y(y)$ (and hence $\hat{Y}(y)$) if and only if $F(z, c, y) = 0$ and $c \leq B(y)$.*

In particular, if $F(z, c, y) = 0$ and $c = B(y)$ then z and c define, respectively, the upper bounds on emission and consumption for the strictly efficient frontier of $Y(y)$. Assumption 2.3 implies that these upper bounds increase with increase in the endowment. It also implies that the trade-off between emission and consumption along the strictly efficient frontier, which is given by $-\frac{F_z}{F_c}$, is positive.

2.4. Some examples of the technology set.

The following is an example of a homothetic technology that we will develop further in Section 6. The feasible sets of emission and consumption radially expand with increase in the economic resource base.

Example 1. Suppose the image of functions F and B in Assumption 2.3 are

$$\begin{aligned} F(z, c, y) &= \frac{(c - z)^2}{c} + 1.5c - y \quad \text{if } c > z \\ &= 1.5c - y \quad \text{if } c \leq z \\ B(y) &= \frac{y}{1.5}, \end{aligned}$$

and $Y(y)$ is defined as in (2.4) for all $y \geq 0$. Panels (i) and (ii) of Figure 1 show the sets $\{\langle z, c, \rangle \in \mathbf{R}^2_+ \mid F(z, c, y) \leq 0\}$ for different values of y ranging from 2.5 to 22.5. Note that

$-\frac{F_z}{F_c} > 0$ whenever $c > z$ and $-\frac{F_z}{F_c} = 0$ whenever $c \leq z$. Given the form of function B, it can be shown that (2.4) implies $Y(y) = \{\langle z, c, \rangle \in \mathbf{R}^2_+ | \ F(z, c, y) \leq 0\}$, for all $y \geq 0$. Panel (iii) of Figure 1 shows the strictly efficient frontiers of this technology for different values of the endowment.

In much of the literature on EKC, emission generation is directly linked to consumption. In particular, works such as Andreoni and Levinson [2001] treat emission as a consumption externality, *i.e.,* emission is assumed to be caused by consumption. Further, it is assumed that the initial endowment can either be consumed or be used in abatement efforts (denoted by a) to mitigate emission. Hence, it is as if function B in Assumption 2.3, which specifies the upper bound on consumption for every level of resource, takes the form $B(y) = y$. Thus, if c amount of the endowment y is consumed, then the amount used for abatement is $a = y - c$. Net emission is a function of the amount of consumption and the amount of abatement, and is often represented by a function such as $\bar{g} : \mathbf{R}^2_+ \longrightarrow \mathbf{R}_+$ with image $\bar{g}(c, a)$. Function $\bar{g}$ identifies the strictly efficient level of emission associated with given levels of consumption and abatement. Define a function $g : \mathbf{R}^2_+ \longrightarrow \mathbf{R}_+$ with image $g(c, y) = \bar{g}(c, y - c)$. Under Assumption 2.3, we have the following relation between functions F and g:

$$F(z, c, y) \leq 0 \iff z \geq g(c, y) \tag{2.5}$$

This implies g is quasi-convex with $g_y < 0$ and $g_c > 0$.

<u>*Example 2.*</u> The model in Andreoni and Levinson [2001] defines an abatement function that makes the extent of mitigation of emission a function of both the scale of gross emission and the amount of endowment devoted to abatement. Since emission is a consumption externality in this paper, the scale of gross emission is reflected by the level of consumption. The abatement technology is, hence, represented by the function $A : \mathbf{R}^2_+ \longrightarrow \mathbf{R}_+$ with image $A(c, a)$, where a denotes the resource devoted to abatement. Define a mapping $\bar{A} : \mathbf{R}^2_+ \longrightarrow \mathbf{R}_+$ with image $\bar{A}(c, y) := A(c, y - c)$. The function g is defined as

$$g(c, y) = \bar{g}(c, y - c) = c - A(c, y - c) = c - \bar{A}(c, y), \tag{2.6}$$

hence, in this case, it follows from (2.5) that $F(z, c, y) = c - \bar{A}(c, y) - z$ and $B(y) = y$. Panels (iv) to (vi) of Figure 1 plot the sets $\{\langle z, c, \rangle \in \mathbf{R}^2_+ | \ F(z, c, y) \leq 0\}$, $Y(y)$, and the strictly efficient frontier of $Y(y)$, respectively, for different values of y ranging from 0 to 9.7, when $A(c, (y - c))$ takes the form $c(y - c)$ as in a numerical example in Andreoni and Levinson.

3. Necessary conditions of welfare maximisation.

Necessary conditions of welfare maximisation are conditions on preference-related and production-related shadow prices at the optimum. In the most general case, which allows for non-convexities and non-smoothness in the economically relevant sets and their frontiers, respectively, shadow prices can be classically derived from the sets of normals to appropriate tangent cones.[7] Section 3.1 reviews these concepts; Section 3.2 states the properties of the tangent cones of the two types of sets relevant for our analysis, namely, the preferences and the technology; Section 3.3 derives the sets of preference-based and technological shadow prices from the normal cones to these sets; and in Section 3.4 the necessary and sufficient conditions of welfare maximisation are stated in terms of these sets of shadow prices.

3.1. Tangent and normal cones: definitions and some relevant properties.

The definition of the tangent cone employed in this paper is attributable to Clarke [1975, 1983, 1989].[8] Let $\mathcal{B} \subset \mathbf{R}^n$ and $b \in cl\ \mathcal{B}$.[9]

Definition: The *tangent cone* for $\mathcal{B}$ relative to b is the set
$T(\mathcal{B}, b) = \big\{x \in \mathbf{R}^n \mid$ for every sequence $\{t^\nu\} \longrightarrow 0$ and sequence $\{b^\nu\} \longrightarrow b$ with $b^\nu \in cl\ \mathcal{B}$, $\exists$ a sequence $\{x^\nu\} \longrightarrow x$ such that $b^\nu + t^\nu x^\nu \in cl\ \mathcal{B}$ for all large enough $\nu\big\}$.

The set $\{b\} + T(\mathcal{B}, b)$ offers a local approximation of set $\mathcal{B}$ at the point b. The tangent cone $T(\mathcal{B}, b)$ is a closed convex cone. The normal cone to $T(\mathcal{B}, b)$, denoted by $N(\mathcal{B}, b)$, is the negative polar of $T(\mathcal{B}, b)$, denoted by $T(\mathcal{B}, b)^-$.[10] It is a closed convex cone. Some important results based on the interior of a tangent cone will be employed in the analysis below. Lemma 3.1 characterises this cone. Intuitively, Lemma 3.1 implies that the interior of the tangent cone of any set, relative to a point in the closure of the given set, is the cone formed by all directions of change that lead points in the set, which are in a local neighbourhood around the given point, to other points that lie in the interior of the set.

Lemma 3.1: *The interior of the tangent cone of $\mathcal{B}$ relative to b is the set*
$int\ T(\mathcal{B}, b) = \big\{x \in \mathbf{R}^n \big| \exists\ \epsilon > 0,\ \eta > 0,\ \delta > 0$ *such that* $\{b'\} + \lambda cl\ \mathcal{N}_\epsilon(x) \subseteq cl\ \mathcal{B}\ \forall\ \lambda \in [0, \eta]$ *and* $\forall\ b' \in (cl\ \mathcal{B} \cap\ cl\ \mathcal{N}_\delta(b))\big\}$.

[7] Seminal contributions in the study of equilibria and welfare properties of non-smooth and non-convex economies include Guesnerie [1975], Beato [1976], Brown and Heal [1979], and Bonnisseau and Cornet [1988].

[8] We refer you to the literature for a more detailed study of these concepts and proofs of the results in this subsection. See, for instance, Rockafellar [1978], Rockafellar and Wets [2009], Khan and Vohra [1987], Cornet [1990], and Quinzii [1993].

[9] Given a set $\mathcal{B} \subset \mathbf{R}^n$, we denote the closure of $\mathcal{B}$ as $cl\ \mathcal{B}$ and the interior of $\mathcal{B}$ as $int\ \mathcal{B}$. Given a scalar $\epsilon > 0$ and $b \in \mathbf{R}^n$, we denote $\mathcal{N}_\epsilon(b)$ as the $\epsilon-$neighbourhood of b. $cl\ \mathcal{B}$ and $int\ \mathcal{B}$ denote the closure and interior of of the set $\mathcal{B}$, respectively.

[10] The *negative polar cone* of $\mathcal{B}$ is the set $\mathcal{B}^- = \{v \in \mathbf{R}^n \big| v \cdot b \leq 0\ \forall\ b \in \mathcal{B}\}$.

Lemma 3.2 describes its relation to the normal cone, namely, both the tangent cone and its interior have the same normal cone.

Lemma 3.2: *$N(\mathcal{B}, b) = int\ T(\mathcal{B}, b)^-$ and $x \cdot v < 0$ for all $x \in int\ T(\mathcal{B}, b)$ and $v \in N(\mathcal{B}, b)$.*

Lemmas A.1 to A.6 in the appendix provide some additional results based on these concepts that are relevant for our analysis. In particular, Lemma A.5 specialises the concepts of tangent and normal cones to the smooth case, while Lemma A.6 specialises these concepts to the case when $\mathcal{B}$ is a convex set.

3.2. Tangent cones of preferences and technology.

Given any $\langle z, c\rangle \in \mathbf{R}^2_+$, let the no-worse-than and better-than sets relative to this point be, respectively, defined as

$$\succeq (z,c) := \left\{\langle z', c'\rangle \in \mathbf{R}^2_+ \middle|\ u(z', c') \geq u(z,c)\right\} \text{ and } \succ (z,c) := \left\{\langle z', c'\rangle \in \mathbf{R}^2_+ \middle|\ u(z', c') > u(z,c)\right\}. \tag{3.1}$$

Assumption 2.1 implies local non-satiation (LNS) and continuity of preferences.[11] Debreu [1959] shows that together these imply the following property of preferences that will be exploited in many of the theorems below.

Remark 3.1: *Under Assumption 2.1, $\succeq (z,c) = cl\ \succ (z,c)$ for any $\langle z, c\rangle \in \mathbf{R}^2_+$.*

Lemma 3.5, below, which we prove in the appendix, follows from the monotonicity properties of function u under Assumption 2.1 and Lemma 3.1. Assumption 2.1 implies that, starting from any emission-consumption combination with a strictly positive amount of emission, any direction of change that increases consumption and decreases emission is welfare improving, *i.e.,* leads to points that are in $\succ (z, y)$, which belongs to the interior of $\succeq (z, y)$. Hence, it follows from Lemma 3.1 that the cone formed by all such directions of change, namely, $\mathbf{R}_{--} \times \mathbf{R}_{++}$, is a subset of the interior of the tangent cone of $\succeq (z,c)$ relative to $\langle z, c\rangle$.

Lemma 3.5: *Under Assumption 2.1, $\mathbf{R}_{--} \times \mathbf{R}_{++} \subset int\ T(\succeq (z,c), \langle z, c\rangle)$ for all $\langle z, c\rangle \in \mathbf{R}^2_+$ such that $z \neq 0$.*

Similarly, Lemma 3.1, Assumption 2.2, and the disposability properties of set $Y(y)$ for $y \geq 0$ (which follow from its definition in (2.2)) imply that, starting from any point in $Y(y)$ with a strictly positive amount of consumption, any direction of change, which decreases the

[11] u satisfies local non-satiation if for every $\langle z, c\rangle \in \mathbf{R}^2_+$ and for every $\epsilon > 0$, we have $\mathcal{N}_\epsilon(z,c) \cap \succ (z,c) \neq \emptyset$.

consumption output and increases emission, takes points that lie in the local neighbourhood of the given point to other points in $Y(y)$. Hence, such a direction of change belongs to the interior of the tangent cone. The proof of Lemma 3.6, below, is similar to the proof of Lemma 3.5.

Lemma 3.6: *Suppose Assumption 2.2 holds and $Y(y)$ is as defined in (2.2) for all $y \geq 0$. Then for all $\langle z, c\rangle \in Y(y)$ such that $y > 0$, we have $\mathbf{R}_{++} \times \mathbf{R}_{--} \subset int\ T(Y(y), \langle z, c\rangle)$.*

3.3. The sets of shadow prices of emission.

From every normal cone associated with the preferences or the technology, we can derive a of set of shadow prices of emission. For every $\langle z, c\rangle \in \mathbf{R}^2_+$, define the set of preference-based shadow prices of emission as

$$S^P(\succeq(z,c), \langle z, c\rangle) = \left\{ -\frac{v_1}{v_2} \in \mathbf{R}_+ \cup \{\infty\} \;\middle|\; \langle v_1, v_2\rangle \in -N(\succeq(z,c), \langle z, c\rangle) \right\}. \tag{3.2}$$

To every $\langle z, c\rangle \in Y(y)$ for $y \geq 0$, assign a set of technological shadow prices of emission as[12]

$$S^Y(Y(y), \langle z, c\rangle) = \left\{ -\frac{v_1}{v_2} \in \mathbf{R}_+ \cup \{\infty\} \;\middle|\; \langle v_1, v_2\rangle \in N(Y(y), \langle z, c\rangle) \right\}. \tag{3.3}$$

Remark 3.2: *Since a normal cone is a closed and convex cone, the associated set of shadow prices will be a closed interval.*

To understand the set of preference-based shadow prices, let us distinguish between two extreme shadow prices at any emission-consumption combination: first, the maximum reduction in consumption that can offset the gain in welfare due to a unit reduction in emission (this is also called the marginal willingness to pay (MWTP) for emission reduction) and, second, the minimum increase in consumption that can compensate the welfare loss due to a unit increase in emission (this is also called the marginal willingness to accept (MWTA) compensation for increase in emission). In general, the MWTP and the MWTA may differ, *e.g.,* if indifference curves are kinked, then at a kink, the MWTP will not be the same as the MWTA.[13] Remark 3.2 implies that the set of preference-based shadow prices of emission is the set of all convex combinations of the two. Similarly, the relevant trade-offs in production can be the minimum reduction in the output of the consumption good when the producing unit is required to reduce a unit of its emission (this is called the marginal abatement cost (MAC)) or the maximum increase

[12] Note, a shadow price takes value ∞, when $v_2 = 0$.

[13] Intuitively, MWTP will correspond to the left derivative of u, while MWTA will correspond to the right derivative of u at the given emission-consumption combination.

in the output of the consumption good when the producing unit is allowed to generate an extra unit of emission (we call this the marginal return (MR) from increase in emission). As in the case of preferences, when the frontier of the production technology is not smooth, MAC and MR may differ at a point on the frontier. The set of technological shadow prices at a point on the frontier will be the set of all convex combinations of the MAC and the MR.

If u is continuously differentiable, Lemma A.5 implies that, for every $\langle z, c\rangle \in \mathbf{R}^2_+$, we have

$$-N(\succeq (z,c), \langle z,c\rangle) = \left\{v \in \mathbf{R}^2 \;|v = \kappa_u \nabla_{\langle z,c\rangle} u(z,c),\; \kappa_u \geq 0\right\}, \tag{3.4}$$

and defining the *preference MRS* at any point $\langle z,c\rangle \in \mathbf{R}^2_{++}$ as $r(z,c) := -\frac{u_z(z,c)}{u_c(z,c)}$, we obtain from (3.2) and (3.4)[14]

$$S^P(\succeq (z,c), \langle z,c\rangle) = \{r(z,c)\} \;\forall\; \langle z,c\rangle \in \mathbf{R}^2_{++}. \tag{3.5}$$

Under Assumption 2.3, for every $\langle z,c\rangle \in Y(y)$, Lemma A.4 implies that

$$N\left(Y(y), \langle z,c\rangle\right) = N(\Gamma_F(y), \langle z,c\rangle) + N(\Gamma_B(y), \langle z,c\rangle), \tag{3.6}$$

where $\Gamma_F(y) = \{\langle z', c'\rangle \in \mathbf{R}^2_+ |\; F(z', c', y) \leq 0\}$ and $\Gamma_B(y) = \{\langle z', c'\rangle \in \mathbf{R}^2_+ |\; c' \leq B(y)\}$. In particular, if $\langle z,c\rangle$ is a strictly efficient point of $Y(y)$ then (3.6), combined with Lemma A.5, implies

$$\begin{aligned} N(Y(y), \langle z,c\rangle) &= \left\{v \in \mathbf{R}^2 \;|v = \kappa_F \nabla_{\langle z,c\rangle} F(z,c,y),\; \kappa_F \geq 0\right\} \qquad \text{if } c < B(y) \text{ and} \\ &= \left\{v \in \mathbf{R}^2 \;|v = \kappa_F \nabla_{\langle z,c\rangle} F(z,c,y),\; \kappa_F \geq 0\right\} + \left\{v \in \mathbf{R}^2 \;|v = \kappa_B \langle 0,1\rangle,\; \kappa_B \geq 0\right\} \\ &\qquad\qquad \text{if } c = B(y). \end{aligned} \tag{3.7}$$

Define the *technological MRS* at any point $\langle z,c,y\rangle \in \mathbf{R}^3_{++}$ as $s(z,c,y) := -\frac{F_z(z,c,y)}{F_c(z,c,y)}$. In this smooth case, if $\langle z,c\rangle$ is a strictly efficient point of $Y(y)$ then (3.3) and (3.7) imply[15]

$$\begin{aligned} S^Y(Y(y), \langle z,c\rangle) &= \{s(z,c,y)\} \qquad \text{if } c < B(y) \text{ and} \\ &= \left\{p \in \mathbf{R}_+ \cup \{\infty\} |\; p = \frac{-\kappa_F F_z(c,y)}{\kappa_F F_c(z,c,y) + \kappa_B} \text{ for some } \kappa_B \geq 0, \kappa_F \geq 0\right\} \text{ if } c = B(y). \end{aligned} \tag{3.8}$$

3.4. *Necessary conditions of welfare maximisation.*

Theorem 3.1, whose proof can be found in the appendix, provides a necessary characterisation of a solution to problem (2.3) in a general case which allows for non-smoothness and

[14] Note, under Assumption 2.1, $u_y(z,c) \neq 0$.

[15] Note, under Assumption 2.3, $F_c(z,c,y) \neq 0$.

non-convexities. It states that, at an optimum, the normal cone of the technology and the negative of the normal cone of the preferences have a non-empty intersection. At point **B** in Panel (ii) of Figure 2, the normal cone to the technology is the convex cone enclosed by the two red arrows, while the negative of the normal cone to the preferences is the blue arrow. These two cones do not intersect, and hence point **B** is not an optimum for resource level y'. Point **C** is optimal as the normal cones of preferences and technology coincide at this point.

Theorem 3.1: *Suppose Assumptions 2.1 and 2.2 hold and* $\langle \overset{*}{z}, \overset{*}{c} \rangle \in \varphi(y)$ *for* $y \geq 0$ *with* $\overset{*}{z} \neq 0$ *and* $\overset{*}{c} \neq 0$*. Then there exists* $\overset{*}{v} \in \mathbf{R}^2 \setminus \{0^2\}$ *such that* $\overset{*}{v} \in N(Y(y), \langle \overset{*}{z}, \overset{*}{c} \rangle)$ *and* $-\overset{*}{v} \in N\left(\succeq (\overset{*}{z}, \overset{*}{c}), \langle \overset{*}{z}, \overset{*}{c} \rangle\right)$*. Equivalently,* $S^Y\left(Y(y), \langle \overset{*}{z}, \overset{*}{c} \rangle\right) \bigcap S^P\left(\succeq (\overset{*}{z}, \overset{*}{c}), \langle \overset{*}{z}, \overset{*}{c} \rangle\right) \neq \emptyset$*.*

Thus, at an optimum, the intersection of the sets of preference and technology-based shadow prices is non-empty. Specialising this theorem to the differentiable case, we obtain Theorem 3.2.

Theorem 3.2: *Suppose Assumptions 2.1 and 2.3 hold and* u *is continuously differentiable. If* $\langle z, y \rangle \in \varphi(y) \cap \mathbf{R}^2_{++}$ *for* $y \geq 0$*, then* $F(z, c, y) = 0$ *and the following hold:*

$$(i) \;\; c < B(y) \implies r(z,c) = s(z,c,y) \quad \textit{and} \quad (ii) \;\; c = B(y) \implies r(z,c) \leq s(z,c,y).$$

Proof: Theorem 3.1 implies that there exists $v \in \mathbf{R}^2 \setminus \{0^2\}$ such that $v \in -N\left(\succeq (z,c), \langle z,c \rangle\right)$ and $v \in N(Y(k), \langle z, c \rangle)$. The former, coupled with (3.4), implies there exists a scalar $\kappa_u > 0$ such that

$$v = \kappa_u \begin{bmatrix} u_z(z,c) \\ u_y(z,c) \end{bmatrix} \tag{3.9}$$

Remark 2.1 implies that $\langle z, c \rangle$ is a strictly efficient point of $Y(y)$. Hence, Remark 2.3 implies $F(z,c,y) = 0$ and $c \leq B(y)$. Two cases arise:
(i) Suppose $c < B(y)$. Then, (3.7) implies that there exists $\kappa_F > 0$ such that

$$v = \kappa_F \begin{bmatrix} F_z(z,c,y) \\ F_c(z,c,y) \end{bmatrix}. \tag{3.10}$$

Hence, (3.9) and (3.10) imply $r(z,c) = s(z,c,y)$.
(ii) Suppose $c = B(y)$. Then, since (3.9) implies $v_1 \neq 0$, (3.7) implies that there exists $\kappa_B \geq 0$ and $\kappa_F > 0$ such that

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \kappa_F \begin{bmatrix} F_z(z,c,y) \\ F_y(z,c,y) \end{bmatrix} + \kappa_B \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{3.11}$$

From (3.9) and (3.11) it follows that

$$\frac{r(z,c)}{s(z,c,y)} = \frac{1}{1 + \frac{\kappa_B}{\kappa_F F_y(z,c,y)}} \leq 1. \blacksquare \tag{3.12}$$

As in Stokey [1998], Theorem 3.2 distinguishes between the two types of solutions of Problem (2.3), which are covered by its conclusions (i) and (ii). The ability of a society to consume is bounded by its resources. In our model, given $y \geq 0$ amount of resource, the maximal feasible consumption is $B(y)$. Case (i) corresponds to an interior solution of Problem (2.3), where the upper bound on feasible consumption is not binding at the optimum. In that case, a necessary condition of welfare maximisation is the equality of the MRSs in preferences and production. On the other hand, in case (ii), we have a corner solution to problem (2.3), *i.e.,* at the optimum, the upper bound on feasible consumption is binding. This is because, here, the MAC (which is also the technological MRS) is more than the MWTP for emission reduction (which is also the preference MRS) at every point on the strictly efficient frontier of the feasible set.

4. Necessary and sufficient conditions for an environmental Kuznets curve.

An environmental Kuznets curve (EKC) is a possible outcome of the comparative statics of the welfare maximisation problem (2.3), where the locus of the optimal emission level is traced when the level of the resource changes. In our general model, the welfare maximisation problem can yield multiple solutions. For defining an EKC, we focus on the locus of a particular solution, namely, the one with the least amount of emission. For $y \geq 0$, define

$$z^{min}(y) := \min\{z \in \mathbf{R}_+ \mid \exists\, c \in \mathbf{R}_+ \text{ such that } \langle z, c\rangle \in \varphi(y)\}$$

This paves the path for the following definition of an EKC.

Definition: The mapping φ exhibits an EKC if there exists $\overset{*}{y} > 0$ such that

(i) $z^{min}(y') > z^{min}(y) > 0$ whenever $0 < y < y' < \overset{*}{y}$ and

(ii) $z^{min}(y) > z^{min}(y') > 0$ whenever $0 < \overset{*}{y} < y < y'$.

Thus, φ exhibits an EKC if there is threshold level of resource, below (respectively, above) which the minimal level of optimal emission increases (respectively, decreases) with increase in the resource.

The effect of a change in the level of the resource on the optimum can be decomposed into two effects: (i) an *emission-held-fixed effect (EHFE)*, where the emission policy does not adjust to the change in resource, while consumption adjusts to solve a *restricted* welfare maximisation subject to the initial optimal level of emission and the new level of the resource and (ii) a *difference-in-the-shadow-prices effect (DISPE)*, where emission adjusts optimally given the differences in the preference and technology-based shadow prices at the outcome reached by the EHFE. The comparative statics of Problem (2.3) yields an EKC if and only if the sets of preference and technology-based shadow prices at the outcome reached by the EHFE bear certain relations to each other. In Section 4.1, we state some properties of the normal cones to preferences and technology, which have implications for the sets of shadow prices. Employing these properties, in Section 4.2, below, we state the necessary and sufficient conditions for an EKC. Section 4.3 specialises these results to the differentiable case, which provides a greater understanding.

4.1. Some relevant properties of normal cones of preferences and technology.

Lemma 4.1, below, presents the signs of the elements of every vector that lies in the negative of a normal cone of preferences. In particular, it implies that, if Assumption 2.1 holds and preferences are convex, then both elements of any vector in the negative of the associated normal cone are non-zero, and have opposite signs. Hence, the preference shadow price, which is the negative of their ratio, is positive and a finite number.

Lemma 4.1: *Suppose Assumption 2.1 holds and* $\langle z, y\rangle \in \mathbf{R}^2_+$. *Suppose* $v = \langle v_1, v_2\rangle \in -N(\succeq (z,c), \langle z,c\rangle)$. *Then the following are true:*

(i) $v_1 \le 0$ *and* $v_2 \ge 0$.

(ii) If $\langle -1, 0\rangle \in$ *int* $T(\succeq (z,c), \langle z,c\rangle)$ *and* $v \ne 0^2$, *then* $v_1 < 0$.

(iii) If $\langle 0, 1\rangle \in$ *int* $T(\succeq (z,c), \langle z,c\rangle)$ *and* $v \ne 0^2$, *then* $v_2 > 0$.

(iv) If u *is quasi-concave and* $z \ne 0$, *then* $\langle -1, 0\rangle \in$ *int* $T(\succeq (z,c), \langle z,c\rangle)$ *and* $\langle 0, 1\rangle \in$ *int* $T(\succeq (z,c), \langle z,c\rangle)$.

Similarly, Lemma 4.2 describes the sign of the elements of every vector that lies in a normal cone of the technology. In particular, if Assumption 2.2 holds and the technology is convex, then every technological shadow price of emission is non-negative and finite.

Lemma 4.2: *Suppose Assumption 2.2 holds,* $y \ge 0$, $Y(y)$ *is as defined in (2.2), and* $\langle z, c\rangle \in Y(y)$. *If* $v = \langle v_1, v_2\rangle \in N(Y(y), \langle z,c\rangle)$, *then the following are true:*

(i) $v_1 \le 0$ *and* $v_2 \ge 0$.

(ii) If $\langle 1, 0\rangle \in$ *int* $T(Y(y), \langle z,c\rangle)$ *and* $v \ne 0^2$ *then* $v_1 < 0$.

(iii) If $\langle 0, -1\rangle \in$ *int* $T(Y(y), \langle z,c\rangle)$ *and* $v \ne 0^2$ *then* $v_2 > 0$.

(iv) If $Y(y)$ *is convex for all* $y \ge 0$, *then* $\langle 0, -1\rangle \in$ *int* $T(Y(y), \langle z,c\rangle)$ *for all* $\langle z,c\rangle \in Y(y)$ *such that* $z > 0$ *and* $c > 0$.

Proofs of both Lemmas 4.1 and 4.2 are in the appendix. Under the assumptions of these lemmas, in the following remark, the normal cones to preferences and the technology are recovered back from the sets of preference-based and technological shadow prices.

Remark 4.1: *Suppose Assumption 2.1 holds and u is quasi-concave. Then, for all $\langle z, c\rangle \in \mathbf{R}^2_+$, if $p \in S^P(\succeq (z,c), \langle z,c\rangle)$ then $0 < p < \infty$ and*

$$-N(\succeq (z,c), \langle z,c\rangle) = \big\{\langle v_1, v_2\rangle \in \mathbf{R}_{--} \times \mathbf{R}_{++} \mid \exists\ p \in S^P(\succeq (z,c), \langle z,c\rangle) \text{ and } \kappa \geq 0 \text{ such that } \langle v_1, v_2\rangle = \kappa\langle -p, 1\rangle\big\}.$$

Suppose Assumption 2.2 holds, the mapping Y is as defined in (2.2), and $Y(y)$ is convex for all $y \geq 0$. Then, for all $\langle z, c\rangle \in Y(y)$, if $p \in S^Y(Y(y), \langle z,c\rangle)$ then $0 \leq p < \infty$ and

$$N(Y(y), \langle z,c\rangle) = \big\{\langle v_1, v_2\rangle \in \mathbf{R}_{-} \times \mathbf{R}_{++} \mid \exists\ p \in S^Y(Y(y), \langle z,c\rangle) \text{ and } \kappa \geq 0 \text{ such that } \langle v_1, v_2\rangle = \kappa\langle -p, 1\rangle\big\}.$$

4.2. Necessary and sufficient conditions for an EKC.

The comparative statics of problem (2.3) yields an EKC if and only if the sets of preference-based and technological shadow prices at the outcome reached by the EHFE bear certain relations to each other. To state these conditions on the shadow prices it will be helpful to define, for every $y \geq 0$, the maximum level of consumption that is possible when the emission level is also held fixed at a level, say, $z \geq 0$:

$$c^m(z,y) := \max\{c \in \mathbf{R}_+ \mid c \in Y(z,y)\}, \text{ where } Y(z,y) := \{c \in \mathbf{R}_+ \mid \langle z,c\rangle \in Y(y)\}\ \forall\ \langle z,y\rangle \in \mathbf{R}^2_+.$$

Remark 4.2, below, states that $c^m(z,y)$ is the solution of a restricted welfare maximisation problem, where both the resource and emission levels are held fixed. Moreover, if the emission is fixed at an optimum of the unrestricted problem (2.3) with resource level $y \geq 0$, then the consumption level that solves the restricted problem for resource level y and the chosen optimal level of emission is also the consumption level that solves the unrestricted problem.

Remark 4.2: *Under Assumption 2.1, we have*

$$c^m(z,y) = \arg\max_c \{u(z,c) \mid \langle z,c\rangle \in Y(y)\} \quad \text{and} \quad \Big\langle\ z^{min}(y),\ c^m\left(z^{min}(y),\ y\right)\ \Big\rangle \in \varphi(y). \tag{4.1}$$

If endowment increases from $y' \geq 0$ to $y' > y$, the initial optimum $\mathbf{A} := \langle z^{min}(y), c^m(z^{min}(y),\ y)\rangle$ changes to a new optimum $\mathbf{C} := \langle z^{min}(y'), c^m(z^{min}(y'), y')\rangle$. This change can be split into (i) EHFE, which implies moving from $\mathbf{A}$ to outcome $\mathbf{B} := \langle z^{min}(y), c^m(z^{min}(y), y')\rangle$ of the restricted welfare maximisation Problem (4.1), where emission is held fixed at $z^{min}(y)$, while the endowment is held at the new level y' and (ii) DISPE, which leads outcome $\mathbf{B}$ of EHFE to the new optimum $\mathbf{C}$. See Figure 2.

Theorem 4.1, which is one of our main results and whose proof can be found in the appendix, provides a complete characterisation of an EKC. It states that the solution to Problem (2.3) exhibits an EKC if and only if there exists a threshold level of resource such that, at any level of resource below (respectively, above) the threshold, the outcome reached by the EHFE because of an increase in the resource is one where the set of preference-based shadow prices lies completely below (respectively, above) the set of technological shadow prices. From Remark 4.2, it follows that the set of preference-based shadow prices lies completely below (respectively, above) the set of technological shadow prices if and only if the negative of the normal cone of preferences lies completely above (respectively, below) the normal cone of the technology. In Panel (ii) of Figure 2, the normal cone of the technology lies below the negative of the normal cone of preferences; hence the set of preference-based shadow prices lies below the set of technological shadow prices.

Theorem 4.1: *Suppose Assumptions 2.1 and 2.2 hold; $Y(y)$ is convex for all $y \geq 0$; the function u is quasi-concave; and $\langle z, c\rangle \in \varphi(y)$ for $y > 0$ implies $z > 0$ and $c > 0$. Then φ exhibits an EKC if and only if there exists $\overset{*}{y} > 0$ such that (i) and (ii) below are true.*
(i) if $0 < y < y' < \overset{*}{y}$ then

$$\max\ S^P\left(\succeq(\boldsymbol{z}, \boldsymbol{c^m}), \langle \boldsymbol{z}, \boldsymbol{c^m}\rangle\right) < \min\ S^Y\left(Y(y'), \langle \boldsymbol{z}, \boldsymbol{c^m}\rangle\right) \tag{4.2}$$

(ii) if $y' > y > \overset{*}{y}$ then

$$\min\ S^P\left(\succeq(\boldsymbol{z}, \boldsymbol{c^m}), \langle \boldsymbol{z}, \boldsymbol{c^m}\rangle\right) > \max\ S^Y\left(Y(y'), \langle \boldsymbol{z}, \boldsymbol{c^m}\rangle\right). \tag{4.3}$$

where $\boldsymbol{z} := z^{min}(y)$ and $\boldsymbol{c^m} := c^m(z^{min}(y), y')$.

4.3. Necessary and sufficient conditions for EKC: the differentiable case.

In this section, we specialise Theorem 4.1 to the differentiable case for a further intuitive understanding of this result. In the differentiable case, (3.5) showed that the set of preference-based shadow price is a singleton, and the concepts of MWTP, MWTA, and the preference MRS coincide. The following corollary is a direct application of the comparative-static results in Theorem 4.1 to the differentiable case. It states that the optimal emission level will rise (respectively, fall) with an increase in the resource if and only if, at the outcome reached by the EHFE, the preference MRS is smaller (respectively, bigger) than the technological MRS. In other words, DISPE will imply a rise in emission at such an outcome if and only if the MWTA compensation for an increase in emission is smaller than the MR in production from an increase in emission, and DISPE will imply a fall in emission level if and only if the MWTP for a decrease in emission is bigger than the MAC. Moreover, the proof of this corollary shows that, because of the continuity of the MRSs in preferences and technology, corner solutions of Problem (2.3), which are characterised in Theorem 3.2, are a possibility only along the upward sloping part of an EKC.

Corollary of Theorem 4.1: *Suppose Assumptions 2.1 and 2.3 are true, u is continuously differentiable and quasi-concave, and $\langle z, c\rangle \in \varphi(y)$ for $y > 0$ implies $z > 0$ and $c > 0$. Then φ exhibits an EKC if and only if there exists $\overset{*}{y} > 0$ such that*

(i) if $0 < y < y' < \overset{*}{y}$ then

$$r(\boldsymbol{z}, \boldsymbol{c^m}) < s(\boldsymbol{z}, \boldsymbol{c^m}, y') \quad \textit{and} \quad r(\boldsymbol{z}, \boldsymbol{c}) \leq s(\boldsymbol{z}, \boldsymbol{c}, y) \tag{4.4}$$

(ii) if $y' > y \geq \overset{*}{y}$ then

$$r(\boldsymbol{z}, \boldsymbol{c^m}) > s(\boldsymbol{z}, \boldsymbol{c^m}, y') \quad \textit{and} \quad r(\boldsymbol{z}, \boldsymbol{c}) = s(\boldsymbol{z}, \boldsymbol{c}, y). \tag{4.5}$$

where $\boldsymbol{z} := z^{min}(y)$, $\boldsymbol{c} = c^m(\boldsymbol{z}, y)$, and $\boldsymbol{c^m} := c^m(\boldsymbol{z}, y')$.

Proof: For any $\bar{y} > 0$, since $F(z, c^m(z, \bar{y}), \bar{y}) = 0$, from the implicit function theorem, we have $\frac{\partial c^m(z,\bar{y})}{\partial z} = -\frac{F_z(z,c^m(z,\bar{y}),\bar{y})}{F_c(z,c^m(z,\bar{y}),\bar{y})} > 0$. Thus, c^m is increasing in z. Define $\hat{z}$ and z' such that they solve $F(\hat{z}, B(y), y) = 0$ and $F(z', B(y'), y') = 0$. Assumption 2.3 implies that $\hat{z} < z'$. Also, $c^m(\boldsymbol{z}, y) \leq B(y) \equiv c^m(\hat{z}, y)$. Hence, since c^m is increasing in z, we have $\boldsymbol{z} \leq \hat{z} < z'$. Hence, $c^m(\boldsymbol{z}, y') < c^m(z', y') \equiv B(y')$, *i.e.,* $\boldsymbol{c^m} < B(y')$. The first inequalities of (4.4) and (4.5) follow as direct applications of Theorem 4.1, (3.5), and (3.8). Suppose $0 < y < y' < \overset{*}{y}$ and $r(\boldsymbol{z}, \boldsymbol{c}) > s(\boldsymbol{z}, \boldsymbol{c}, y)$. Remark 4.2 implies $r(\boldsymbol{z}, \boldsymbol{c}) \equiv r(\boldsymbol{z}, c^m(\boldsymbol{z}, y))$ and $s(\boldsymbol{z}, \boldsymbol{c}, y) = s(\boldsymbol{z}, c^m(\boldsymbol{z}, y), y)$. Hence, the continuity of functions r and s imply that there exists $\delta > 0$ such that for all $y' \in (y, y+\delta)$, we have $r(\boldsymbol{z}, c^m(\boldsymbol{z}, y')) - s(\boldsymbol{z}, c^m(\boldsymbol{z}, y'), y') > 0$, which is a contradiction to the first inequality of (4.4). Hence, the second inequality of (4.4) is true. Suppose $y' > y \geq \overset{*}{y}$. Theorem 3.2 implies $r(\boldsymbol{z}, \boldsymbol{c}) \leq s(\boldsymbol{z}, \boldsymbol{c}, y)$. If $r(\boldsymbol{z}, \boldsymbol{c}) < s(\boldsymbol{z}, \boldsymbol{c}, y)$, then the continuity of functions r and s imply that there exists $\delta > 0$ such that for all $y' \in (y, y+\delta)$, we have $r(\boldsymbol{z}, c^m(\boldsymbol{z}, y')) - s(\boldsymbol{z}, c^m(\boldsymbol{z}, y'), y') < 0$, which is a contradiction to the first inequality of (4.5). Hence, the second equality of (4.5) is true. ■

The corollary, above, characterises an EKC in terms of how the optimal emission-consumption combination changes when the resource level *rises*; *e.g.,* from $y > 0$ to $y' > y$. Remark 4.3, below, does the same for the case when the resource level *falls*, say, from $y > 0$ to $y' < y$. If φ exhibits an EKC then, starting at an optimal corresponding to a level of resource that is smaller than the threshold $\overset{*}{y}$ in the definition of an EKC, a decrease in the resource implies that the EHFE takes us to an outcome where the MWTP for a decrease in emission is bigger than the MAC; hence, the DISPE will imply a reduction in the emission level. The argument for the case when the initial level of the resource is bigger than $\overset{*}{y}$ is similar.

Remark 4.3: *Suppose the assumptions of Corollary of Theorem 4.1 hold, φ exhibits an EKC, and $\overset{*}{y} > 0$ is such that (i) and (ii) in the definition of an EKC hold. Then*
(i) $0 < y' < y \leq \overset{*}{y}$ *and* $Y(\boldsymbol{z}, y') \neq \emptyset$ *implies*

$$r(\boldsymbol{z}, \boldsymbol{c^m}) > s(\boldsymbol{z}, \boldsymbol{c^m}, y'), \ \ \textit{and} \tag{4.6}$$

(ii) $y > y' > \overset{*}{y}$ *and* $Y(\boldsymbol{z}, y') \neq \emptyset$ *implies*

$$r(\boldsymbol{z}, \boldsymbol{c^m}) < s(\boldsymbol{z}, \boldsymbol{c^m}, y'). \tag{4.7}$$

where $\boldsymbol{z} := z^{min}(y)$ *and* $\boldsymbol{c^m} := c^m(\boldsymbol{z}, y')$.

5. EKC and the responses of shadow prices to changes in the resource level.

Employing Corollary to Theorem 4.1 and Remark 4.3, for the differentiable case, Theorem 5.1, below, provides a characterisation of an EKC in terms of the relative responsiveness of the preference-based and technological MRSs to changes in the level of the resource. As will be seen in the following sections, this characterisation is helpful for the identification of preference and feasible-set structures which can potentially yield EKCs and for putting into the context many existing cases of EKC seen in the literature.

Remark 5.1 *Suppose Assumption 2.3 is true. Since $F(z, c^m(z, y), y) = 0$, from the implicit function theorem, we have $\frac{\partial c^m(z,y)}{\partial y} = -\frac{F_y(z,c,y)}{F_c(z,c,y)} > 0$, where $c = c^m(z, y) > 0$.*

If u and F are twice continuously differentiable then the changes in the preference and technological MRSs due to a change Δy in the resource, assuming no change in the emission policy, are given, respectively, by

$$\begin{aligned} \Delta R(z, y, \Delta y) &:= r\left(z, c^m(z, y + \Delta y)\right) - r\left(z, c^m(z, y)\right) \quad \text{and} \\ \Delta S(z, y, \Delta y) &:= s\left(z, c^m(z, y + \Delta y), y + \Delta y\right) - s\left(z, c^m(z, y), y\right). \end{aligned} \tag{5.1}$$

Employing the chain rule of differentiation and Remark 5.1, we can define the responsiveness of preference-based and technological MRSs to a change in the resource, assuming no change in the emission policy, as the derivatives:

$$dR(z, y) := \lim_{\Delta y \longrightarrow 0} \frac{\Delta R(z, y, \Delta y)}{\Delta y} = \frac{\partial r(z, c)}{\partial c}\frac{\partial c^m(z, y)}{\partial y} = \frac{\partial r(z, c)}{\partial c}\left(-\frac{F_y(z, c, y)}{F_c(z, c, y)}\right) \text{ and} \tag{5.2}$$

$$dS(z,y) := \lim_{\Delta y \longrightarrow 0} \frac{\Delta S(z,y,\Delta y)}{\Delta y} = \frac{\partial s(z,c,y)}{\partial c}\frac{\partial c^m(z,y)}{\partial y} + \frac{\partial s(z,c,y)}{\partial y}$$
$$= \frac{\partial s(z,c,y)}{\partial c}\left(-\frac{F_y(z,c,y)}{F_c(z,c,y)}\right) + \frac{\partial s(z,c,y)}{\partial y}. \tag{5.3}$$

Theorem 5.1: *Suppose Assumptions 2.1 and 2.3 hold and $\langle z,c\rangle \in \varphi(y)$ for $y>0$ implies $z>0$ and $c>0$. Suppose u and F are twice continuously differentiable. If φ exhibits an EKC, then there exists $\overset{*}{y} > 0$ such that*

(i) for all y such that $0 < y < \overset{*}{y}$, we have

$$dR(\boldsymbol{z},y) \le dS(\boldsymbol{z},y) \qquad \text{if} \quad r(\boldsymbol{z},c^m(\boldsymbol{z},y)) = s(\boldsymbol{z},c^m(\boldsymbol{z},y),y) \text{ and}$$
$$r(\boldsymbol{z},c^m(\boldsymbol{z},y')) < s(\boldsymbol{z},c^m(\boldsymbol{z},y'),y') \ \forall\ y' \in [y,y+\epsilon] \text{ for some } \epsilon > 0$$
$$\text{if} \quad r(\boldsymbol{z},c^m(\boldsymbol{z},y)) < s(\boldsymbol{z},c^m(\boldsymbol{z},y),y), \tag{5.4}$$

(ii) for all y such that $y > \overset{*}{y}$, we have

$$dR(\boldsymbol{z},y)) \ge dS(\boldsymbol{z},y), \quad \text{and} \tag{5.5}$$

(iii) $y = \overset{*}{y}$ implies

$$dR(\boldsymbol{z},y) = dS(\boldsymbol{z},y), \tag{5.6}$$

where $\boldsymbol{z} := z^{min}(y)$. If there exists $\overset{*}{y} > 0$ such that (5.4) and (5.5) hold as strict inequalities, then φ exhibits an EKC.

Proof: **(i)** Suppose $r(\boldsymbol{z},c^m(\boldsymbol{z},y)) = s(\boldsymbol{z},c^m(\boldsymbol{z},y),y)$. From (5.1), it follows that the difference quotient is

$$\frac{\Delta R(\boldsymbol{z},y,\Delta y) - \Delta S(\boldsymbol{z},y,\Delta y)}{\Delta y} = \frac{r\left(\boldsymbol{z},c^m(\boldsymbol{z},y+\Delta y)\right) - s\left(\boldsymbol{z},c^m(\boldsymbol{z},y+\Delta y),y+\Delta y\right)}{\Delta y}.$$

From (4.4) it follows that its right limit is non-positive, *i.e.,* $\lim_{\Delta y \longrightarrow 0+} \frac{\Delta R - \Delta S}{\Delta y} \le 0$. Similarly, (4.6) implies that its left limit is also non-positive, *i.e.,* $\lim_{\Delta y \longrightarrow 0-} \frac{\Delta R - \Delta S}{\Delta y} \le 0$. Hence, $dR(\boldsymbol{z},y) - dS(\boldsymbol{z},y) \le 0$. The continuity of $r(\boldsymbol{z},c^m(\boldsymbol{z},\hat{y})) - s(\boldsymbol{z},c^m(\boldsymbol{z},\hat{y}),\hat{y})$ in $\hat{y}$ explains the second part of (i), where $r(\boldsymbol{z},c^m(\boldsymbol{z},y)) - s(\boldsymbol{z},c^m(\boldsymbol{z},y),y) < 0$.

Part **(ii)** can be proved in a manner similar to part (i) by employing (4.5) to find the right limit and (4.7) to find the left limit of the difference quotient. Part **(iii)** can be proved by employing (4.4) to find the left limit of the difference quotient and (4.5) to find the right limit. ■

The intuition is as follows. At an interior optimum, Theorem 3.2 showed that the preference and technological MRS are equal. Hence, starting from an initial *interior* optimum for a resource level that lies to the left of $\overset{*}{y}$, $dR - dS \leq 0$ implies that the EHFE associated with an increase in endowment yields an outcome where MWTA compensation is smaller than the MR from an increase in emission, so that DISPE recommends an increase in emission. The opposite is true for a resource level that lies to the right of $\overset{*}{y}$. At the threshold $\overset{*}{y}$, this difference is zero.

6. Homotheticity and EKC.

It is often claimed that homotheticity of both preferences and the technology precludes an EKC.[16] Here we show that, depending on the way preferences are modelled, homotheticity could permit an EKC.

6.1. Homothetic preferences.

Sections 6.1.1 and 6.1.2 show two ways of modelling homotheticity of preferences. In the first case, we show that r is monotonically increasing in consumption. Hence, Remark 5.1 and (5.2) imply that $dR > 0$ in this case. In the second case, r is non-monotonic in consumption – in particular, there is a region in the consumption space where r is increasing in consumption and another region where it is decreasing in consumption.

6.1.1. Homothetic preferences over environmental quality and consumption.

Plassman and Khanna [2006], among others, assume that there is a bound on the maximum feasible environmental quality ω, and the observed environmental quality q deteriorates with increase in emission: $q = \omega - z$. Preferences over environmental quality and consumption are defined by a utility function $\bar{u} : [0, \omega] \times \mathbf{R}_+ \longrightarrow \mathbf{R}$ with image $\bar{u}(q, c)$. Given $\bar{u}$, we can derive the preferences over emission and consumption, which are, here, represented by function u : $[0, \omega] \times \mathbf{R}_+ \longrightarrow \mathbf{R}$ defined as

$$u(z, c) := \bar{u}(\omega - z, c). \tag{6.1}$$

If $\bar{u}$ is continuously differentiable and increasing in q and c, then u is continuously differentiable and satisfies Assumption 2.1. The MRS between environmental quality and consumption is

$$r^{\bar{u}}(q, c) := \frac{\bar{u}_q(q, c)}{\bar{u}_c(q, c)} = r(z, c) > 0. \tag{6.2}$$

Panels (i) and (ii) of Figure 3 plot indifference curves of function $\bar{u}$ and function u derived from $\bar{u}$. If $\bar{u}$ is twice continuously differentiable and strictly quasi-concave, then the determinant of

[16] See, *e.g.,* Lopez [1994].

the bordered Hessian of function $\bar{u}$, evaluated at $\langle q, c\rangle$ and denoted by $Q^{\bar{u}}(q, c)$, is positive. It can be shown that

$$Q^{\bar{u}}(q,c) = \bar{u}_c^2(q,c)\left[\bar{u}_q(q,c)\frac{\partial\frac{\bar{u}_q(q,c)}{\bar{u}_c(q,c)}}{\partial c} - \bar{u}_c(q,c)\frac{\partial\frac{\bar{u}_q(q,c)}{\bar{u}_c(q,c)}}{\partial q}\right]. \tag{6.3}$$

Note that, if $\bar{u}$ is homothetic in q and c, then u is *not* homothetic in z and c.[17] We demonstrate below that, if $\bar{u}$ is homothetic, then the preference MRS, r, is increasing in consumption.

Define a ratio function $\rho : \mathbf{R}_{++} \times \mathbf{R}_{+} \longrightarrow \mathbf{R}$ with image $\rho(q, c) = \frac{c}{q}$. Starting from an environmental quality-consumption combination $\langle q, c\rangle \in \mathbf{R}^2_{++}$, consider a direction of change (doc), which increases the consumption good but leaves environmental quality unchanged, namely, $\langle dq_1, dc_1\rangle = \langle 0, 1\rangle$. The effect of such a doc on the ratio function is

$$\rho_q(q,c)dq_1 + \rho_c(q,c)dc_1 = \frac{1}{q}. \tag{6.4}$$

Suppose $\langle dq_2, dc_2\rangle \in \mathbf{R}^2$ is a direction of change that leaves utility unchanged. Then

$$\bar{u}_q(q,c)dq_2 + \bar{u}_c(q,c)dc_2 = 0 \implies dc_2 = -\frac{\bar{u}_q(q,c)}{\bar{u}_c(q,c)}dq_2. \tag{6.5}$$

In addition, suppose the doc $\langle dq_2, dc_2\rangle$ also results in the same change in the ratio as induced by $\langle dq_1, dc_1\rangle$. This implies

$$\frac{1}{q} = \left(-\rho_c(q,c)\frac{\bar{u}_q(q,c)}{\bar{u}_c(q,c)} + \rho_q(q,c)\right)dq_2 \implies dq_2 = \frac{\bar{u}_c(q,c)}{q\left(-\rho_c(q,c)\bar{u}_q(q,c) + \rho_q(q,c)\bar{u}_c(q,c)\right)} < 0. \tag{6.6}$$

Since $\langle dq_1, dc_1\rangle$ and $\langle dq_2, dc_2\rangle$ lead to the same change in the ratio, homotheticity of preferences implies that they must lead to the same change in the MRS between environmental quality and consumption.

$$\frac{\partial r^{\bar{u}}(q,c)}{\partial q}dq_1 + \frac{\partial r^{\bar{u}}(q,c)}{\partial c}dc_1 = \frac{\partial r^{\bar{u}}(q,c)}{\partial q}dq_2 + \frac{\partial r^{\bar{u}}(q,c)}{\partial c}dc_2. \tag{6.7}$$

Hence, (6.3), (6.5), (6.6), and (6.7) imply

$$\frac{\partial r^{\bar{u}}(q,c)}{\partial c} = \frac{1}{\bar{u}_c(q,c)}\left(\bar{u}_c(q,c)\frac{\partial r^{\bar{u}}(q,c)}{\partial q} - \bar{u}_q(q,c)\frac{\partial r^{\bar{u}}(q,c)}{\partial c}\right)dq_2 = \frac{-Q^{\bar{u}}}{\bar{u}_c^3(q,c)}dq_2 > 0. \tag{6.8}$$

The intuition can be seen in Panel (i) of Figure 3. Because of homotheticity, the MRS between environmental quality and consumption is the same at points B and C which lie on the same ray through the origin. Hence, the MRS increases as we move from point A to point B by holding the environmental quality fixed and increasing consumption. Thus, Lemma 6.1, which follows from (6.8), (6.6), and (6.2), states that, if $\bar{u}$ is homothetic, then the preference MRS between emission and consumption r is increasing in consumption.

[17] Rather, u is homothetic in $\omega - z$ and c.

Lemma 6.1: *Suppose $\bar{u}$ is twice continuous differentiable, strictly quasi-concave, and increasing and homothetic in its arguments. Suppose u is derived from $\bar{u}$ as in (6.1). Then, for all $\langle z, c\rangle \in \mathbf{R}^2_{++}$ such that $0 < z < \omega$ and $q = \omega - z$, we have*

$$\frac{\partial r(z,c)}{\partial c} = -\left(\frac{1}{q\left(-\rho_c(q,c)\bar{u}_q(q,c) + \rho_q(q,c)\bar{u}_c(q,c)\right)}\right)\frac{Q^{\bar{u}}(q,c)}{\bar{u}^2_c(q,c)} > 0.$$

If, in addition, Assumption 2.3 is true, $\langle z, c\rangle \in Y(y)$ for $y > 0$, and $F(z,c,y) = 0$, then $dR(z,y) = \frac{\partial r(z,c)}{\partial c}\left(-\frac{F_y(z,c,y)}{F_c(z,c,y)}\right) > 0$.

6.1.2. Homothetic preferences over emission and consumption.

Lopez [1994], on the other hand, considers the case where preferences defined over emission and consumption are homothetic, *i.e.*, function u is homothetic.[18] To illustrate how the preference MRS r varies with level of consumption, we consider an example of such preferences.

Example 3. Suppose preferences are represented by a twice continuously differentiable utility function that satisfies Assumption 2.1.

$$u(z,c) = c - \frac{z^2}{c} \qquad \forall \langle z, c\rangle \in \mathbf{R}^2_{++}. \tag{6.9}$$

This implies that preferences are homothetic in consumption and emission. The contours of this utility function can be seen in Panel (iv) of Figure 3. For this utility function, it can be verified that

$$r(z,c) = \frac{2cz}{c^2 + z^2} \qquad \text{and}$$

$$\frac{\partial r(z,c)}{\partial c} = \frac{-2c^2 z + 2z^3}{\left(c^2+z^2\right)^2} \implies \frac{\partial r(z,c)}{\partial c} \le 0 \iff z \le c.$$

Thus, the preference MRS r is increasing in consumption when $c < z$ and is decreasing in consumption when $c > z$. Remark 5.1 and (5.2) will, hence, imply that dR will be non-positive if and only if $c \ge z$.

Remark 6.1 states that the qualitative properties of preferences seen in Example 3 are true for all homothetic preferences defined in the space of emission and consumption.

Remark 6.1: *Suppose u is twice continuously differentiable and homothetic and there exists $\langle \bar{z}, \bar{c}\rangle \in \mathbf{R}^2_{++}$ such that $r(\bar{z},\bar{c}) = \frac{\bar{c}}{\bar{z}} =: \bar{\rho}$.*[19] *Then*

(i) $u(z,c) = u(\bar{z},\bar{c})$ *for all* $\langle z,c\rangle \in \mathbf{R}^2_{++}$ *such that* $\frac{c}{z} = \bar{\rho}$

(ii) $r(z,c) > \frac{c}{z}$ *and* $\frac{\partial r(z,c)}{\partial c} > 0$ *for all* $\langle z,c\rangle \in \mathbf{R}^2_{++}$ *such that* $\frac{c}{z} < \bar{\rho}$

(iii) $r(z,c) < \frac{c}{z}$ *and* $\frac{\partial r(z,c)}{\partial c} < 0$ *for all* $\langle z,c\rangle \in \mathbf{R}^2_{++}$ *such that* $\frac{c}{z} > \bar{\rho}$

(iv) $r(z,c) = \frac{c}{z}$ *and* $\frac{\partial r(z,c)}{\partial c} = 0$ *for all* $\langle z,c\rangle \in \mathbf{R}^2_{++}$ *such that* $\frac{c}{z} = \bar{\rho}$

---

[18] Note, if we use u to derive $\bar{u}$ that defines preferences over environmental quality and consumption as in Plassman and Khanna [2006], then $\bar{u}$ will not be homothetic if u is homothetic.

[19] In Example 3 above $\bar{\rho} = 1$.

6.2. Homothetic technology.

To describe a homothetic technology, *i.e.,* one where the feasible sets corresponding to different levels of endowment are radial translates of one another in the space of emission and consumption, suppose there exists a function $\mathcal{Y} : \mathbf{R}^2_{++} \longrightarrow \mathbf{R}_+$ with image $\mathcal{Y}(z,c)$ such that

$$F(z,c,y) = 0 \iff y = \mathcal{Y}(z,c). \tag{6.10}$$

Applying the implicit function theorem, we have $s\,(z,c,\mathcal{Y}(z,c)) = -\frac{\mathcal{Y}_z(z,c)}{\mathcal{Y}_c(z,c)}$. We will say that the technology is homothetic if function $\mathcal{Y}$ is homothetic. Employing this definition of a homothetic technology, Lemma 6.2, below, follows:

Lemma 6.2: *Suppose Assumption 2.3 is true, F is twice continuously differentiable, and function $\mathcal{Y}$ is defined as in (6.10). If $\mathcal{Y}$ is homothetic then, for any $\langle z,c,y\rangle \in \mathbf{R}^3_{++}$ such that $y = \mathcal{Y}(z,c)$ ($\iff$ $c = c^m(z,y)$), we have $dS(z,y) \geq 0$.*

This lemma can be proved exactly along the lines of arguments in Section 6.1. The intuition is demonstrated in Panel (iii) of Figure 3 where, due to homotheticity, the slope at point B is the same as the slope at point C. Hence, with a convex technology, as we move from point A to point B due to an increase in the endowment with no change in the emission level, the slope of the technology, *i.e,* the MAC increases.

6.3. Homotheticity and EKC.

Suppose preferences are homothetic in the space of emission and consumption and the technology is convex and homothetic. Then, $\langle z,c\rangle \in \varphi(y)$ implies that $\frac{c}{z} \geq r(z,c)$.[20] Hence, Remarks 5.1 and 6.1 and (5.2) imply that $dR(z,y) \leq 0$. Lemma 6.2 implies that $dS(z,y) > 0$. Hence, $dR(z,c) - dS(z,c) \leq 0$ whenever $\langle z,c\rangle \in \varphi(y)$. Theorem 5.1, hence, rules out an EKC in this case. Specifically, since the EHFE of an increase in endowment results always in outcomes where the MWTP is no bigger than the MAC, the DISPE will always imply that the optimal emission level is non-decreasing in the level of the endowment.

Now suppose assumptions of Lemmas 6.1 and 6.2 hold. In particular, this means that the preferences are homothetic in environmental quality and consumption. This implies that function $\bar{u}$ is homothetic. Lemmas 6.1 and 6.2 imply that we cannot ambiguously sign the difference $dR - dS$ at the optimum $\varphi(y)$ for any $y > 0$, as both dR and dS are positive. This

[20] This is because if $\langle z,c\rangle$ is such that $\frac{c}{z} < r(z,c)$, then convexity of the technology (or the concavity of the frontier of $Y(y)$) will imply that the slope of the technology at $\langle z,c\rangle$ satisfies $s(z,c,y) \leq \frac{c}{z} < r(z,c)$, which contradicts the conclusions of Theorem 1.

indicates that, perhaps, for some configuration of values of parameters an EKC could arise. In the example below, which is a continuation of Example 1 in Section 2.4, we show that there is a threshold level of resource, such that at any optimum corresponding to a resource level that lies to the left (respectively, right) of the threshold, $dR - dS$ is negative (respectively, positive). Hence, from Theorem 5.1, it follows that this example generates an EKC.

Example 1 continued. Suppose preferences are homothetic in environmental quality and consumption:

$$u(z,c) \equiv \left((\omega - z)^{\sigma} + c^{\sigma}\right)^{\frac{1}{\sigma}}, \quad z < \omega, \quad \sigma < 1, \sigma \neq 0.$$

For the parameter values $\omega = 3$ and $\sigma = .3$, the contours of this preference structure in the environmental quality-consumption space and emission-consumption space are seen in Panels (i) and (ii) of Figure 3. The contours of the technology were shown in Panels (i) to (iii) of Figure 1. The signs of dR and dS will be as suggested by Lemmas 6.1 and 6.3.

Given the parameter values above, Problem (2.3) yields interior solutions. Define functions $\hat{z} : \mathbf{R}_+ \longrightarrow \mathbf{R}_+$ and $\hat{c} : \mathbf{R}_+ \longrightarrow \mathbf{R}_+$ such that $\langle \hat{z}(y), \hat{c}(y)\rangle = \varphi(y)$. The blue curve in Panel (vi) of Figure 3 plots the graph of $\hat{z}$, which, from Theorem 1, is the set $\left\{\langle z, y\rangle \in \mathbf{R}^2_{++} \middle|\ r(z,y) - s(z, c^m(z,y), y) = 0\right\}$. Clearly, it exhibits an EKC. Also define a function $\Delta : \mathbf{R}^2_{++} \longrightarrow \mathbf{R}_+$ with image $\Delta(z,y) = dR(z,y) - dS(z,y)$. The red curve in Panel (vi) of Figure 3 plots the set $\left\{\langle z, y\rangle \in \mathbf{R}^2_{++} \middle|\ \Delta(z,y) = 0\right\}$. Panel (v) of Figure 3 plots the contours of function Δ. From this plot, it follows that, on all points above (respectively, below) the red curve in Panel (vi), Δ takes is negative (respectively, positive). As seen in Panel (vi), the rising (respectively, falling) part of the graph of $\hat{z}$ corresponds to Δ being negative (respectively, positive). This is as predicted by Theorem 5.1.

7. EKC as a primarily technological phenomenon.

Here, we continue with Example 2 in Section 2.4, which is due to Andreoni and Levinson [2001]. They argue that φ can exhibit an EKC if the abatement technology exhibits increasing returns in both consumption and the resource allocated towards abatement.

Assumption 7.1: Function A is increasing, twice continuously differentiable, homogeneous of degree $L > 1$, and $A(c, 0,) = A(0, a) = 0$. The function $\bar{A}$ is strictly concave in c for any given $y \in \mathbf{R}_+$.

Under Assumption 7.1, we show that there is a subset of technologically feasible combinations of emission, endowment, and consumption levels where dS is positive and there is also a feasible subset of such combinations where dS is negative. Theorem 5.1 then implies that an EKC can arise if, in the former subset, dR is non-positive, and dR is non-negative in the latter subset. In particular, if preferences are linear (as in the numerical examples of Andreoni and Levinson), then $dR = 0$ and the resulting EKC is fully attributed to a technological factor, namely, increasing returns to abatement.

If $\langle z, c, y\rangle \in \mathbf{R}^3_{++}$ is such that $0 < c < y$ and $F(z, c, y) = 0$ ($\iff$ $c = c^m(z, y)$), then (2.5) and (2.6) imply

$$s(z, c, y) = -\frac{F_z(z, c, y)}{F_c(z, c, y)} = \frac{1}{1 - \bar{A}_c(c, y)}.$$

Hence, Remark 5.1 and (5.3) imply that the total effect of an increase in the resource on the MAC when emission is held fixed can be split into (i) an indirect effect, where MAC changes due to a change in consumption following the increase in the resource and (ii) a direct effect of a change in the resource on the MAC. Thus,

$$dS(z, y) = \frac{\partial s(z, c, y)}{\partial c}\frac{\partial c^m(z, y)}{\partial y} + \frac{\partial s(z, c, y)}{\partial y} = \frac{\partial \frac{1}{1-\bar{A}_c(c,y)}}{\partial c}\left(\frac{\bar{A}_y(c, y)}{1 - \bar{A}_c(c, y)}\right) + \frac{\partial \frac{1}{1-\bar{A}_c(c,y)}}{\partial y}, \quad (7.1)$$

where (suppressing arguments of functions) we have

$$\frac{\partial s}{\partial c}\frac{\partial c^m}{\partial y} = \frac{\partial \frac{1}{1-\bar{A}_c}}{\partial c}\frac{\bar{A}_y}{1 - \bar{A}_c} = \frac{\bar{A}_y \bar{A}_{cc}}{(1 - \bar{A}_c)^3} \quad \text{and} \quad \frac{\partial s}{\partial y} = \frac{\partial \frac{1}{1-\bar{A}_c}}{\partial y} = \frac{\bar{A}_{cy}}{(1 - \bar{A}_c)^2}. \quad (7.2)$$

Note, if $z = c - \bar{A}(c, y) > 0$ then $c > \bar{A}(c, y)$. Combined with strict concavity of $\bar{A}$ in c, this implies[21]

$$1 > \frac{\bar{A}(c, y)}{c} > \bar{A}_c(c, y) \implies 1 - \bar{A}_c(c, y) > 0. \quad (7.3)$$

To study the sign of dS, we first sign the indirect effect of the increase in resource, $\frac{\partial s}{\partial c}\frac{\partial c^m}{\partial y}$. Noting that $\bar{A}_{cc} < 0$ as $\bar{A}$ is strictly concave in c, (7.2) and (7.3) imply $\frac{\partial s}{\partial c}\frac{\partial c^m}{\partial y} < 0$.

From (7.2) and (7.3), it follows that the sign of the direct effect of an increase in resource on the MAC, $\frac{\partial s}{\partial y}$, depends on the sign of $\bar{A}_{cy}$. Assumption 7.1 implies that there exists a strictly concave function $\Psi : \mathbf{R}_+ \longrightarrow \mathbf{R}_+$ such that

$$A(c, y - c) = \bar{A}(c, y) = y^L \Psi(\frac{c}{y}) \quad \text{and} \quad \bar{A}(y, 0) = \bar{A}(0, y) = 0. \quad (7.4)$$

Hence, $\Psi'' < 0$ and $\Psi(1) = \Psi(0) = 0$. Hence, there exists $\hat{c} > 0$ such that $\Psi'(\frac{\hat{c}}{y}) = 0$ and $\Psi'(\frac{c'}{y}) > 0$ if and only if $c' \leq \hat{c}$. (7.4) also implies that

$$\bar{A}_{cy}(c, y) = y^{L-2}\left[(L - 1)\Psi'(\frac{c}{y}) - \frac{c}{y}\Psi''(\frac{c}{y})\right].$$

[21] Strict concavity of $\bar{A}$ in c implies that average abatement is bigger than marginal abatement.

Thus, the nature of the returns to scale of the abatement technology affects dS mainly by affecting $\bar{A}_{cy}$, which is the response of $\bar{A}_c$ to an increase in the endowment. When it exhibits increasing returns, we have $L > 1$, and $\Psi' > 0$ implies $\bar{A}_{cy} > 0$, while $\bar{A}_{cy} < 0$ implies that $\Psi' < 0$.

Thus, the signs of the direct and indirect effects imply that the sign of dS is ambiguous. Employing (7.1) and (7.4) it can be shown that

$$dS = \frac{y^{L-2}\left((L-1)\Psi'\left[1 - y^{L-1}\Psi'\right] - \frac{\Psi'' L}{y}\left[\frac{c}{L} - y^L\Psi\right]\right)}{(1 - y^{L-1}\Psi')^3}. \tag{7.5}$$

Since $1 - \bar{A}_c = 1 - y^{L-1}\Psi'$, if net emission level is positive then (7.3) implies that $1 - y^{L-1}\Psi' > 0$. Hence, (7.5) implies

$$dS \geq 0 \iff (L-1)\Psi'\left[1 - y^{L-1}\Psi'\right] - \frac{\Psi'' L}{y}\left[\frac{c}{L} - y^L\Psi\right] \geq 0. \tag{7.6}$$

Under Assumption 7.1, Theorem 7.1 identifies a set of combinations $\langle z, c, y\rangle \in \mathbf{R}^3_{++}$ which satisfy $\Psi' \geq 0$ and $\frac{c}{L} - y^L\Psi \geq 0$ so that $dS \geq 0$. In this region, the endowment levels are bounded from above. It also identifies a set of combinations $\langle z, c, y\rangle \in \mathbf{R}^3_{++}$ which satisfy $\Psi' < 0$ and $\frac{c}{L} - y^L\Psi < 0$ so that $dS < 0$. In this region endowment levels are bounded from below.

Theorem 7.1: *Suppose Assumption 2.3 holds and there exists a mapping $g : \mathbf{R}^2_+ \longrightarrow \mathbf{R}_+$ which satisfies (2.5) and (2.6). If Assumption 8.1 is true then there exist subsets $\bar{\Gamma} \subset \mathbf{R}^3_{++}$ and $\underline{\Gamma} \subset \mathbf{R}^3_{++}$ and $\bar{y} > 0$ and $\underline{y} > 0$ such that*

$$\begin{aligned} dS(z,y) \geq\ & 0, \quad 0 < y \leq \bar{y}, \quad c = c^m(z,y) \quad \forall\ \langle z,c,y\rangle \in \bar{\Gamma} \\ dS(z,y) <\ & 0, \quad y > \underline{y}, \qquad\ \ c = c^m(z,y) \quad \forall\ \langle z,c,y\rangle \in \underline{\Gamma} \end{aligned}$$

See the appendix for the proof of this theorem. To illustrate this theorem, consider the example in Andreoni and Levinson. Preferences are assumed to be linear in consumption and emission, hence $dR = 0$. The abatement technology exhibits increasing returns with $L = 2$ and

$$A(c, y-c) \equiv \bar{A}(c,y) = c(y-c) = y^2\Psi\left(\frac{c}{y}\right), \text{ where } \Psi\left(\frac{c}{y}\right) = \frac{c}{y} - \left(\frac{c}{y}\right)^2.$$

The contours of this technology in the emission-consumption space are shown in Panels (iv) to (vi) of Figure 1. In this case, $\bar{A}_{cy}(c,y) = 1$ and $\bar{A}_{cc}(c,y) = -2$.

$$s(z,c,y) = \frac{1}{1-(y-2c)} \quad\text{and}\quad dS(z,y) = \frac{1-y}{(1-y+2c)^3}.$$

It can be verified that $z > 0$ implies $1 - y + 2c > 0$. Hence, $dS \geq 0$ if and only if $y \leq 1$. Thus, in this example:

$$\bar{\Gamma} = \left\{\langle z, c, y\rangle \in \mathbf{R}^3_{++} \;\middle|\; y \leq 1 \text{ and } z = c - \bar{A}(c, y)\right\} \text{ and } \underline{\Gamma} = \left\{\langle z, c, y\rangle \in \mathbf{R}^3_{++} \;\middle|\; y > 1 \text{ and } z = c - \bar{A}(c, y)\right\}.$$

Given linear preferences, we have $dR - dS = -dS$. The contours of $dR - dS$ are drawn in Panel (i) of Figure 4. The red curve in Panel (ii) of this figure shows all points where $dR - dS = 0$, while the blue curve plots the optimal emission level as a function of the endowment. The blue curve shows that this example exhibits an EKC with the threshold level of endowment $\overset{*}{y} = 1$. To the left (respectively, right) of the red curve, $dR - dS < 0$ (respectively, $dR - dS > 0$) and hence the blue curve is rising (respectively, falling) in this region. This is in tune with the conclusions of Theorem 5.1.

8. EKC as a primarily preference-based phenomenon.

Here we explore the plausibility of a preference structure for which there exists a region in the consumption space where dR is negative and a region where it is non-negative such that, if there exists a technology for which dS is non-negative in the former region and non-positive in the latter (*e.g.,* if the technology was linear), we obtain an EKC.

Lemma 8.1, below, which is proved in the appendix, links the sign of $\frac{\partial r}{\partial c}$, which is the responsiveness of the MWTP to changes in consumption, to the income effect on a budget-constrained utility maximising choice of emission. Consider a hypothetical utility maximisation exercise, where a consumer *receives* a price $-p_z > 0$ for bearing the emission. Suppose the consumption good is the numeraire, so its price is normalized to be one. Suppose the consumer's income is denoted by $m \in \mathbf{R}_{++}$. Consumer's utility maximization problem is defined as

$$\max_{z,c} \{u(z, c) \mid p_z z + c \leq m\}, \tag{8.1}$$

and the resulting Marshallian demands are $\langle z^M(p_z, m),\ c^M(p_z, m)\rangle$.

Lemma 8.1: *Suppose Assumption 2.1 holds and u is twice continuously differentiable and strictly quasi-concave. If $\langle z, c\rangle \in \mathbf{R}^2_{++}$ solves (8.1) for some $\langle p_z, m\rangle \in \mathbf{R}_{--} \times \mathbf{R}_{++}$ then the following is true:*

$$\frac{\partial z^M(p_z, m)}{\partial m} = -\frac{u_c^3}{Q^u(z, c)}\frac{\partial r(z, c)}{\partial c} \quad \textit{and} \quad \frac{\partial c^M}{\partial m} = \frac{u_c^3}{Q^u(z, c)}\frac{\partial r(z, c)}{\partial z}.$$

Definition: Given an emission-consumption combination, $\langle z, c\rangle \in \mathbf{R}^2_{++}$, we say that emission exhibits inferiority (respectively, normality) at $\langle z, c\rangle$ if $\frac{\partial r(z,c)}{\partial c} \geq 0$ (respectively, $\frac{\partial r(z,c)}{\partial c} < 0$). Consumption exhibits normality at $\langle z, c\rangle$ if $\frac{\partial r(z,c)}{\partial z} \geq 0$.

Remark 8.1 shows that these definitions of inferiority and normality of goods coincides with their traditional definitions in terms of the signs of the income effects.

Remark 8.1: *Suppose the assumptions of Lemma 8.1 hold. If $\langle z, c\rangle \in \mathbf{R}^2_{++}$ solves (8.1) for some $\langle p_z, m\rangle \in \mathbf{R}_{--} \times \mathbf{R}_{++}$, then the following are true:*

(i) If emission exhibits inferiority at $\langle z, c\rangle$ then $\frac{\partial z^M(p_z,m)}{\partial m} \leq 0$ and if it exhibits normality at $\langle z, c\rangle$ then $\frac{\partial z^M(p_z,m)}{\partial m} > 0$

(ii) If consumption exhibits normality at $\langle z, c\rangle$ then $\frac{\partial y^M(p_z,m)}{\partial m} \geq 0$.

Figure 5 demonstrates the link between income effects on emission and the responsiveness of MWTP to changes in consumption good level. In Panel (i), the income consumption curve (ICC) or the locus of points with equal MWTP passing through the point $\langle z,' c'\rangle$ is positively sloped. It is clear that this implies $\frac{\partial r(z',c')}{\partial c} < 0$, *i.e.*, at this point the MWTP is decreasing in the level of consumption (holding the emission level fixed at z'). Thus, emission exhibits normality at $\langle z', c'\rangle$, both in terms of our definition and the conventional definition. Note, this is true despite emission being a 'bad' good. In Panel (ii), emission is an inferior good at $\langle z', c'\rangle$ as the ICC passing through this point is non-positively sloped, while $\frac{\partial r(z',c')}{\partial c} > 0$.

Usually, it is common in the literature to assume that, globally, consumption is a normal good, while emission is an inferior good. We hope that our discussion in Section 6.1.2, Remark 8.1, and Figure 5 have convinced the reader that normality of emission is also a plausible phenomenon. Suppose preferences are such that they exhibit both normality and inferiority of emission, albeit in different regions of the consumption space. In particular, suppose the region where they exhibit normality is freely disposable and bounded in consumption.

Assumption 8.1: u is such that there exists an open set $\mathcal{M} \subset \mathbf{R}^2_{++}$ such that

(i) emission exhibits normality at all $\langle z, c\rangle \in \mathcal{M}$ and

(ii) emission exhibits inferiority at all $\langle z, c\rangle \in \mathbf{R}^2_{++}$ such that $\langle z, c\rangle \notin \mathcal{M}$.

(iii) $\langle z, c\rangle \in \mathcal{M}$ and $0 \leq \bar{c} \leq c$ implies $\langle z, \bar{c}\rangle \in \mathcal{M}$, and

(iii) there exists c^b such that $c \leq c^b$ for all $\langle c, z\rangle \in \mathcal{M}$.

The implication of Assumption 8.1 is clear: In the region $\mathcal{M}$, ICCs will be positively sloped, while they will be non-positively sloped elsewhere.

Example 4. Suppose Assumption 8.1 holds and $\mathcal{M} = \{\langle z, c\rangle \in \mathbf{R}^2_{+} |\ c < 0.5\}$. Suppose u is such that the locus of emission-consumption combinations where the MWTP is held fixed, say at 0.033, is the set $ICC(0.033)$ given by

$$ICC(0.033) := \{\langle c, z\rangle \in \mathbf{R}^2_+ \mid z = -c^2 + c + 1 \text{ and } c \in [0, 1.61803]\}. \tag{8.2}$$

As seen in Panel (iii) of Figure 5, the ICC is backward bending. In the upward sloping part of the ICC, emission exhibits normality, while in the backward bending part it exhibits inferiority. The consumption good is globally normal. Although Figure 5 indicates that a backward bending ICC is not pathological, it is analytically difficult to write down functional forms for well-behaved utility functions that exhibit such a property.[22] If, in addition, the technology is linear with $s(z, c, y) = .033$ for all $y \geq 0$ and $c = c^m(z, y)$, then $dS(z, c) = 0$. At the same time, this implies that $dR(z, c) < 0$ if $\langle z, c\rangle \in \mathcal{M}$ and $dR(z, c) \geq 0$ if $\langle z, c\rangle \notin \mathcal{M}$. The locus traced by φ for different levels of endowment can be inferred from $ICC(.033)$. Conclusions of Theorem 5.1 hold and we can infer that φ exhibits an EKC.

9. EKC as an outcome of economic growth: A few remarks.

In the growth model below, capital stock is denoted by $k \geq 0$, $\rho \geq 0$ is the social rate of time preference, and $\delta \geq 0$ is the depreciation rate of capital.[23] The total output produced by the economy at any time point is allocated between investment in capital and a resource, denoted by y, available for consumption c and abatement a. Net emission z depends on the amounts of consumption and abatement.

$$\begin{gathered}\max \int_0^\infty e^{-\rho t} u(z(t), c(t)) dt \\ \text{subject to} \\ \dot{k}(t) = f(k(t)) - \delta k(t) - y(t), \quad y(t) = c(t) + a(t), \quad z(t) = \bar{g}(c(t), a(t)), \\ k(t) \geq 0,\ c(t) \geq 0,\ a(t) \geq 0,\ z(t) \geq 0, k(0) = k_0.\end{gathered} \tag{9.1}$$

Given $k(0) = k_0$, let the optimal trajectories of the state and control variables be denoted by $k^o(t, k_0)$, $z^o(t, k_0)$, $c^o(t, k_0)$, and $a^o(t, k_0)$. The optimal trajectory of resource available for consumption and abatement is given by $y^o(t, k_0) := f(k^o(t, k_0)) - \delta k^o(t, k_0) - \dot{k}^o(t, k_0)$. It is clear that $y^o(t, k_0) = c^o(t, k_0) + a^o(t, k_0)$. Lemma 9.1, which is proved in the appendix, shows that, given the amount of resource that is available to be distributed between consumption and abatement at any time point t along the optimal trajectory, the optimal consumption and emission levels at time point t also solve the static optimisation problem (2.3).

[22] But we can construct such utility functions by splicing two or more functional forms. Generally, indifference curves of such utility functions will be kinked.

[23] The model permits both exogenous and endogenous growth.

Lemma 9.1: *Suppose $g(c, y) := \bar{g}(c, y - c)$ for all $y \geq 0$ and $c \geq 0$. For any $t \geq 0$ and $k_0 \geq 0$, $\langle z^o(t, k_0), c^o(t, k_0)\rangle$ solves the static optimisation problem (2.3) with $Y(y) = \{\langle z, c\rangle \in \mathbf{R}^2_+ \mid z \geq g(c, y)$ and $c \leq y\}$ and $y = y^o(t, k_0)$.*

Theorem 9.1 follows in a straightforward way. It states a set of conditions under which the optimal trajectory of emission has an inverted-U shape.

Theorem 9.1: *Suppose mapping φ that solves Problem (2.3) exhibits an EKC for utility function u and mapping Y with image $Y(y) = \{\langle z, c\rangle \in \mathbf{R}^2_+ \mid z \geq \bar{g}(c, y - c),\ c \leq y\}$ for all $y \geq 0$. Suppose trajectories k^o and y^o are continuous, increasing, and unbounded functions of time and k_0 is such that $y^o(0, k_0) < \overset{*}{y}$, where $\overset{*}{y} > 0$ satisfies (i) and (ii) in the definition of an EKC. Then there exists $\overset{*}{t} > 0$ such that $y^o(\overset{*}{t}, k_0) = \overset{*}{y}$ and*
(1) $z^o(t, k_0) < z^o(t', k_0)$ for all t, t' such that $0 \leq t < t' \leq \overset{*}{t}$ and
(2) $z^o(t, k_0) > z^o(t', k_0)$ for all t, t' such that $t' > t \geq \overset{*}{t}$.

Thus, conclusions of Theorem 9.1 will hold for the preference and technology combinations specified in Sections 6, 7, and 8. In particular, Egli and Steger [2007] provides an illustration of this theorem, where a dynamic version of the model in Andreoni and Levinson [2001] (discussed in Section 7 above) is studied.

10. Conclusions.

This paper derives a set of necessary and sufficient conditions that characterise the EKC in a general framework. We conjecture that this characterisation could potentially be used to test for an EKC using data based on revealed choices of economies employing parametric or non-parametric techniques. As seen in our examples, parametric specifications of preferences and technology often impose a-priori structures on how the preference and technology-based shadow prices respond to growth. Non-parametric techniques could, hence, be helpful when we would rather that these responses be determined purely by data. We conjecture that our general definition of shadow prices based on the normal cone will be especially helpful to conduct the test for an EKC in such situations.

The conditions reveal that the reasons for an EKC can differ greatly across countries. For some countries, the rising part of the EKC could be explained by the standard intuitive reasoning that, at subsistence levels, the MWTP for emission reduction is decreasing while the MAC is rising, *i.e.,* both preference and technological factors work towards increasing the emission level. However, the rising part of an EKC can also be a result of a globally rising MWTP provided the MAC is also rising at a faster rate. It can also be explained when both MWTP and MAC are falling, provided the former falls faster. Similarly, both preference and technological factors can work towards generating the falling part of the EKC, *i.e.,* this region can be characterised by rising MWTP and falling MAC. However, it can also be explained when both the MWTP and the MAC are rising (respectively, falling) provided the rise (respectively, the fall) in the former (respectively, the latter) is relatively more. Our examples substantiated some of these cases. We can also infer from the above analysis that, if there are emissions such as green house gasses for which an EKC is not empirically evident, then this could be because the the world's valuation of the environment (the MWTP for emission reduction) is, on average, not sufficiently increasing with growth and/or the technological costs (MAC) of emission reduction are not falling sufficiently with growth.

APPENDIX

Lemma A.1: *Let $\mathcal{B}^i \subset \mathbf{R}^n$ and int $\mathcal{B}^i \neq \emptyset \ \forall \ i = 1, \ldots, m$. If $b \in \bigcap_i cl \ \mathcal{B}^i$ then $\bigcap_i int \ T(\mathcal{B}^i, b^i) \subseteq int \ T(\cap_i \mathcal{B}^i, b)$.*

Lemma A.2: *Let $\mathcal{B}^i$ be a convex subset of $\mathbf{R}^n$ for all $i = 1, \ldots, m$. If $b \in \bigcap_i cl \ \mathcal{B}^i$ then $\bigcap_i int \ T(\mathcal{B}^i, b^i) = int \ T(\cap_i \mathcal{B}^i, b)$.*

Lemma A.3: *Let $K_0, \ldots, K_{m-1}$ be m open and non-empty convex cones and K_m be a convex cone with vertex 0^n in $\mathbf{R}^n$. Then $\bigcap_{i=0}^m K_i = \emptyset$ if and only if for all $i = 0, \ldots, m$, $\exists \ q^i \in K_i^P$ with $q^i \neq 0^n$ for some $i = 0, \ldots, m$ such that $\sum_i q^i = 0^n$.*

Lemma A.4: *Suppose $\mathcal{B} = \{b \in \mathbf{R}^n \mid f^1(b) \leq a_1, \ f^2(b) \leq a_2\}$, where $f^i : \mathbf{R}^n \longrightarrow \mathbf{R}$ is continuous for $i = 1, 2$. If $\bar{b} \in \mathcal{B}$ then $N(\mathcal{B}, \bar{b}) = \sum_{i=1}^2 N(\mathcal{B}^i, \bar{b})$, where $\mathcal{B}^i = \{b \in \mathbf{R}^n \mid f^i(b) \leq a_i\}$ for $i = 1, 2$.*

Lemma A.5: *Suppose $\mathcal{B} = \{b \in \mathbf{R}^n \mid f(b) \leq a\}$, where $f : \mathbf{R}^n \longrightarrow \mathbf{R}$ is differentiable and $a \in \mathbf{R}$. If $\bar{b} \in \mathcal{B}$ and $f(\bar{b}) = a$, then $T(\mathcal{B}, \bar{b}) = \{x \in \mathbf{R}^n \mid \nabla_b f(\bar{b}) \cdot x \leq 0\}$ and $N(\mathcal{B}, \bar{b}) = \{v \in \mathbf{R}^n \mid v = \kappa \nabla_b f(\bar{b}), \ \kappa \geq 0\}$.[24] If $f(\bar{b}) < a$, then $T(\mathcal{B}, \bar{b}) = \mathbf{R}^n$ and $N(\mathcal{B}, \bar{b}) = 0^n$.*

[24] $\nabla_b f(\bar{b})$ stands for the gradient of function f with respect to b evaluated at $\bar{b}$.

Lemma A.6: *Suppose $\mathcal{B}$ is convex. If $b \in cl\ \mathcal{B}$ then $\mathcal{B} \subseteq \{b\} + T(\mathcal{B}, b)$ and $int\ \mathcal{B} \subseteq \{b\} + int\ T(\mathcal{B}, b)$.*

Proof of Lemma 3.5: Let $x := \langle x_1, x_2 \rangle \in \mathbf{R}_{--} \times \mathbf{R}_{++}$. Since $\mathbf{R}_{--} \times \mathbf{R}_{++}$ is open, x is an interior point of it. Hence, there exists $\epsilon > 0$ such that $\mathcal{N}_\epsilon(x) \subset \mathbf{R}_{--} \times \mathbf{R}_{++}$. Pick $\delta > 0$ and $\eta > 0$ such that $z - \delta + \eta(x_1 - \epsilon) > 0$. Let $\langle z', c' \rangle \in \mathcal{N}_\delta(z, c) \cap \succeq (z, c)$. Pick $\lambda \in (0, \eta]$ and $a = \langle a_1, a_2 \rangle \in \mathcal{N}_\epsilon(x)$. Then, we have $z - \delta \leq z'$ and $x_1 - \epsilon \leq a_1$. Hence, recalling that $x_1 < 0$, we have $0 < z - \delta + \eta(x_1 - \epsilon) \leq z - \delta + \lambda(x_1 - \epsilon) \leq z' + \lambda(x_1 - \epsilon) \leq z' + \lambda a$. Hence, $z' + \lambda a_1 > 0$. Also, since $a_2 > 0$, we have $c' + \lambda a_2 > 0$. Hence, $\langle z', c' \rangle + \lambda a \in \mathbf{R}^2_{++}$ and, recalling that $a_1 < 0$, we have $z' + \lambda a_1 < z'$ and $c' + \lambda a_2 > c'$. Hence, under Assumption 2.1, $\langle z', c' \rangle + \lambda a \in \succ (z, c)$. Since $\langle z', c' \rangle$, a, and λ were chosen arbitrarily from $\mathcal{N}_\delta(z, c) \cap \succeq (z, c)$, $\mathcal{N}_\epsilon(x)$, and $(0, \eta]$, respectively, we have $\{\langle z', c' \rangle\} + \lambda \mathcal{N}_\epsilon(x) \subset \succ (z, c)$ for all $\lambda \in [0, \eta]$ for any $\langle z', c' \rangle \in \mathcal{N}_\delta(z, c) \cap \succeq (z, c)$. Hence, from Lemma 3.1 it follows that $x \in int\ T(\succeq (z, c))$. ■

Proof of Theorem 3.1: Since $\langle \overset{*}{z}, \overset{*}{c} \rangle \in \varphi(y)$, we have $V := \succ (\overset{*}{z}, \overset{*}{c}) \cap Y(y) = \emptyset$. Define

$$\hat{V} := int\ T\left(\succeq (\overset{*}{z}, \overset{*}{c}), \langle \overset{*}{z}, \overset{*}{c} \rangle\right) \ \cap\ int\ T\left(Y(y), \langle \overset{*}{z}, \overset{*}{c} \rangle\right)$$

and

$$\bar{V} := \ cl\ \succ (\overset{*}{z}, \overset{*}{c}) \cap Y(y).$$

Remark 3.1 implies that

$$\bar{V} = \succeq (\overset{*}{z}, \overset{*}{c}) \cap Y(y)$$

and $\langle \overset{*}{z}, \overset{*}{c} \rangle \in \bar{V}$. We will show that $\hat{V} = \emptyset$. Suppose not. Then there exists $x \in \hat{V}$. From the characterisation of the interior of a tangent cone in Lemma 3.1, there exist $\epsilon_1 > 0$, $\epsilon_2 > 0$, $\eta_1 > 0$, and $\eta_2 > 0$ such that $\{\langle \overset{*}{z}, \overset{*}{c} \rangle\} + \lambda cl\ \mathcal{N}_{\epsilon_1}(x) \subset \succeq (\overset{*}{z}, \overset{*}{c})$ for all $\lambda \in [0, \eta_1]$ and $\{\langle \overset{*}{z}, \overset{*}{c} \rangle\} + \lambda cl\ \mathcal{N}_{\epsilon_2}(x) \subset Y(y)$ for all $\lambda \in [0, \eta_2]$. Define $\epsilon = \min\{\epsilon_1, \epsilon_2\}$ and $\eta = \min\{\eta_1, \eta_2\}$. Then $\{\langle \overset{*}{z}, \overset{*}{c} \rangle\} + \lambda cl\ \mathcal{N}_\epsilon(x) \subset \bar{V}$ for all $\lambda \in [0, \eta]$. Pick $\lambda \in [0, \eta]$. Define $b := \{\langle \overset{*}{z}, \overset{*}{c} \rangle\} + \lambda x$. Then $\{\langle \overset{*}{z}, \overset{*}{c} \rangle\} + \lambda cl\ \mathcal{N}_\epsilon(x) \equiv cl\ \mathcal{N}_{\lambda\epsilon}(b) \subset \bar{V}$. In particular, Remark 3.1 implies $b \in \succeq (\overset{*}{z}, \overset{*}{c}) = cl\ \succ (\overset{*}{z}, \overset{*}{c})$. Hence, b is a limit point of $\succ (\overset{*}{z}, \overset{*}{c})$. Hence, there exists $b' \in \mathcal{N}_{\lambda\epsilon}(b) \cap \succ (\overset{*}{z}, \overset{*}{c}) \subset \mathcal{N}_{\lambda\epsilon}(b) \subset \bar{V}$. Hence, $b' \in \succ (\overset{*}{z}, \overset{*}{c}) \cap Y(y) = V$, which contradicts $V = \emptyset$. Hence, $\hat{V} = \emptyset$. By Lemmas 3.5 and 3.6, $int\ T\left(\succeq (\overset{*}{z}, \overset{*}{c}), \langle \overset{*}{z}, \overset{*}{c} \rangle\right)$ and $int\ T\left(Y(y), \langle \overset{*}{z}, \overset{*}{c} \rangle\right)$ are nonempty. Hence, by Lemma A.3, there exists $\overset{*}{v} \in N\left(Y(y), \langle \overset{*}{z}, \overset{*}{y} \rangle\right)$ and $\hat{\overset{*}{v}} \in N\left(\succeq (\overset{*}{z}, \overset{*}{y}), \langle \overset{*}{z}, \overset{*}{y} \rangle\right)$ such that $\overset{*}{v} + \hat{\overset{*}{v}} = 0^2$. Hence, $\hat{\overset{*}{v}} = -\overset{*}{v}$ and conclusions of the theorem follow from (3.2) and (3.3). ■

Proof of Lemma 4.1: (i) Consider a sequence $\{\langle -1, x_2^\nu \rangle\} \longrightarrow \langle -1, 0 \rangle$ such that $x_2^\nu > 0$ for all ν. Then, Lemma 3.5 implies that $\langle -1, x_2^\nu \rangle \in int\ T(\succeq (z, c), \langle z, c \rangle)$ for all ν. Suppose $v =$

$\langle v_1, v_2\rangle \in -N(\succeq (z,c), \langle z,c\rangle)$. Then, the definition of a normal cone implies $\langle -1, x_2^\nu\rangle \cdot \langle v_1, v_2\rangle > 0$ for all ν. Hence, continuity implies $\{\langle -1, x_2^\nu\rangle \cdot \langle v_1, v_2\rangle\} \longrightarrow \langle -1, 0\rangle \cdot \langle v_1, v_2\rangle = -v_1 \geq 0$. Hence, $v_1 \leq 0$. Consider a sequence $\{\langle x_1^\nu, 1\rangle\} \longrightarrow \langle 0, 1\rangle$ such that $x_1^\nu < 0$ for all ν. Then, Lemma 3.5 implies that $\langle x_1^\nu, 1\rangle \in int\ T(\succeq (z,c), \langle z,c\rangle)$ for all ν. Suppose $v = \langle v_1, v_2\rangle \in -N(\succeq (z,c), \langle z,c\rangle)$. Then, the definition of a normal cone implies $\langle x_1^\nu, 1\rangle \cdot \langle v_1, v_2\rangle > 0$ for all ν. Hence, continuity implies $\{\langle x_1^\nu, 1\rangle \cdot \langle v_1, v_2\rangle\} \longrightarrow v_2 \geq 0$.

(ii) Suppose $\langle -1, 0\rangle \in int\ T(\succeq (z,c), \langle z,y\rangle)$. Then $\langle -1, 0\rangle$ is an interior point of $int\ T(\succeq (z,c), \langle z,c\rangle)$. Hence, there exists $\epsilon > 0$ such that $\mathcal{N}_\epsilon(-1,0) \subset int\ T(\succeq (z,c), \langle z,c\rangle)$. Pick $0 < \lambda < \epsilon$. Then $\langle -1, -\lambda\rangle \in \mathcal{N}_\epsilon(-1,0) \subset int\ T(\succeq (z,c), \langle z,c\rangle)$. Suppose $v = \langle v_1, v_2\rangle \in -N(\succeq (z,c), \langle z,c\rangle)$ is such that $v \neq 0^2$. Suppose $v_1 = 0$. Then part (i) implies $v_2 > 0$, and we have $\langle v_1, v_2\rangle \cdot \langle -1, -\lambda\rangle = -\lambda v_2 < 0$. This contradicts $v \in -N(\succeq (z,c), \langle z,c\rangle)$. Hence, $v_1 \neq 0$. Hence, part (i) implies $v_1 < 0$.

Proof of **(iii)** is similar to proof of (ii).

(iv) Let $0 < \lambda < z$. Assumption 2.1 implies that $\langle z,c\rangle + \langle -\lambda, 0\rangle \in \succ (z,c) \subset int\ \succeq (z,c)$ and $\langle z,c\rangle + \langle 0,1\rangle \in \succ (z,c) \subset int\ \succeq (z,c)$. If u is quasi-concave, then Lemma A.6 implies $int\ \succeq (z,c) \subset \{\langle z,c\rangle\} + int\ T(z,c)$. Hence, $\langle -\lambda, 0\rangle \in int\ T(z,c)$ and $\langle 0,1\rangle \in int\ T(z,c)$. Since $int\ T(z,c)$ is a cone, we have $\langle -1, 0\rangle \in int\ T(z,c)$. Hence, parts (ii) and (iii) imply $v_1 < 0$ and $v_2 > 0$. ■

Proof of Lemma 4.2: Proofs of parts (i) to (iii) are similar to Lemma 4.1. Below we present the proof of part (iv). Since Assumption 2.2 and Remark 2.2 imply that $0^2 \in Y(y)$ and $Y(y)$ is a convex set, we have $\lambda\langle z,c\rangle \in Y(y)$ for all $\lambda \in (0,1)$. Pick one such λ. The disposability properties of $Y(y)$ imply $\langle z, \lambda c\rangle \in Y(y)$. We first show that there exists $\epsilon > 0$ such that $\mathcal{N}_\epsilon(z, \lambda c) \subset Y(y)$. Choose $\epsilon_1 > 0$ such that $\epsilon_1 < \min_\kappa \left\{\|\langle z, \lambda c\rangle - \kappa\langle z,c\rangle\| \,\middle|\, \kappa \in [\lambda, 1]\right\}$.[25] Then $\epsilon_1 < \|\langle z, \lambda c\rangle - \kappa\langle z,c\rangle\| \leq \|\langle z, \lambda c\rangle - \lambda\langle z,c\rangle\| = (1-\lambda)z$ for all $\kappa \in [\lambda, 1]$. Hence, $z - \epsilon_1 > \lambda z$. Choose $\epsilon > 0$ such that $\epsilon < \min\{\lambda c, \epsilon_1\}$. Then for all $\langle z', c'\rangle \in \mathcal{N}_\epsilon(z, \lambda c)$, we have $c' > 0$, $z' > z - \epsilon > \lambda z > 0$, and $c' < c$.[26] We show that $\mathcal{N}_\epsilon(z, \lambda c) \subset Y(y)$. If $\langle z', c'\rangle \in \mathcal{N}_\epsilon(z, \lambda c)$ is such that $z' \geq z$, then $\langle z', c'\rangle \in [\{\langle z,c\rangle\} + (\mathbf{R}_+ \times \mathbf{R}_-)] \cap \mathbf{R}^2_+$. Hence, given the disposability properties of $Y(y)$, $\langle z', y'\rangle \in Y(y)$. Suppose $\langle z', c'\rangle \in \mathcal{N}_\epsilon(z, \lambda c)$ is such that $z' < z$. Define μ such that $c' = \mu c$. Since $0 < c' < c$, we have $0 < \mu < 1$. Hence, the disposability properties of $Y(y)$ implies $\langle z, \mu c\rangle \in Y(y)$. Hence, $\langle z', c'\rangle$ is a convex combination of $\mu\langle z,c\rangle$ and $\langle z, \mu c\rangle$. Hence, convexity of $Y(y)$ implies $\langle z', c'\rangle \in Y(y)$. Hence, $\mathcal{N}_\epsilon(z, \lambda c) \subset Y(y)$. Define $\langle x_1, x_2\rangle = \langle z, \lambda c\rangle - \langle z,c\rangle$.

[25] Minimization of a continuous function over a compact set is well defined.

[26] This is because $c' < \lambda c + \epsilon$ for all $\langle z', c'\rangle \in \mathcal{N}_\epsilon(z,c)$ and $(1-\lambda)c = \|\langle z,c\rangle - \langle z, \lambda c\rangle\| \geq \|\kappa\langle z,c\rangle - \langle z, \lambda c\rangle\| > \epsilon = \|\langle z, \lambda c + \epsilon\rangle - \langle z, \lambda c\rangle\|$ for all $\kappa \in [\lambda, 1]$. Hence, $\lambda c + \epsilon < c$.

Then Lemma A.6 implies $\langle x_1, x_2\rangle \in T(Y(y), \langle z, c\rangle)$. Since $\mathcal{N}_\epsilon(z, \lambda c) = \{\langle z, c\rangle\} + \mathcal{N}_\epsilon(x_1, x_2)$ and $\mathcal{N}_\epsilon(z, \lambda c) \subset Y(y)$, Lemma A.6 implies $\mathcal{N}_\epsilon(x_1, x_2) \subset T(Y(y), \langle z, c\rangle)$. Hence, $\langle x_1, x_2\rangle \equiv \langle 0, (\lambda - 1)c\rangle \in int\, T(Y(y), \langle z, c\rangle)$. Since $int\, T(Y(y), \langle z, c\rangle)$ is a cone, we have $\langle 0, -1\rangle \in int\, T(Y(y), \langle z, c\rangle)$. ■

Proof of Theorem 4.1: Sufficiency of (i) and (ii) for φ to exhibit EKC:
Suppose there exists $\overset{*}{y} > 0$ such that (i) and (ii) in the theorem hold. Consider y and y' such that $0 < y < y' < \overset{*}{y}$. Since (i) is true, Remarks 3.2 and 4.1 imply that

$$-N\left(\succeq (\boldsymbol{z}, \boldsymbol{c^m}), \langle \boldsymbol{z}, \boldsymbol{c^m}\rangle\right) \;\cap\; N\left(Y(y'), \langle \boldsymbol{z}, \boldsymbol{c^m}\rangle\right) = \{0^2\}.$$

Hence, Lemma A.3 implies that

$$\hat{V} := int\; T\left(\succeq (\boldsymbol{z}, \boldsymbol{c^m}), \langle \boldsymbol{z}, \boldsymbol{c^m}\rangle\right) \bigcap\; int\; T\left(Y(y'), \langle \boldsymbol{z}, \boldsymbol{c^m}\rangle\right) \neq \emptyset.$$

Lemma A.1, hence, implies

$$\tilde{V} := int\; T\left(\succeq (\boldsymbol{z}, \boldsymbol{c^m}) \cap Y(y'), \langle \boldsymbol{z}, \boldsymbol{c^m}\rangle\right) \neq \emptyset.$$

Let $b := \langle b_1, b_2\rangle \in \tilde{V}$. By definition of the interior of a tangent cone there exist $\epsilon > 0$ and $\eta > 0$ such that $\{\langle \boldsymbol{z}, \boldsymbol{c^m}\rangle\} + \lambda cl\; \mathcal{N}_\epsilon(b) \subset \succeq (\boldsymbol{z}, \boldsymbol{c^m}) \cap Y(y')$ for all $\lambda \in [0, \eta]$. Pick $\lambda \in [0, \eta]$. Then Remark 3.1 implies that $\mathcal{N}_{\lambda\epsilon}(\langle \boldsymbol{z}, \boldsymbol{c^m}\rangle + \lambda b) \subset \succeq (\boldsymbol{z}, \boldsymbol{c^m}) \cap Y(y') = cl\; \succ (\boldsymbol{z}, \boldsymbol{c^m}) \cap Y(y')$. Since $\mathcal{N}_{\lambda\epsilon}(\langle \boldsymbol{z}, \boldsymbol{c^m}\rangle + \lambda b) \subset Y(y')$ is an open subset of $Y(y')$, we have $\mathcal{N}_{\lambda\epsilon}(\langle \boldsymbol{z}, \boldsymbol{c^m}\rangle + \lambda b) \subset cl\; \succ (\boldsymbol{z}, \boldsymbol{c^m}) \cap int\; Y(y')$. Since $\langle \boldsymbol{z}, \boldsymbol{c^m}\rangle + \lambda b$ is a limit point of $\succ (\boldsymbol{z}, \boldsymbol{c^m})$, there exists $\langle \tilde{z}, \tilde{c}\rangle \in \mathcal{N}_{\lambda\epsilon}(\langle \boldsymbol{z}, \boldsymbol{c^m}\rangle + \lambda b)$ such that $\langle \tilde{z}, \tilde{c}\rangle \in \succ (\boldsymbol{z}, \boldsymbol{c^m}) \cap int\; Y(y')$. Hence, $\succ (\boldsymbol{z}, \boldsymbol{c^m}) \cap int\; Y(y') \neq \emptyset$. Define $p^P := \max\; S^P\left(\succeq (\boldsymbol{z}, c^m(\boldsymbol{z}, y')), \langle \boldsymbol{z}, c^m(y', \boldsymbol{z})\rangle\right)$ and $p^Y := \min\; S^Y\left(Y(y'), \langle \boldsymbol{z}, c^m(\boldsymbol{z}, y')\rangle\right)$. Then (i) implies $p^P < p^Y$ and, under the maintained assumptions, Lemma 4.1 and Remark 4.1 imply that $p^Y > p^P > 0$. Pick any $\langle \tilde{z}, \tilde{c}\rangle \in \succ (\boldsymbol{z}, \boldsymbol{c^m}) \;\cap\; int\; Y(y')$. Define $\langle x_1, x_2\rangle := \langle \tilde{z}, \tilde{c}\rangle - \langle \boldsymbol{z}, \boldsymbol{c^m}\rangle$. Convexity of $Y(y')$ and quasi-concavity of u imply from Lemma A.6 that $\succ (\boldsymbol{z}, \boldsymbol{c^m}) \subset \succeq (\boldsymbol{z}, \boldsymbol{c^m}) \subset \{\langle \boldsymbol{z}, \boldsymbol{c^m}\rangle\} + int\; T\left(\succeq (\boldsymbol{z}, \boldsymbol{c^m}), \langle \boldsymbol{z}, \boldsymbol{c^m}\rangle\right)$ and $int\; Y(y') \subset \{\langle \boldsymbol{z}, \boldsymbol{c^m}\rangle\} + int\; T\left(Y(y'), \langle \boldsymbol{z}, \boldsymbol{c^m}\rangle\right)$. Hence, $\langle x_1, x_2\rangle \in int\; T\left(\succeq (\boldsymbol{z}, \boldsymbol{c^m}), \langle \boldsymbol{z}, \boldsymbol{c^m}\rangle\right)$ and $\langle x_1, x_2\rangle \in int\; T\left(Y(y'), \langle \boldsymbol{z}, \boldsymbol{c^m}\rangle\right)$. Hence, $\langle x_1, x_2\rangle \in \hat{V}$. Hence, Lemma 3.2 and Remark 4.1 imply $\langle x_1, x_2\rangle \cdot \langle -p^P, 1\rangle > 0$ and $\langle x_1, x_2\rangle \cdot \langle -p^Y, 1\rangle < 0$. Which implies $p^P x_1 < x_2 < p^Y x_1$ and hence $(p^P - p^Y)x_1 < 0$, which implies $x_1 > 0$ and $x_2 > 0$. Hence, $\tilde{z} > \boldsymbol{z}$ and $\tilde{c} > \boldsymbol{c^m}$. Thus, the set of all emission-consumption combinations that yield greater welfare than $\langle \boldsymbol{z}, \boldsymbol{c^m}\rangle$ in the set $Y(y')$ is not empty, and a move from $\langle \boldsymbol{z}, \boldsymbol{c^m}\rangle$ to any such point implies an increase in the level of emission. In particular, it implies $z^{min}(y') > \boldsymbol{z}$, a property φ is required to satisfy if it exhibits EKC.

Proof of sufficiency in the case where $0 < \overset{*}{y} < y < y'$ and (ii) holds is similar.

Necessity of (i) and (ii) for φ to exhibit EKC:

Suppose φ exhibits EKC and $\overset{*}{y} > 0$ satisfies (i) and (ii) in the definition of an EKC. Consider y and y' such that $0 < y < y' < \overset{*}{y}$. Remark 4.2 implies $\langle \boldsymbol{z}, c^m(\boldsymbol{z}, c)\rangle \in \varphi(y)$. From Assumption 2.2 we have $Y(y) \subset Y(y')$ and, hence, $u(\boldsymbol{z}, c^m(\boldsymbol{z}, y)) \le u(\boldsymbol{z}, \boldsymbol{c^m})$. By the definition of an EKC, $\boldsymbol{z} < z^{min}(y')$. Hence, $\langle \boldsymbol{z}, \boldsymbol{c^m}\rangle \notin \varphi(y')$. Hence,

$$\succ (\boldsymbol{z}, \boldsymbol{c^m}) \ \cap \ Y(y') \neq \emptyset.$$

Assumption 2.1 implies $\succ (z, c^m(z, y')) \subseteq int \ \succeq (z, c^m(z, y'))$. Combined with the convexity of $Y(y')$ and quasi-concavity of u, Lemma A.6 implies $\succ (\boldsymbol{z}, \boldsymbol{c^m}) \subset \ \{\langle \boldsymbol{z}, \boldsymbol{c^m}\rangle\} + int\, T\left(\succeq (\boldsymbol{z}, \boldsymbol{c^m}), \langle \boldsymbol{z}, \boldsymbol{c^m}\rangle\right)$ and $Y(y') \subset \{\langle \boldsymbol{z}, \boldsymbol{c^m}\rangle\} + T\left(Y(y'), \langle \boldsymbol{z}, \boldsymbol{c^m}\rangle\right)$. Hence,

$$\left[\succ (\boldsymbol{z}, \boldsymbol{c^m}) \ \cap \ Y(y')\right] - \{\langle \boldsymbol{z}, \boldsymbol{c^m}\rangle\} \subset int\, T\left(\succeq (\boldsymbol{z}, \boldsymbol{c^m}), \langle \boldsymbol{z}, \boldsymbol{c^m}\rangle\right) \ \cap \ T\left(Y(y'), \langle \boldsymbol{z}, \boldsymbol{c^m}\rangle\right) \neq \emptyset. \quad \text{(A.1)}$$

Hence, Lemma A.3 implies

$$-N\left(\succeq (\boldsymbol{z}, \boldsymbol{c^m}), \langle \boldsymbol{z}, \boldsymbol{c^m}\rangle\right) \ \cap \ N\left(Y(y'), \langle \boldsymbol{z}, \boldsymbol{c^m}\rangle\right) = \{0^2\}.$$

Hence, Remarks 3.2 and 4.1 imply that either (4.2) is true or (4.3) is true. Suppose (4.3) is true and let $p^P := \max\ S^P\left(\succeq (\boldsymbol{z}, \boldsymbol{c^m}), \langle \boldsymbol{z}, \boldsymbol{c^m}\rangle\right)$ and $p^Y := \min S^Y\left(Y(y'), \langle \boldsymbol{z}, \boldsymbol{c^m}\rangle\right)$. Then, under the maintained assumptions, (4.3), Lemma 4.2, and Remark 4.1 imply $p^P > p^Y \ge 0$; $\langle -p^P, 1\rangle \in -N\left(\succeq (\boldsymbol{z}, \boldsymbol{c^m}), \langle \boldsymbol{z}, \boldsymbol{c^m}\rangle\right)$; and $\langle -p^Y, 1\rangle \in N\left(Y(y'), \langle \boldsymbol{z}, \boldsymbol{c^m}\rangle\right)$. Pick any $\langle \tilde{z}, \tilde{c}\rangle \in \succ (\boldsymbol{z}, \boldsymbol{c^m}) \ \cap \ Y(y')$. Define $x = \langle x_1, x_2\rangle := \langle \tilde{z}, \tilde{c}\rangle - \langle \boldsymbol{z}, \boldsymbol{c^m}\rangle$. Then (A.1) implies $\langle x_1, x_2\rangle \in int\, T\left(\succeq (\boldsymbol{z}, \boldsymbol{c^m}), \langle \boldsymbol{z}, \boldsymbol{c^m}\rangle\right) \cap T\left(Y(y'), \langle \boldsymbol{z}, \boldsymbol{c^m}\rangle\right)$. Hence, the definition of a normal cone and Lemma 3.2 imply $\langle x_1, x_2\rangle \cdot \langle -p^P, 1\rangle > 0$ and $\langle x_1, x_2\rangle \cdot \langle -p^Y, 1\rangle \le 0$. Which implies $p^P x_1 < x_2 \le p^Y x_1$ and hence $(p^P - p^Y)x_1 < 0$, which implies $x_2 \le 0$ and $x_1 < 0$. Hence, $\tilde{z} < \boldsymbol{z}$ and $\tilde{c} \le \boldsymbol{c^m}$. Thus, any move from $\langle \boldsymbol{z}, \boldsymbol{c^m}\rangle$ that increases welfare and is also feasible in the set $Y(y')$ implies a decrease in the level of emission. In particular, it implies $z^{min}(y') < \boldsymbol{z}$, which is a contradiction to φ exhibiting an EKC. Hence, (4.2) is true.

The case where $0 < \overset{*}{y} < y < y'$ implies (4.3) can be analogously proved. ■

Proof of Theorem 7.1: (7.4) and strict concavity of $\bar{A}$ in c implies that there exists a function $\hat{c} : \mathbf{R}_+ \longrightarrow \mathbf{R}_+$ with image $\hat{c}(y)$ such that for any $y \in \mathbf{R}_{++}$, the function $\bar{A}$ attains a maximum at $\hat{c}(y) \in (0, y)$, *i.e.*, $\hat{c}(y)$ solves

$$\bar{A}_c(\hat{c}(y), y) = y^{L-1}\Psi'\left(\frac{\hat{c}(y)}{y}\right) = 0 \text{ and} \quad \text{(A.2)}$$

$$c \leq \hat{c}(y) \iff \Psi'(\frac{c}{y}) \geq 0. \tag{A.3}$$

Define $\bar{y}$ such that

$$\frac{1}{L} = \bar{y}^{L-1}\Psi'(0) \equiv \bar{A}_c(0, \bar{y}) \implies \bar{y} = \left(\frac{1}{\Psi'(0)L}\right)^{\frac{1}{L-1}}. \tag{A.4}$$

Then $\frac{1}{L}$ is the slope of the function $\bar{A}(c, \bar{y}) \equiv \bar{y}^L\Psi(\frac{c}{\bar{y}})$ at $c = 0$. Strict concavity of $\bar{A}$ in c implies

$$\frac{\bar{y}^L\Psi(\frac{c}{\bar{y}})}{c} < \frac{1}{L} \implies \bar{y}^L\Psi(\frac{c}{\bar{y}}) < \frac{c}{L}\ \forall\ c \in (0, \bar{y}].$$

Since $\bar{A}(c, y) \leq \bar{A}(c, \bar{y})$ for all $y \leq \bar{y}$, we also have

$$y^L\Psi(\frac{c}{y}) \leq \bar{y}^L\Psi(\frac{c}{\bar{y}}) < \frac{c}{L}\ \forall\ y \leq \bar{y} \text{ and } c \in (0, y]. \tag{A.5}$$

Define y' such that

$$1 = {y'}^{L-1}\Psi'(0) \equiv \bar{A}_c(0, y') \implies y' = \left(\frac{1}{\Psi'(0)}\right)^{\frac{1}{L-1}}. \tag{A.6}$$

Then the slope of the function $\bar{A}(c, y') \equiv {y'}^L\Psi(\frac{c}{y'})$ at $c = 0$ is 1. Strict concavity of $\bar{A}$ in c implies

$${y'}^L\Psi(\frac{c}{y'}) < c \quad \forall\ c \in (0, y'].$$

Hence, for all $y \leq y'$ and $c \in (0, y]$

$$y^L\Psi(\frac{c}{y}) \leq {y'}^L\Psi(\frac{c}{y'}) < c \implies z = c - \bar{A}(c, y) > 0. \tag{A.7}$$

Note, from (A.4), (A.6), and the fact that $L > 1$ it follows that $y' > \bar{y}$. Define

$$\bar{\Gamma} := \left\{\langle z, y, k\rangle \in \mathbf{R}^3 \middle|\ 0 < y \leq \bar{y},\ 0 < c \leq \hat{c}(y),\ z = c - y^L\Psi(\frac{c}{L})\right\}.$$

Then (A.7) implies $\bar{\Gamma} \subset \mathbf{R}^3_{++}$, and (7.5), (A.3), and (A.5) imply that $dS \geq 0$ whenever $\langle z, c, y\rangle \in \bar{\Gamma}$. Moreover, $\langle z, c, y\rangle \in \bar{\Gamma}$ implies $y \leq \bar{k}$.

Define $\psi > 0$ such that $\Psi'(\psi) = 0$. From (A.2), it follows that, for any $y > 0$, $\hat{c}(y) = \psi y$. Define $\hat{y}$ such that $\hat{y} > 0$ and $\hat{y}$ solves

$$\frac{\hat{c}(\hat{y})}{L} - \hat{y}^L\Psi(\frac{\hat{c}(\hat{y})}{\hat{y}}) = 0.$$

This is equivalent to

$$\frac{\psi\hat{y}}{L} - \hat{y}^L\Psi(\psi) = 0 \implies \hat{y}\left[\frac{\psi}{L} - \hat{y}^{L-1}\Psi(\psi)\right] = 0.$$

Then $\hat{y} = \left(\frac{\psi}{L\Psi(\psi)}\right)^{\frac{1}{L-1}}$ and, for all $y > \hat{y}$, we have $\left[\frac{\psi}{L} - y^{L-1}\Psi(\psi)\right] < \left[\frac{\psi}{L} - \hat{y}^{L-1}\Psi(\psi)\right] = 0$. Hence,

$$\frac{\hat{\hat{c}}(y)}{L} - y^L\Psi(\frac{\hat{\hat{c}}(y)}{y}) \equiv y\left[\frac{\psi}{L} - y^{L-1}\Psi(\psi)\right] < 0 \iff \frac{1}{L} < \frac{y^L\Psi(\frac{\hat{\hat{c}}(y)}{y})}{\hat{\hat{c}}(y)}. \tag{A.8}$$

For all $y > \hat{y}$, (A.8) and the strict concavity of $\bar{A}$ in c imply that the average abatement $\frac{\bar{A}(c,y)}{c}$ varies from $y^{L-1}\Psi'(0) > \frac{1}{L}$ to 0 when $c \in (0, y]$. The continuity of $\frac{\bar{A}(c,y)}{c}$ implies that there exists a function $\tilde{c} : \mathbf{R}_+ \longrightarrow \mathbf{R}_+$ with image $\tilde{c}(y)$ such that the following holds:

$$\frac{1}{L} = y^L\frac{\Psi(\frac{\tilde{c}(y)}{y})}{\tilde{c}(y)}. \tag{A.9}$$

Strict concavity of $\bar{A}(c, y)$ in c implies that for all $0 \le c < \tilde{c}(y)$ and $y > \hat{y}$,

$$\frac{1}{L} < y^L\frac{\Psi(\frac{c}{y})}{c}. \tag{A.10}$$

For all $y > y'$, the strict concavity of $\bar{A}$ in c implies that the average abatement $\frac{\bar{A}(c,y)}{c}$ varies from $y^{L-1}\Psi'(0) > 1$ to 0 when $c \in (0, y]$. The continuity of $\frac{\bar{A}(c,y)}{c}$ implies that there exists a function $\bar{c} : \mathbf{R}_+ \longrightarrow \mathbf{R}_+$ with image $\bar{c}(y)$, such that the following holds:

$$1 = y^L\frac{\Psi(\frac{\bar{c}(y)}{y})}{\bar{c}(y)} \tag{A.11}$$

and for all $c > \bar{c}(y)$,

$$1 > y^L\frac{\Psi(\frac{c}{y})}{c} \implies z = c - y^L\Psi(c) > 0. \tag{A.12}$$

For all $y > y'$, define $c^{max}(y) = \max\{\hat{\hat{c}}(y), \bar{c}(y)\}$. For all $y > \hat{y}$, strict concavity of $\bar{A}(c, y)$ and (A.8) and (A.9) imply $\tilde{c}(y) > \hat{\hat{c}}(y)$. For all $y > \hat{y}$, strict concavity of $\bar{A}(c, y)$ and definitions of $\bar{c}(y)$ and $\tilde{c}(y)$ in (A.9) and (A.11) imply $\tilde{c}(y) > \bar{c}(y)$. Hence, $\tilde{c}(y) > c^{max}(y)$. Now define

$$\underline{\Gamma} := \left\{\langle z, c, y\rangle \in \mathbf{R}^3 \middle|\, y > \max\{\hat{y}, y'\},\ c^{max}(y) < c \le \tilde{c}(y),\ z = c - y^L\Psi(\frac{c}{y})\right\}.$$

Since $c > c^{max}(y) \ge \bar{c}(y)$ for all $\langle z, c, y\rangle \in \underline{\Gamma}$, (A.12) implies $z > 0$. Hence, $\underline{\Gamma} \subset \mathbf{R}^3_{++}$. Since $y > \hat{y}$ and $c < \tilde{c}(y)$ for all $\langle z, c, y\rangle \in \underline{\Gamma}$, (A.10) holds. Since $c > \hat{\hat{c}}(y)$ for all $\langle z, c, y\rangle \in \underline{\Gamma}$, (A.3) implies that $\Psi' < 0$. Hence, (7.6) implies $dS < 0$ and $y > \underline{y} := \max\{\hat{y}, y'\}$ for all $\langle z, c, y\rangle \in \underline{\Gamma}$. ■

Proof of Lemma 8.1: Strict quasiconcavity of u implies that $Q^u \equiv Q^u(y, z) > 0$. The Lagrangian of utility maximization (8.1) is: $L = u(z, c) - \kappa[p_z z + c - m]$. The FOCs for an interior optimum are

$$u_c - \kappa = 0, \quad u_z - \kappa p_z = 0, \quad \text{and} \quad -[c + p_z z - m = 0]. \tag{A.13}$$

Differentiating the FOCs with respect to κ, c, z, and m yields:

$$\begin{bmatrix} 0 & -1 & -p_z \\ -1 & u_{cc} & u_{cz} \\ -p_z & u_{cz} & u_{zz} \end{bmatrix} \begin{bmatrix} d\kappa \\ dy \\ dz \end{bmatrix} = - \begin{bmatrix} -z \\ 0 \\ -\kappa \end{bmatrix} dm. \tag{A.14}$$

Noting from FOCs (A.13) that $\kappa = u_c$ and $p_z = \frac{u_z}{\kappa}$ and employing Cramer's rule on (A.14), we obtain

$$\begin{aligned} \frac{\partial z^M}{\partial m} &= \frac{u_c}{Q^u}[u_c u_{cz} - u_z u_{cc}] = -\frac{u_c^3}{Q^u}\frac{\partial r}{\partial c} \text{ and} \\ \frac{\partial c^M}{\partial m} &= -\frac{u_c}{Q^u}[u_c u_{zz} - u_z u_{cz}] = \frac{u_c^3}{Q^u}\frac{\partial r}{\partial z}. \blacksquare \end{aligned} \tag{A.15}$$

Proof of Lemma 9.1: The current value Hamiltonian of Problem (9.1) is

$$H^c(z, c, a, k, \lambda) = u(z, c) + \lambda\,[f(k) - \delta k - c - a] \text{ where } z = \bar{g}(c, a).$$

The Hamiltonian optimised over the control variables is obtained as

$$\begin{aligned} h^c(k, \lambda) &:= \max_{z,c,a} \left\{ u(z, c) + \lambda\,[f(k) - \delta k - c - a] \;\middle|\; z = \bar{g}(c, a) \right\} \\ &= \max_{z,c,a} \left\{ u(z, c) - \lambda\,[c + a] \;\middle|\; z = \bar{g}(c, a) \right\} \;+\; \lambda\,[f(k) - \delta k] \\ &= v(\lambda) + \lambda\,[f(k) - \delta k]\,, \text{ where} \end{aligned} \tag{A.16}$$

$$v(\lambda) := \max_{z,c,a} \left\{ u(z, c) - \lambda\,[c + a] \;\middle|\; z = \bar{g}(c, a) \right\} \tag{A.17}$$

The proof of this lemma is by contradiction. Suppose $\langle \bar{z}, \bar{c} \rangle$ solves (2.3) and $\langle \bar{z}, \bar{c} \rangle \neq \langle z^o(t, k_0),\; c^o(t, k_0) \rangle$. Then $\bar{z} = g(\bar{c}, y^o(t, k_0)) \equiv \bar{g}\,(\bar{c},\; y^o(t, k_0) - \bar{c})$. Since $z^o(t, k_0) = \bar{g}(c^o(t, k_0), a^o(t, k_0)) = \bar{g}(c^o(t, k_0), y^o(t, k_0) - c^o(t, k_0))$ and $c^o(t, k_0) \leq y^o(t, k_0)$, we have $\langle z^o(t, k_0),\; c^o(t, k_0) \rangle \in Y\,(y^o(t, k_0))$. Hence, since $\langle \bar{z}, \bar{c} \rangle$ solves (2.3) and $\langle z^o(t, k_0),\; c^o(t, k_0) \rangle$ does not, it must be the case that $u\,(\bar{z}, \bar{c}) > u\,(z^o(t, k_0), c^o(t, k_0))$. Define $\bar{a} := y^o(t, k_0) - \bar{c}$. Then $\bar{c} + \bar{a} = y^o(t, k_0) = c^o(t, k_0) + a^o(t, k_0)$. But then we also have $u\,(z^o(t, k_0), c^o(t, k_0)) - \lambda^o(t, k_0)\,[c^o(t, k_0) + a^o(t, k_0)] < u(\bar{z}, \bar{c}) - \lambda^o(t, k_0))\,[\bar{c} + \bar{c}]$ and $\bar{z} = \bar{g}(\bar{c}, \bar{a})$. This contradicts $\langle z^o(t, k_0), c^o(t, k_0), a^o(t, k_0) \rangle$ solves (A.17) for $\lambda = \lambda^o(t, k_0)$. $\blacksquare$

REFERENCES

Andreoni, J. and A. Levinson, "The simple analytics of the environmental Kuznets curve," *Journal of public economics*, 80: 269–286, 2001.

Beato P., "The existence of marginal cost pricing equilibria with increasing returns," *Quarterly journal of economics,* 97: 669–689, 1982.

Bonnisseau, J. M. and B. Cornet, "Valuation equilibrium and Pareto Optimum in non-convex economies," *Journal of mathematical economics*, 17: 293–315, 1988.

Brown, D., and G. Heal, "Equity, efficiency, and increasing returns," *Review of economic studies,* 46: 571–85, 1979.

Caputo M. R., *Foundations of dynamic economic analysis: Optimal control theory and application,* Cambridge University Press, 2005.

Clarke, F. H., "Generalized gradients and applications," *Transactions of the American mathematical society,* 205: 247–262, 1975.

Clarke, F. H., *Optimization and nonsmooth analysis,* Wiley, New York, 1983.

Clarke, F. H., *Methods of dynamic and nonsmooth optimization,* Capital City Press, Montpelier, Vermont, 1989.

Cornet, B., "Existence of equilibria in economies with increasing returns," in: B. Cornet, H. Tulkens (Eds.), Contributions to Operations Research and Economics: The Twentieth Anniversary of CORE, MIT, Cambridge, Massachusetts, 1989.

Dasgupta, S., B. Laplante, H. Wang, and D. Wheeler, "Confronting the environmental Kuznets curve," *Journal of economic perspectives* 16 (1): 147–168, 2002.

Debreu, G., *Theory of Value,* Cowles Foundation Monograph 17, Yale University, 1959.

Dinda, S., "Environmental Kuznets curve hypothesis: A survey," *Ecological economics,* 49: 431–455, 2004.

Egli H. and T. M. Steger, " A dynamic model of the environmental Kuznets curve: Turning point and public policy," *Environment and resource economics,* 36: 15–34, 2007.

Färe, R., S. Grosskopf, C. A. K. Lovell, and S. Yaisawarng, "Derivation of shadow prices for undesirable outputs: A distance function approach," *Review of economics and statistics*, 75(2): 374–380, 1993.

Färe, R., S. Grosskopf, M. Norris, and Z. Zhang, "Productivity growth, technical progress, and efficiency change in industrialised countries," *American economic review*, 84(1): 66-83, 1994.

Grossman, G. M. and A. B. Krueger, "Environmental impacts of a North-American free trade agreement," *The U. S. - Mexico free trade agreement*, P. Garber, ed., The MIT press, Cambridge Massachusetts, 1993.

Grossman, G. M. and A. B. Krueger, "Economic growth and the environment," *Quarterly journal of economics*, 110: 353–377, 1995.

Guesnerie, R., "Pareto optimality in non-convex economies," *Econometrica*, 43: 1–30, 1975.

Holz-Eakin, D. and T. M. Selden, "Stoking the fires? CO_2 emissions and economic growth," *Journal of public economics*, 57: 85–101, 1995.

John, A. and R. Pecchenino, "An overlapping generations model of growth and the environment," *Economic journal*, 104: 1393–1410, 1994.

Jones, L. E. and R. E. Manuelli, "Endogenous policy choice: the case of pollution and growth," *Review of economic dynamics*, 4: 369–405, 1994.

Khan, M. A. and R. Vohra, "An extension of the second welfare theorem to economies with nonconvexities and public goods," *Quarterly journal of economics,* 102: 223–242, 1987.

Kijima M., K. Nishide, and A. Ohyama, "Economic models for the environmental Kuznets curve: A survey," *Journal of economic dynamics and control*, 34: 1187–1201, 2010.

Lieb, C.M., "The environmental Kuznets curve and satiation: a simple static model," *Environment and development economics,* 7(3): 429–448, 2002.

Lopez, R., "The environment as a factor of production: the effects of economic growth and trade liberalization," *Journal of environmental economics and management*, 27: 163–184, 1994.

Lopez, R. and S. Mitra, "Corruption, pollution, and the Kuznets environment curve," *Journal of environmental economics and management*, 40 (2): 137– 150, 2000.

Mason R. and T. Swanson, "A Kuznets curve analysis of ozone-depleting substances and the Montreal protocol," *Oxford economic papers*, 55: 1–24, 2003.

McConnell, K.E., "Income and the demand for environmental quality," *Environment and development economics,* 2: 383– 399, 1997.

Murty, S., "On the environmental Kuznets curve with fossil-fuels induced emission: Theory and some illustrative examples," DP1407 – Department of Economics Discussion Paper series, University of Exeter Business School, 2014.

Panayotou, T., "Empirical tests and policy analysis of environmental degradation at different stages of economic development," ILO, Technology and Employment Programme, Geneva, 1993

Plassman F. and N. Khanna, "Preferences, technology, and the environment: Understanding the environmental Kuznets curve hypothesis," *American journal of agricultural economics,* 88(3): 632–643, 2006.

Quinzii M., *Increasing returns and economic efficiency,* Oxford University Press, 1993.

[32] R. T. Rockafellar R. T., "Clarke's tangent cones and the boundaries of closed sets in $\mathbf{R}^n$," *Nonlinear analysis: Theory and applications,* 3: 145–154, 1978.

Rockafellar R. T. and R. J-B Wets, *Variational analysis*, 2009.

Selden, T. M. and D. Song, "Environmental quality and development: is there a Kuznets curve for air pollution emissions?" *Journal of environmental economics and management*, 27: 147–162, 1994.

Selden, T. and D. Song, "Neoclassical growth, the J curve for abatement, and the inverted-U curve for pollution." *Journal of environmental economics and management* 29: 162–168, 1995.

Shafik, N. and S. Bandyopadhyay, "Economic growth and environmental quality: Time series and cross-country evidence," Background paper for the World Development Report. The World Bank, Washington, DC, 1992.

Stokey, N. L., "Are there limits to growth?" *International economic review*, 39: 1–31, 1998.

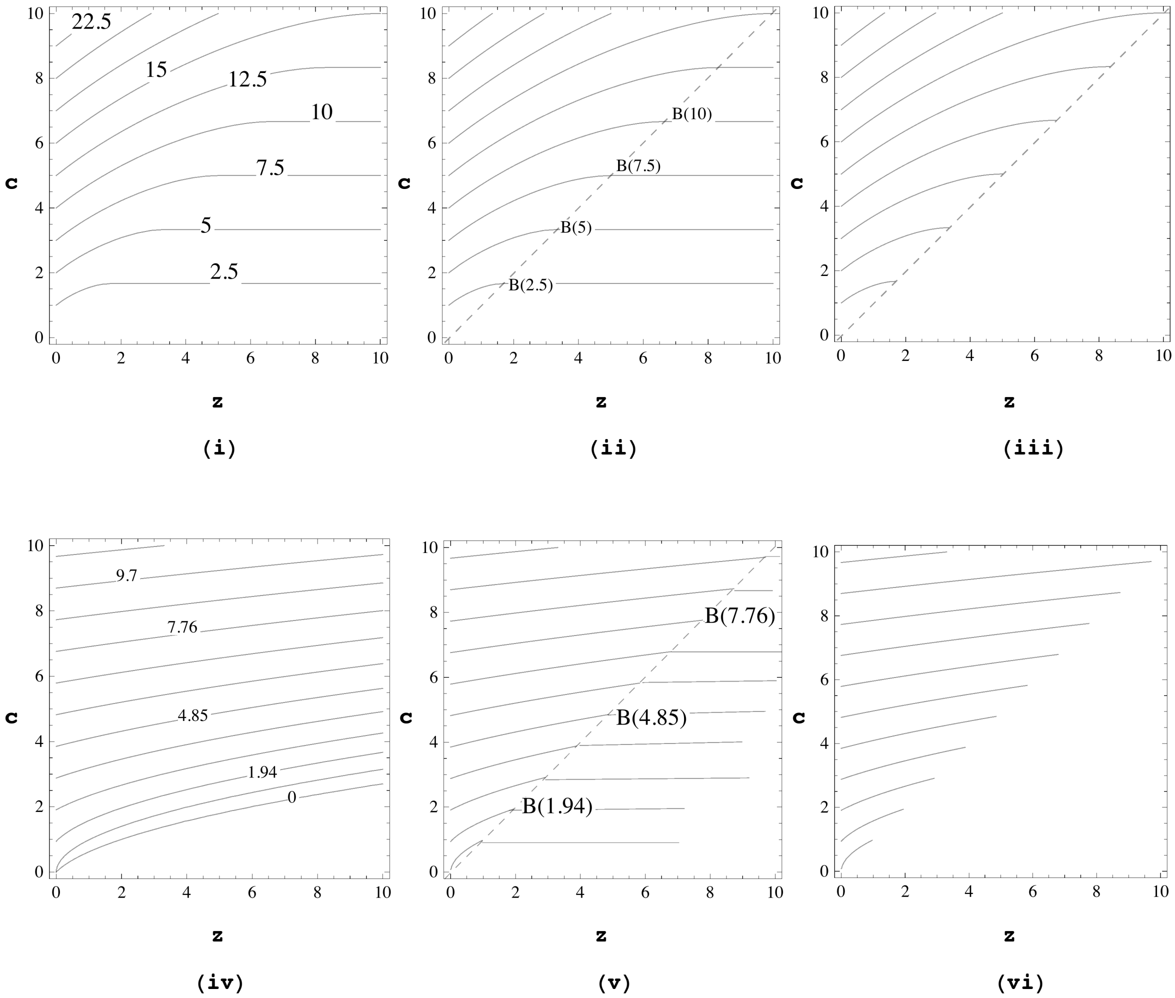
Stern, D. I., "The rise and fall of the environmental Kuznets curve," *World development,* 32: 1419–1439, 2004.

Varian, H. R., "The non-parametric approach to demand analysis," *Econometrica*, 50(4): 945–974, 1982.

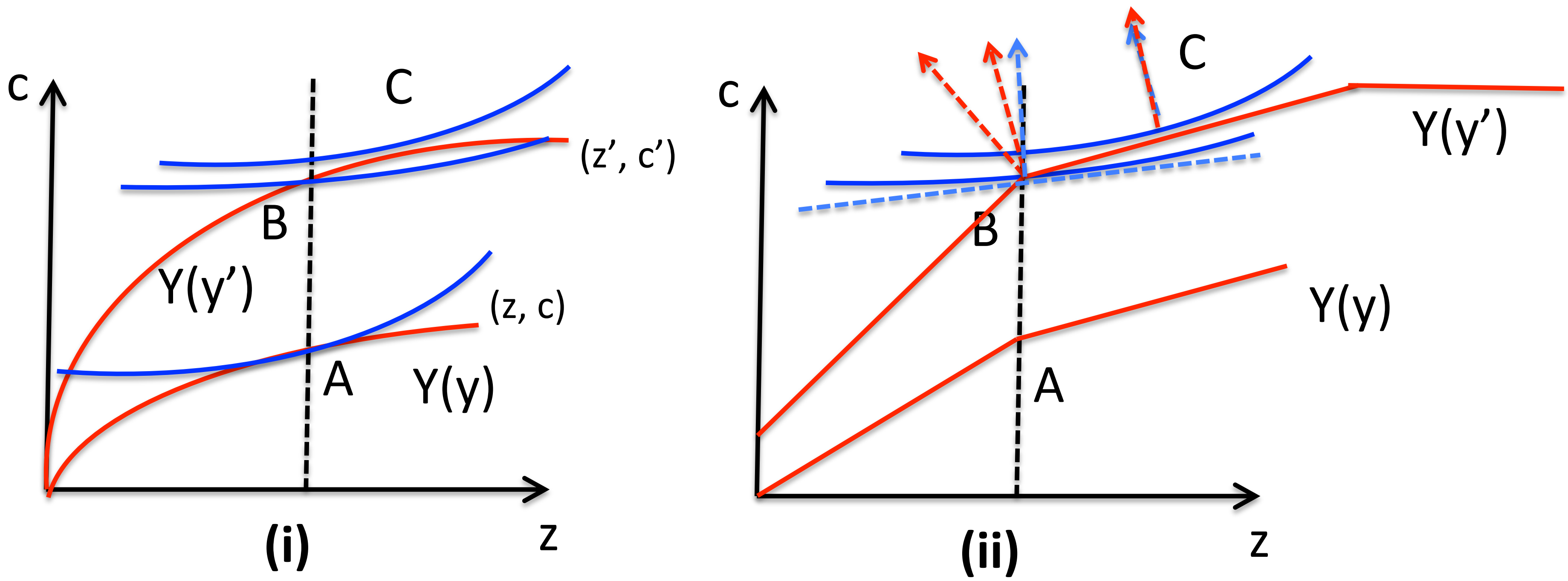
Varian, H. R., "The non-parametric tests of consumer behaviour," *Review of economic studies*, 50(1): 99–110, 1983.

Varian, H. R., "The non-parametric approach to production analysis," *Econometrica*, 52(3): 579–598, 1984.

World Development Report, World Bank, Oxford University Press, New York, 1992.



22.5
15
12.5
10
7.5
5
2.5
c
z
(i)
B(10)
B(7.5)
B(5)
B(2.5)
c
z
(ii)
c
z
(iii)
9.7
7.76
4.85
1.94
0
c
z
(iv)
B(7.76)
B(4.85)
B(1.94)
c
z
(v)
c
z
(vi)



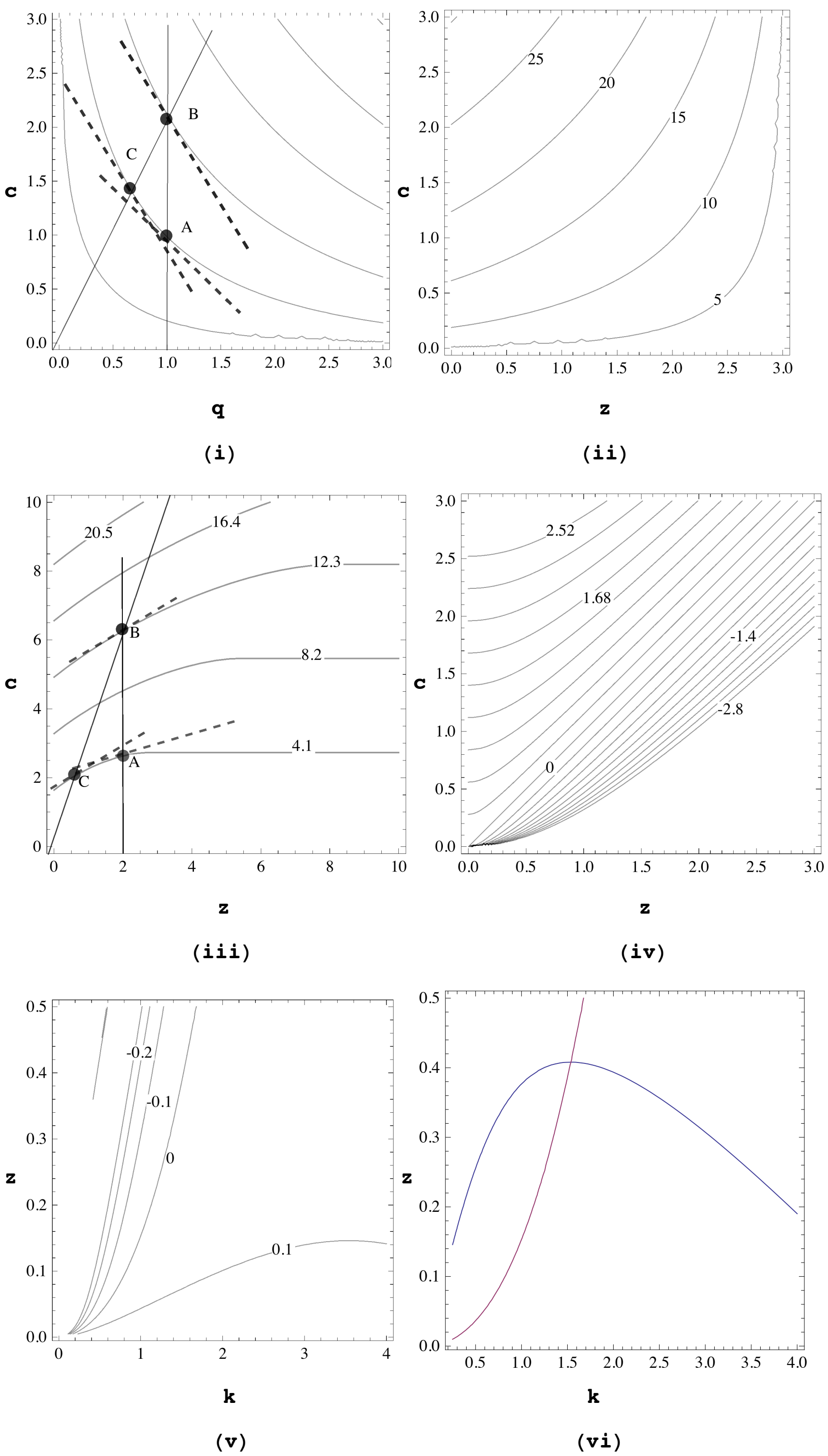
y'>y

EHFE: A to B

DISPE: B to C

at B, MWTA < MR

Figure 2



c
q
(i)
B
C
A
c
z
(ii)
25
20
15
10
5
c
z
(iii)
20.5
16.4
12.3
8.2
4.1
B
A
C
c
z
(iv)
2.52
1.68
-1.4
-2.8
0
z
k
(v)
-0.2
-0.1
0
0.1
z
k
(vi)

Figure 3

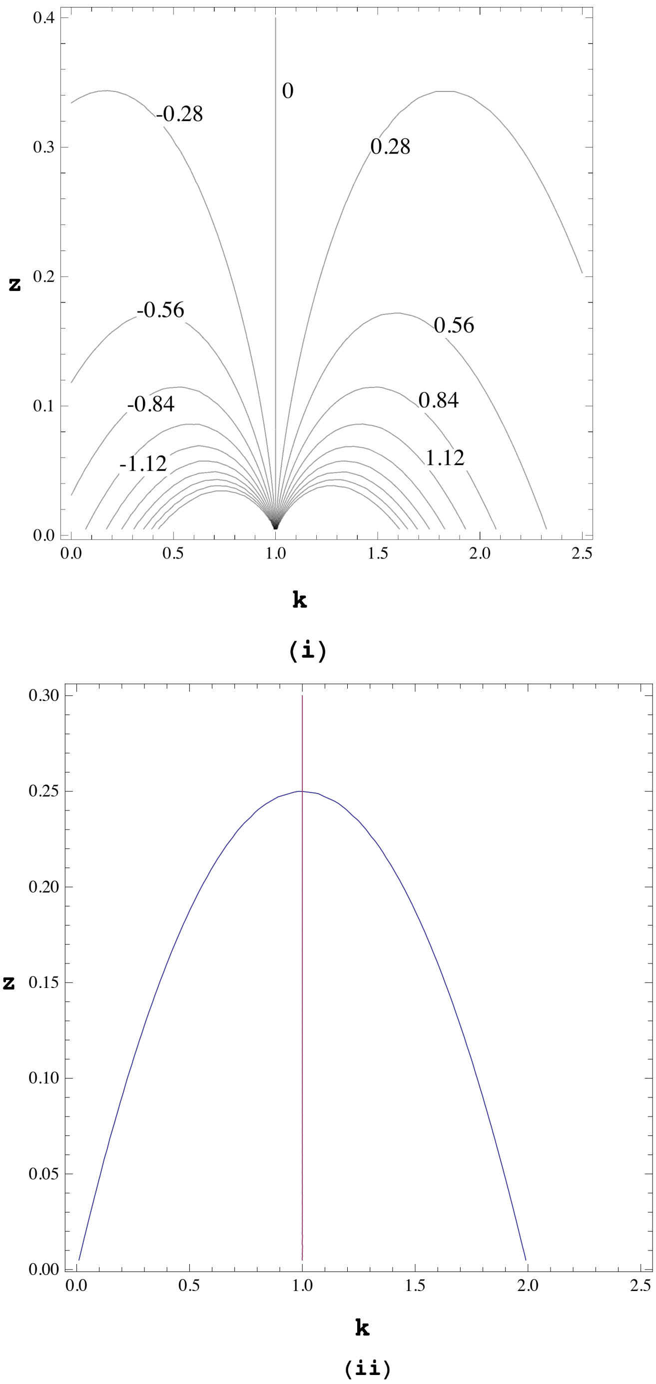


Figure 4

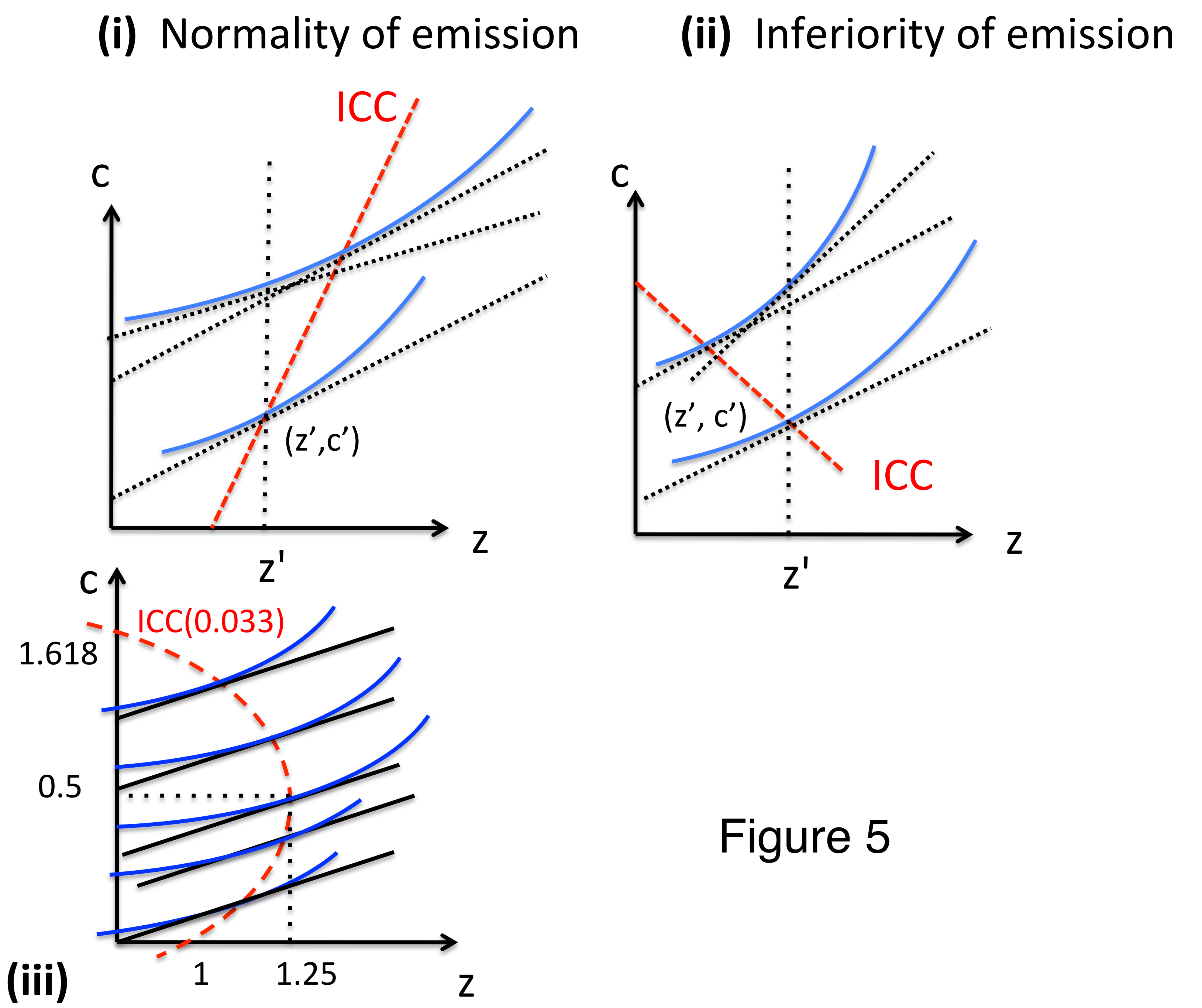

(i) Normality of emission
ICC
c
(z',c')
z
z'
(ii) Inferiority of emission
c
(z', c')
ICC
z
z'
c
ICC(0.033)
1.618
0.5
(iii)
1
1.25
z

Figure 5