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Tatiana Kirsanova and Jack Rogers

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Fixed versus Variable Rate Debt Contracts and Optimal Monetary Policy *

Tatiana Kirsanova† University of Glasgow

Jack Rogers‡ University of Exeter

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Abstract

What role does the proportion of fixed versus variable rate debt contracts play in the macroeconomy? We explore this issue by integrating borrowing-constrained households with a quantity-optimising banking sector that lends under either fixed or variable rates. Our framework is then used to investigate the relationships between the structure of debt contracts and monetary policy. In particular, we study the propagation of productivity shocks in the non-durable sector under Ramsey monetary policy. The introduction of overlapping debt contracts tempers the effect of the financial multiplier and reduces the deterministic component of social welfare, but we also show that an appropriate design of debt contracts, including both their length and their interest rate composition, can reduce volatility of the key economic variables, in such a way that the financial sector can play a stabilising role in the economy. We demonstrate that an intermediate ratio of fixed- and variable-rate debt contracts is socially optimal.

Key Words: Optimal Monetary Policy, Fixed Rate Debt, Durable Goods, Collateral Constraints, Financial Accelerator

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†Address: Adam Smith Building, University of Glasgow, Glasgow, G12 8RT; e-mail: tatiana.kirsanova@glasgow.ac.uk

‡Address: University of Exeter, School of Business and Economics, Streatham Court, Rennes Drive Exeter EX4 4PU; e-mail: j.r.rogers@exeter.ac.uk

1 Introduction

Developments in housing markets have received a great deal of attention from monetary policy-makers, as many developed countries experienced a significant rise in house prices prior to the financial crisis of 2007-2010. Combined with growth in the availability of retail financial products this allowed easier access to equity, and hence spending, fuelling further house price rises, and so on. This mechanism (henceforth referred to as the 'financial accelerator') that links house prices to consumption via collateral and borrowing, is considered to have been an important source of macroeconomic instability, particularly in the UK and the US. There is, however, an institutional characteristic that is strikingly different: the majority of mortgage interest payments in the US and many continental European countries are negotiated at long term, nominal fixed rates, whereas in the UK they are mainly variable. It has been proposed (see for example Maclennan et al. (1998), Treasury (2003), Miles (2004) and Calza et al. (2009)) that countries with more variable rate mortgages may be more exposed to monetary policy changes. Policy messages regarding the desirability of fixed versus variable rate mortgages though, have been mixed across the Atlantic. In a speech given at the Credit Union National Association 2004 Governmental Affairs Conference, Alan Greenspan stated: *'American consumers might benefit if lenders provided greater mortgage product alternatives to the traditional fixed-rate mortgage. To the degree that households are driven by fears of payment shocks but are willing to manage their own interest rate risks, the traditional fixed-rate mortgage may be an expensive method of financing a home.'* In contrast, during a speech given in February 2008, the UK chancellor stated: *'...we also want to see greater availability of affordable long-term fixed rate mortgages. For many households, particularly those on low incomes, fixing the level of mortgage repayments for several years makes real sense; and it can also contribute to wider macroeconomic stability.'* Prior to this, the absence of long-term fixed rate mortgages in the UK had also been put forward as one of the main reasons to delay the UK's entry into European Monetary Union (see for example Treasury (2003)).

In this paper we try to shed light on this issue, by investigating the social benefits of fixed versus variable rate debt contracts in an economy suffering from the inherent instability of a financial accelerator mechanism. By integrating a quantity-optimising banking sector that lends under either a fixed or variable rate, into a model with borrowing constrained households, we are able to explore this issue, focusing on the transmission of productivity shocks under fully optimal (Ramsey) monetary policy. We find that an intermediate ratio of fixed- and variable-rate debt contracts is socially optimal.

The economy is populated by households who would like to borrow under the collateral of durable goods, and households who are willing to lend to them. In a similar way to Iacoviello (2005), Kiyotaki and Moore (1997), and Campbell and Hercowitz (2005), we introduce heterogeneity by assuming that some households are relatively impatient. This distinction is a useful device that naturally gives rise to borrowing constraints and debt. Impatient households become constrained borrowers, and patient households become unconstrained lenders within a utility maximising framework, rather than appealing to other behavioural assumptions such as 'rules of thumb'. Throughout the remainder of the paper we refer to patient households as 'unconstrained' and impatient households as 'constrained'.

The financial accelerator mechanism depends on allowing households to increase their borrowing when they provide enough collateral.[1] We employ a two good – two factor model, similar to the one developed in Monacelli (2007)Calza et al. (2009), and Monacelli (2009), where 'durable goods' are used for this purpose. Durable goods can capture houses, and any other long term

[1] The use of a 'financial accelerator' as an explanation for business cycle amplification dates back at least as far as Bernanke et al. (1999). This term is also used in Iacoviello (2005), which includes impatient entrepreneurs tied to collateral-based constraints, and in Aoki et al. (2004) in the context of households with collateral based on house value. In contrast to our Monacelli framework however, Aoki et al. (2004) use rule-of-thumb consumers (who always consume their current income), and there is no analysis of optimal monetary policy.

purchases such as cars, whose associated debt contracts are also often specified in nominal variable or fixed rate terms. We assume that both borrowers and savers work, although all firms belong to savers. In this respect our model is similar to the one developed in Calza et al. (2009), except that we provide a detailed modelling of financial markets.

We relax the standard assumption of perfect competition in the banking sector by introducing monopolistically competitive banks in a very similar way to Graham and Wright (2007). This model has the attractive feature of capturing the simultaneous setting of fixed and variable rates in a tractable, yet forward-looking way.[2] Banks borrow from unconstrained households and lend to constrained households under either fixed or variable rates. All debt contracts are in nominal terms and overlapping à la Calvo (Calvo (1983)). Every time the contract is rewritten, the monopolistically-competitive bank decides on the amount of new lending, but sets the fixed rate optimally as a price-competitive firm[3].

As in all previous literature (for example, Rubio (2009)), we do not endogenise the share of contracts with fixed rates, rather we investigate the welfare consequences of adjusting this proportion from the perspective of a central planner. In contrast to Rubio (2009) however, we model financial markets with overlapping debt contracts and make a clear distinction between fixed and variable rates. Unlike Graham and Wright (2007), we do not assume a fixed size of collateral for each household, but use durable goods as collateral for constrained households. In essence then, this paper combines the endogenous collateral constraint of the Calza et al. (2009) model, with the banking sector that optimally sets debt contracts at fixed and variable rates as in Graham and Wright (2007).

The paper is organised as follows. In the next Section we set up a two good – two sector

[2] See also Gerali et al. (2008), Andres and Arce (2008) for a monopolistically-price-competitive banking sector with infinite lending.

[3] See also Rubio (2009) where there are no overlapping debt contracts but actuarially fair fixed interest rates on debt contracts. Although Calza et al. (2009) define the fixed rate in a non-optimising way, their focus is on a different research question.

model with a financial sector. Section 3 defines optimal policy. We discuss calibration in Section 4 and demonstrate how to find the steady state of the dynamic model in Section B. Section 5 presents results and Section 6 concludes.

2 The Model

The model builds on Monacelli (2007) and Graham and Wright (2007). The economy consists of two types of households, unconstrained and constrained, and two sectors – producing durable and non durable goods respectively – each populated by a large number of monopolisticly competitive firms and by a perfectly competitive final goods producer. The relative impatience of the constrained households results in their preference for current consumption at the expense of future consumption (their marginal utility of current consumption exceeds their marginal utility of saving). They choose to borrow to maximise their utility and the unconstrained households choose to lend. The presence of household debt reflects equilibrium intertemporal trading between the two types of agents, with the unconstrained acting as standard consumption-smoothers. The amount of debt that constrained households can borrow is limited by the amount of collateral they own in the form of durable goods, and in what follows we denote their associated variables with a subscript c, whereas unconstrained household variables are given subscript u.

The financial sector in this economy is represented by banks which compete in quantity but not in price. Each household's debt contract is renogotiated (they 'move house') with constant probability, at which point banks have the opportunity to choose the amount of lending in such a way as to maximise expected profits. At the same time, they either apply the competitive Central Bank's rate to variable rate contracts, or apply the fixed rate, as determined by a no arbitrage condition.

2.1 Preferences

The period utility function for each household of type $j \in \{u, c\}$ is identical:

$$u_{jt} = \frac{X_{jt}^{1-\sigma}}{1-\sigma} - \varkappa \frac{N_{jt}^{1+\phi}}{1+\phi} \tag{1}$$

where X_{jt} and N_{jt} denote composite consumption, and hours of labour, $\frac{1}{\sigma}$ and $\frac{1}{\phi}$ are the elasticities of intertemporal substitution of consumption and labour, and $\varkappa > 0$ is a scale parameter capturing the relative disutility of working compared to consuming.[4]

For all households X_t is a CES consumption aggregator of the form:

$$X_{ut} \equiv \left((1-\alpha)^{\frac{1}{\eta}} C_{ut}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} D_{ut}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{2}$$

$$X_{ct} \equiv \left((1-\alpha)^{\frac{1}{\eta}} C_{ct}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} D_{ct}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{3}$$

where C_t is a Dixit and Stiglitz (1977) (henceforth D-S) aggregator of differentiated non-durable goods, D_t denotes consumption of durable goods, $\alpha > 0$ is the share of durable goods in the composite consumption index, and $\eta > 0$ is the elasticity of substitution between non-durable and durable consumption.

The differentiated goods are produced by monopolistically competitive intermediate goods firms, indexed by $z \in (0, 1)$. Perfectly competitive final goods firms then aggregate varieties into a single consumption good, so that the standard D-S aggregator for non-durable goods is:

$$C_t = \left[\int_0^1 c_t(z)^{\frac{\epsilon-1}{\epsilon}} \, dz\right]^{\frac{\epsilon}{\epsilon-1}}$$

where $\epsilon > 1$ is the elasticity of substitution between intermediate goods. The corresponding non-durable goods price index is given by:

$$P_{nd,t} \equiv \left[\int_0^1 p_t(z)^{1-\epsilon} \, dz\right]^{\frac{1}{1-\epsilon}}$$

[4] Notice that we follow the now conventional approach of building a monetary model in which households do not derive any utility from holding money, see McCallum (2001) and Woodford (2003), Ch3.

and the composite price index is determined as:

$$P_t \equiv \left[(1-\alpha){P_{nd,t}}^{1-\eta} + \alpha {P_{d,t}}^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

where $P_{d,t}$ is the rental price of durable goods. We assume that households are indifferent between buying new or existing stocks of durable goods, which, if we pursue an analogy with housing stock, means that once a constant rate of depreciation is allowed for, there is no qualitative consumption difference between the service flows from old and new houses. Demand for each individual variety of non-durable good is given by:

$$c_t(z) = \left(\frac{p_t(z)}{P_{nd,t}}\right)^{-\epsilon} C_t$$

where $p_t(z)$ is the price of variety z.

The stock of durables is updated according to:

$$D_{t+1} = (1-\delta) D_t + d_t$$

where d_t denotes expenditure on durable goods, and δ is a constant rate of depreciation.

2.2 Households

2.2.1 Savers

The optimisation problem for unconstrained households is standard. Each household maximises their infinite horizon present discounted stream of future utility flows:

$$\max U_{ut} = E_t \sum_{s=t}^{\infty} \beta_u^{s-t} u_{us}(C_{us}, D_{us}, N_{us}) \tag{4}$$

where the period utility function $u_{us}(.)$ is defined by equation (1).

The budget constraint for unconstrained consumers is:

$$\tilde{W}_{ut} \leq A_{ut} + W_{ut}N_{ut} + T_{ut} - P_{nd,t}C_{ut} - P_{d,t}\left(D_{ut} - (1-\delta) D_{ut-1}\right) + E_t\left(Q_{t,t+1}\tilde{D}_{t+1}\right) \tag{5}$$

where $\tilde{W}_{ut}$ represents the value of the household's end of period portfolio, A_{ut} is beginning of period financial wealth and government transfers are denoted by T_{ut}. The nominal wage rate from working N_{ut} hours is W_{ut}. Consumption consists of both non-durable and durable goods spending, priced nominally at $P_{nd,t}$ and $P_{d,t}$ respectively. $\tilde{D}_{t+1}$ are dividends that realise by the beginning of the new period. We define the stochastic discount factor $Q_{t,t+1}$ with the property that the price in period t of any portfolio with random value A_{ut+1} in the following period is given by:

$$\tilde{W}_{ut} = E_t \left(Q_{t,t+1} A_{ut+1} \right)$$

We also denote:

$$\frac{1}{1+R_t} = E_t \left(Q_{t,t+1} \right)$$

If the household holds riskless assets then:

$$A_{ut+1} = (1+R_t) \tilde{W}_{ut}$$

and the budget constraint becomes:

$$A_{ut+1} = (1+R_t) \left(A_{ut} + W_{ut} N_{ut} + T_{ut} - P_{nd,t} C_{ut} - P_{d,t} \left(D_{ut} - (1-\delta) D_{ut-1} \right) \right) + \tilde{D}_{t+1}$$

We also specify a limit on borrowing to prevent 'Ponzi schemes':

$$\tilde{W}_{ut} \geq - \sum_{s=t+1}^{\infty} E_{t+1} \left[Q_{t+1,s} \left(W_{us} N_{us} + T_{us} \right) \right]$$

and introduce the relative price of durables as:

$$q_t = \frac{P_{d,t}}{P_{nd,t}}$$

which allows us to rewrite the budget constraint in real terms:

$$\begin{aligned} \frac{A_{ut+1}}{P_{nd,t}} &= (1+R_t) \left(\frac{A_{ut}}{P_{nd,t-1}} \frac{P_{nd,t-1}}{P_{nd,t}} + \frac{W_{ut}}{P_{nd,t}} N_{ut} + \frac{T_{ut}}{P_{nd,t}} \right. \\ &\qquad \left. - C_{ut} - q_t \left(D_{ut} - (1-\delta) D_{ut-1} \right) \right) + \frac{\tilde{D}_{t+1}}{P_{nd,t}} \end{aligned}$$

Denote $a_{ut} = \frac{A_{ut+1}}{P_{nd,t}}$, $\Pi_t = \frac{P_{nd,t}}{P_{nd,t-1}}$, $w_{ut} = \frac{W_{ut}}{P_{nd,t}}$, $t_{ut} = \frac{T_{ut}}{P_{nd,t}}$, and $\tilde{d}_t = \frac{\tilde{D}_{t+1}}{P_{nd,t}}$ to obtain the simplified real budget constraint:

$$a_{ut} = (1 + R_t) \left(\frac{a_{ut-1}}{\Pi_t} + w_{ut} N_{ut} + t_{ut} - C_{ut} - q_t \left(D_{ut} - (1 - \delta) D_{ut-1} \right) \right) + \tilde{d}_t, \tag{6}$$

To derive first order efficiency conditions for the household maximisation problem we then write down the following Lagrangian:

$$\begin{aligned} L &= \sum_{s=t}^{\infty} \beta_u^{s-t} \Bigg(U_{us} + \Lambda_s \Bigg(a_{us} - (1 + R_s) \left(\frac{a_{us-1}}{\Pi_s} + w_{us} N_{us} + t_{us} \right. \\ & \left. -C_{us} - q_s \left(D_{us} - (1 - \delta) D_{us-1} \right) \right) - \tilde{d}_s \Bigg) \Bigg) \end{aligned}$$

where U_{us} is the objective utility function defined by equation (4), and Λ_s is the shadow value (Lagrange multiplier) associated with the budget constraint. Defining the partial derivative of the objective function with respect to variable x as $\frac{\delta U}{\delta x} = U_x$ (marginal utility), we can write the system of first order efficiency conditions as (see appendix C.1.1 for a complete derivation):

$$\Lambda_t = -\frac{U_{nd,ut}}{(1 + R_t)} \tag{7}$$

$$q_t = \left(\frac{\alpha}{1 - \alpha} \right)^{\frac{1}{\eta}} \frac{D_{ut}^{-\frac{1}{\eta}}}{C_{ut}^{-\frac{1}{\eta}}} + \beta_u (1 - \delta) E_t \left(\frac{X_{ut+1}^{\frac{1}{\eta} - \sigma} C_{ut+1}^{-\frac{1}{\eta}}}{X_{ut}^{\frac{1}{\eta} - \sigma} C_{ut}^{-\frac{1}{\eta}}} q_{t+1} \right) \tag{8}$$

$$w_{ut} = \frac{\varkappa N_{ut}^{\phi}}{(1 - \alpha)^{\frac{1}{\eta}} X_{ut}^{\frac{1}{\eta} - \sigma} C_{ut}^{-\frac{1}{\eta}}} \tag{9}$$

$$1 = \beta_u E_t \left(\frac{X_{ut+1}^{\frac{1}{\eta} - \sigma} C_{ut+1}^{-\frac{1}{\eta}}}{X_{ut}^{\frac{1}{\eta} - \sigma} C_{ut}^{-\frac{1}{\eta}}} \frac{(1 + R_t)}{\Pi_{t+1}} \right) \tag{10}$$

The budget constraint equation (6) determines the path of assets, and conditions (7)-(10) determine the constraint shadow value, consumption of durables, the wage, and consumption of non-durables respectively. Note that with unrestricted access to financial markets, these conditions for unconstrained households are standard. Following Monacelli (2009), equation (8) can be rearranged to give:

$$\frac{U_{d,ut}}{U_{nd,ut}} = q_t - \beta_u (1 - \delta) E_t \left[\frac{U_{nd,ut+1}}{U_{nd,ut}} q_{t+1} \right] \tag{11}$$

This equation shows that unconstrained households will make choices between durables and non-durables in such a way as to equate their marginal rate of substitution (LHS of the equation), to the user cost of durables (RHS of the equation). Taking the analogy with houses, we can consider each of the user cost terms in turn: the higher are expected capital losses due to relative house price falls (high q_t relative to q_{t+1}); the more patient are unconstrained households (high β_u); the higher the rate of depreciation (δ), and the higher the marginal rate of substitution between future and present non-durable consumption, then the less desireable it is to own housing relative to consuming non-durable goods. As this overall opportunity cost associated with buying an extra unit of housing increases, the unconstrained households will want to decrease their relative spending on houses, thus inceasing their marginal utility ($U_{d,ut}$) relative to non-durable goods ($U_{nd,ut}$) until equation 11 is satisfied again.

In this case, with perfect financial markets, movements in the user cost are dominated by expected house price movements (see Erceg and Levin (2006)). Put simply, when house prices are expected to fall, their re-sale value is lower, so their demand falls.

Conditions (7) and (10) determine the stochastic discount factor:

$$Q_{t,t+1} = \beta_u \frac{U_{nd,ut+1}}{U_{nd,ut}} \frac{P_{nd,t}}{P_{nd,t+1}} \tag{12}$$

In contrast to the Monacelli model, we also allow unconstrained households to work. The standard labour supply condition (9) equates the real wage (in units of non-durable consumption) to the marginal rate of substitution between work disutility and non-durable consumption utility. We assume perfect labour mobility, so household optimisation determines the wage, whereas profit maximisation by firms will determine the amount of labour employed in section 2.3.

2.2.2 Borrowers

The utility maximisation problem for constrained households is the same as for unconstrained households:

$$\max U_{ct} = E_t \sum_{s=t}^{\infty} \beta_c^{s-t} u_{cs}(C_{cs}, D_{cs}, N_{cs})$$

where the period utility function $u_{cs}(.)$ is defined by equation (1).

In contrast to the inequality given by equation (5) however, these households face a constraint that, given our assumptions, will be always binding. The relative patience of the unconstrained households ensures that (for small enough deviations from the steady state), they always hold a positive value of real assets, a_{ut}, which in turn is channeled through financial intermediaries to the constrained households. Their real budget constraint is:

$$a_{ct} = \left(1 + R_t^D\right) \left(\frac{a_{ct-1}}{\Pi_t} + C_{ct} + q_t \left(D_{ct} - (1-\delta) D_{ct-1}\right) - w_{ct} N_{ct} + t_{ct} \right) \tag{13}$$

where R_t^D is the interest rate they face on debt, which we will see in section 2.5 on bank behaviour, for a specific time t, need not be the same rate that unconstrained households earn. Similarly, although a_{ct} is the real value of debt sourced from the assets of the unconstrained, its dynamics also depend on the profit-maximising financial sector which channels assets via fixed and variable rate loans, as in G&W. An important difference however, is that, as in the Monacelli model, constrained households face restrictions on borrowing which are proportional to the *variable* collateral value of the durables they own. In other words, we switch the financial accelerator on. We assume that the bank is prepared to lend the nominal amount K_t defined as:

$$K_t = (1-\chi) D_{ct} P_{dt}$$

that depends on the amount of durable goods they possess. The real size of collateral is hence:

$$k_t = (1-\chi) D_{ct} q_t \tag{14}$$

where $(1-\chi)$ is the constant proportion of durable value that can be used as collateral. In the case of mortgages on houses, χ would be the downpayment ratio, or the inverse loan to value ratio, and is hence a measure of how much debt is rationed. Kiyotaki and Moore (1997) provide a careful story for the existence of this type of rationing, with banks who lend to farmers with idiosynchratic technology. The difference between liquidation values in the event of default and the value to the owner of collateral leads to moral hazard, and enforcement costs lead to debt rationing. Here, following G&W and Monacelli, we apply a similar argument: when banks lend more than the proportion $(1-\chi)$ of collateral, the incentive to default kicks in, the probability of default increases, and expected profits fall. Unresticted entry into the financial sector also means that all profit opportunities are exploited, and so banks never lend less than $(1-\chi)$.

Following G&W, we also assume that debt contracts are reconsidered infrequently, with a Calvo-type stickiness parameter ρ, that we interpret as the probability that the household moves house, and hence must re-negotiate its debt contract. At this point the financial intermediary will decide on the maximum nominal quantity it is willing to lend, at either a variable or fixed rate. It is assumed that variable rates are driven down to the perfectly competitive central bank rate, whilst the competitive fixed rates are determined by individual bank optimisation, and hence are subject to short term profits and losses. We demonstrate in section 2.5 that the evolution of real average lending can be described by:

$$a_{ct} = (1-\rho)\frac{a_{ct-1}}{\Pi_t} + \rho\Omega\left[k_t\right] \tag{15}$$

Formula (15) suggests that real debt at time t remains the same as at $t-1$ with probability $(1-\rho)$ when contracts are not reconsidered, and with probability ρ the amount of borrowing is a specific function $\Omega\left[k_t\right]$ of the value of collateral owned at time t. The household also knows that borrowing will be determined by the size of its collateral when it makes consumption decisions, which opens up the possibility of using durable good ownership as a means of expanding debt, and hence consumption, which in turn may fuel durable goods price increases, and so on. This captures the growing phenomenon of Mortgage Equity Withdrawal in the UK, as discussed in Aoki et al. (2004), where it is argued that recent cycles in economic activity have been amplified by this financial accelerator mechanism.

The Lagrangian for constrained households can hence be written as:

$$L = \sum_{s=t}^{\infty} \beta_c^{s-t} \left(\begin{array}{c} U_{cs} + \Theta_s \left(a_{cs} - \left(1 + R_s^D\right) \left(\begin{array}{c} \frac{a_{cs-1}}{\Pi_s} + C_{cs} - w_{cs} N_{cs} + t_{cs} \\ +q_s \left(D_{cs} - (1-\delta) D_{cs-1}\right) \end{array} \right) - \omega_s^A \right) \\ -\Gamma_s \left(a_{cs} - (1-\rho) \frac{a_{cs-1}}{\Pi_s} - \rho \Omega \left[k_t\right] \right) \end{array} \right)$$

where we have two Lagrange multipliers: the first, Θ_s, associated with the real budget constraint, and the second, Γ_s, associated with the collateral constraint. The associated first order conditions are (see appendix C.1.2 for full derivation):

$$\Theta_t = \frac{U_{nd,ct}}{\left(1 + R_t^D\right)} \tag{16}$$

$$U_{nd,ct} q_t = U_{d,ct} + \beta_c (1-\delta) E_t \left(U_{nd,ct+1} q_{t+1}\right) + U_{nd,ct} \rho \Xi_t \Omega_k (1-\chi) q_t \tag{17}$$

$$w_{ct} = -\frac{U_{n,ct}}{U_{nd,ct}} \tag{18}$$

$$\left(1 + R_t^D\right) \Xi_t = 1 - \beta_c E_t \left(\frac{U_{nd,ct+1}}{U_{nd,ct}} \frac{\left(1 + R_t^D\right)}{\Pi_{t+1}} \left(1 - (1-\rho) \Xi_{t+1}\right) \right) \tag{19}$$

where the partial derivatives of the objective function with respect to C_{ct}, D_{ct}, and N_{ct} are written as $U_{nd,ct}$, $U_{d,ct}$, and $U_{n,ct}$ respectively, $\Omega_k = \frac{\partial \Omega[k_t]}{\partial k_t}$, and we have defined the shadow price of the collateral constraint as $\Xi_t = \Gamma_t / U_{nd,ct}$. The budget constraint equation (13) determines the path of non-durable consumption, while conditions (16) to (19) determine the shadow value of the budget constraint, consumption of durables, the wage, and the shadow value of the collateral constraint respectively.

Rearranging condition (17) in the form of condition (11) for unconstrained households we get:

$$\frac{U_{d,ct}}{U_{nd,ct}} = q_t - \beta_c (1-\delta) E_t \left[\frac{U_{nd,ct+1}}{U_{nd,ct}} q_{t+1} \right] - \rho \Xi_t \Omega_k (1-\chi) q_t \tag{20}$$

which is identical to condition (11) for unconstrained households, except for the addition of an extra user cost term $(\rho\Xi_t\Omega_k(1-\chi)q_t))$ that captures how much the collateral constraint restricts the ability of borrowers to purchase debt and acquire new durables. When the shadow value $\Xi_t = 0$, the collateral constraint is not binding, and the nature of constrained household demand for non-durables relative to durables coincides with unconstrained household behaviour. Monacelli (2009) emphasises that, through this new collateral channel, movements in Ξ_t affect constrained borrower behaviour, and can break the normally strong link that exists between asset price movements and durable/non-durable choice. Decreases in Ξ_t indicate a more relaxed collateral constraint, a rise in user cost, and hence a lower demand for durables.

Similarly, in the absence of the collateral constraint, and with the appropriate time preference and borrower interest rates, condition (19) collapses to the standard unconstrained consumption Euler equation (10).

In contrast to the Monacelli model, we have G&W nominal debt contracts which are Calvo-sticky. As debt renegotiation costs approach zero (or the moving probability ρ approaches 1), the stickiness of debt contracts disappears, and we have the Monacelli case in which the full magnitude of the (expected future) collateral constraint Ξ_{t+1} becomes relevant. In this case, condition (19) is identical to equation (14) in Monacelli (2009).

At the opposite extreme, as ρ approaches 0, debt contracts cease to be renegotiated, the effect of changes in the collateral constraint disappears, and the financial accelerator is switched off. Borrowers would no longer be able to use the collateral of their durable good to expand consumption, but would still react to changes in the rate on debt determined by banks, R_t^D. Our model would then closely resemble G&W, although borrowers would still adjust durable and non-durable consumption in response to changes in their relative price q_t.

In general though, the parameter ρ defines our model as an intermediate case between the full accelerator mechanism of Monacelli ($\rho = 1$), and the G&W case with fixed collateral. There are however, some important differences with the model in Monacelli (2007).

First, because of staggered debt contracts, the marginal utility of relaxing the collateral constraint for *an average household* has a relatively small impact if only ρ-share of households can renew debt contracts. This means the collateral constraint can only be weaker or tighter due to changes in price, rather than quantity. It also implies that it is difficult for the household to move this constraint. In other words, although with staggered debt contracts the presence of the collateral constraint in the user cost increases the contemporary demand for durables, the effect is smaller with less frequent adjustment. We shall discuss in Section 2.5 that the dynamic behaviour of $\Upsilon_{Dc,t}$ is also affected by the proportion of FR and VR debt contracts, Ψ.

Second, debt contract arrangements affect the way Ξ_t is determined in equation (19). The effect is twofold. First, infrequent adjustment ($\rho < 1$) implies the future constraint affects the tightness of the current constraint. As the constraint cannot be moved immediately (only with probability ρ) then the higher future Ξ_{t+1} implies a higher net marginal benefit of acquiring a unit of the durable asset today, which in turn allows, by relaxing the collateral constraint at the margin, the purchase of additional current consumption. Second, the proportion of FR and VR debt contracts affects the dynamics of R_t^D and so also affects the tightness of the out-of-steady-state collateral constraint.

2.3 Intermediate Goods Firms

In this section we derive firm behaviour in a standard way, with intermediate-goods firms choosing the quantity of labour to employ, and the prices to set goods at, before they are sold on to a perfectly competitive final-goods sector. Section 2.3.1 presents these derivations for the non-durable goods sector, followed by the durable sector in section 2.3.2. As before, subscript nd indicates employment and output in the non-durable goods sector, and subscript d indicates the durable goods sector. In the non-durable goods sector we have standard imperfect competition, with D-S differentiated goods and nominal price rigidity. For the durable goods market however, given that house purchases are usually negotiated individually, we use the baseline Monacelli framework by assuming perfectly flexible prices.

We split profit maximisation of intermediate-goods firms by dealing with the two problems separately: first, they choose labour to minimise cost intra-temporally, and second, they choose prices to maximise the present value of future profit inter-temporally. Each sector employs two types of labour: those from constrained households with subscript c, and those who are unconstrained with subscript u (also consistent with our earlier notation). There is perfect labour mobility, so for each type of labour, wages are competitively equalised across all firms. We assume that the unconstrained households own all firms, so their discount factor is used to evaluate the present value of expected profits.

2.3.1 Production of Non-Durable Goods

Firms in the non-durable goods sector choose employment and prices to maximise the discounted present value of current and future profits:

$$\max_{\left\{N_{nd,cs}(i),N_{nd,us}(i),p_s^*(i)\right\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} Q_{t,s} \left(y_{nd,s}\left(i\right) p_{nds}\left(i\right) - W_{us} N_{nd,us}\left(i\right) - W_{cs} N_{nd,cs}\left(i\right)\right) \quad (21)$$

subject to a constant returns to scale Cobb-Douglas production technology:

$$y_{ndt}\left(i\right) = Z_{ndt} N_{nd,ut}\left(i\right)^{\nu} N_{nd,ct}\left(i\right)^{1-\nu} \quad (22)$$

where Z_{ndt} is an exogenous technology shock, and the parameter ν captures the relative productivity of unconstrained labour compared to constrained labour. In the case of homogeneous productivity $\nu = 0.5$.

Profit maximisation is also subject to the demand constraint:

$$y_{ndt}\left(i\right) = Y_{ndt} \left(\frac{p_{ndt}\left(i\right)}{P_{ndt}}\right)^{-\epsilon} \quad (23)$$

and Calvo price rigidity:

$$\begin{aligned} p_{nd,t}(i) &= p^*_{nd,t}(i) \\ p_{nd,t+1}(i) &= \begin{cases} p^*_{nd,t+1}(i), & \text{with prob } 1-\theta \\ p_{nd,t+1}(i), & \text{with prob } \theta \end{cases} \end{aligned} \tag{24}$$

where θ is the probability that firms have the opportunity to adjust prices.

Employment Firm i minimises nominal cost:

$$\min_{N_{nd,ct}(i),N_{nd,ut}(i)} \{W_{ut}N_{nd,ut}(i) + W_{ct}N_{nd,ct}(i)\}$$

subject to the production constraint (22).

We can then write down the Lagrangian:

$$L = W_{ut}N_{ndut}(i) + W_{ct}N_{ndct}(i) - P_{nd,t}\xi_t\left(Z_{ndt}N_{ndut}(i)^{\nu}N_{ndct}(i)^{1-\nu} - y_{ndt}(i)\right)$$

where ξ_t is the Lagrange multiplier associated with the production constraint.

The associated first order efficiency conditions associated with choosing optimal quantities of unconstrained and constrained labour are:

$$w_{ut}N_{nd,ut}(i) = \nu\xi_t y_{ndt}(i) \tag{25}$$

$$w_{ct}N_{nd,ct}(i) = (1-\nu)\xi_t y_{ndt}(i) \tag{26}$$

where the real wage is defined as:

$$w_{jt} = \frac{W_{jt}}{P_{nd,t}}$$

It also follows that ξ_t can be derived as:

$$\xi_t = \frac{1}{Z_{ndt}}\left(\frac{w_{ut}}{\nu}\right)^{\nu}\left(\frac{w_{ct}}{1-\nu}\right)^{1-\nu} \tag{27}$$

and substituting ξ_t into conditions (26) and (25) we obtain:

$$
\begin{aligned}
N_{nd,ut}(i) &= \frac{1}{Z_{ndt}} y_{ndt}(i) \frac{w_{ut}^{(\nu-1)}}{\nu^{(\nu-1)}} \frac{w_{ct}^{(1-\nu)}}{(1-\nu)^{(1-\nu)}} \\
N_{nd,ct}(i) &= \frac{1}{Z_{ndt}} y_{ndt}(i) \frac{w_{ut}^{\nu}}{\nu^{\nu}} \frac{w_{ct}^{(-\nu)}}{(1-\nu)^{(-\nu)}}
\end{aligned}
$$

Substitute real wages from conditions (9) and (18) to obtain:

$$
\begin{aligned}
N_{nd,ut}(i) &= \frac{y_{ndt}(i)}{Z_{ndt}} \frac{N_{us}^{\phi(\nu-1)} C_{ut}^{\frac{1}{\eta}(\nu-1)} X_{us}^{\left(\sigma-\frac{1}{\eta}\right)(\nu-1)}}{\nu^{(\nu-1)}} \frac{N_{cs}^{\phi(1-\nu)} C_{ct}^{\frac{1}{\eta}(1-\nu)} X_{cs}^{\left(\sigma-\frac{1}{\eta}\right)(1-\nu)}}{(1-\nu)^{(1-\nu)}} \\
N_{nd,ct}(i) &= \frac{y_{ndt}(i)}{Z_{ndt}} \frac{N_{us}^{\phi\nu} C_{ut}^{\frac{1}{\eta}\nu} X_{us}^{\left(\sigma-\frac{1}{\eta}\right)\nu}}{\nu^{\nu}} \frac{N_{cs}^{\phi(-\nu)} C_{ct}^{\frac{1}{\eta}(-\nu)} X_{cs}^{\left(\sigma-\frac{1}{\eta}\right)(-\nu)}}{(1-\nu)^{(-\nu)}}
\end{aligned}
$$

Next we define price dispersion $\Delta_t = \int \left(\frac{p_{nd,t}(i)}{P_{nd,t}}\right)^{-\epsilon} di$, to allow aggregation of employment:

$$
\begin{aligned}
N_{nd,ut} &= \frac{1}{Z_{ndt}} \left(\frac{N_{cs}^{\phi} C_{ct}^{\frac{1}{\eta}} X_{cs}^{\left(\sigma-\frac{1}{\eta}\right)}}{N_{us}^{\phi} C_{ut}^{\frac{1}{\eta}} X_{us}^{\left(\sigma-\frac{1}{\eta}\right)}} \right)^{(1-\nu)} \frac{\nu^{(1-\nu)}}{(1-\nu)^{(1-\nu)}} Y_{pt} \Delta_{pt} \\
N_{nd,ct} &= \frac{1}{Z_{ndt}} \left(\frac{N_{us}^{\phi} C_{ut}^{\frac{1}{\eta}} X_{us}^{\left(\sigma-\frac{1}{\eta}\right)}}{N_{cs}^{\phi} C_{ct}^{\frac{1}{\eta}} X_{cs}^{\left(\sigma-\frac{1}{\eta}\right)}} \right)^{\nu} \frac{(1-\nu)^{\nu}}{\nu^{\nu}} Y_{pt} \Delta_{pt}
\end{aligned}
$$

and the marginal cost formula can be determined by substituting wages (equation (9)) into equation (27) :

$$
mc_t = \frac{1}{Z_{ndt}} \frac{\varkappa N_{ut}^{\phi\nu} C_{ut}^{\frac{1}{\eta}\nu} X_{ut}^{\left(\sigma-\frac{1}{\eta}\right)\nu} N_{ct}^{\phi(1-\nu)} C_{ct}^{\frac{1}{\eta}(1-\nu)} X_{ct}^{\left(\sigma-\frac{1}{\eta}\right)(1-\nu)}}{(1-\alpha)^{\frac{1}{\eta}} \nu^{\nu} (1-\nu)^{(1-\nu)}} \tag{28}
$$

Price setting The setting of prices is standard, closely following derivations in Woodford (2003), Ch.2. Prices are determined by Calvo-type contracts, with a fixed probability $1-\theta$ that they will be fixed each period, and probability θ that firms will have the opportunity to re-set their prices. Firms will then choose prices to maximise the following expected profit function (21) which can now be written as:

$$
\max_{\{p_s^*(i)\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} Q_{t,s} \left(y_{nds}(i) p_{nds}(i) - y_{nds}(i) MC_s \right) \tag{29}
$$

where $MC_s = \xi_t P_{nd,t}$ is nominal marginal cost. Note that wages here do not depend on index i, since labour of each type is assumed to be perfectly mobile between sectors, so wages for unconstrained and constrained labour are equalised across all firms. We then come to the familiar problem of maximising (29) subject to the constraints (24) and (23)

At time s we only need consider the maximisation problem for the proportion θ^{s-t} of firms that have the opportunity to set their prices at time t. Optimal price setting can therefore be re-stated as (and substituting demand from equation (23)):

$$\max_{\{p_t^*(i)\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} P_{nds} Y_{nds} \left(\left(\frac{p_{ndt}^*(i)}{P_{nds}} \right)^{1-\epsilon} - \left(\frac{p_{ndt}^*(i)}{P_{nds}} \right)^{-\epsilon} \frac{MC_s}{P_{nds}} \right)$$

The associated first order condition is:

$$E_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} P_{nds} y_{nds}(i) \left(\frac{p_{ndt}^*(i)}{P_{nds}} - \mu mc_s \right) = 0 \tag{30}$$

where the steady state mark-up is $\mu = \frac{\epsilon}{(\epsilon-1)}$ and we have defined real marginal cost as $mc_s = \frac{MC_s}{P_{cs}}$.

Since we assume that unconstrained households own all firms, we substitute their stochastic discount factor (equation (12)), and rearrange condition (30) (see appendix (C.2.1)) to obtain our price-setting system for the non-durable goods sector:

$$\frac{1-\theta\Pi_t^{\epsilon-1}}{(1-\theta)} = \left(\frac{G_t}{F_t} \right)^{1-\epsilon} \tag{31}$$

$$G_t = \mu U_{nd,ut} Y_{ndt} mc_t + \theta\beta E \left[{}_t\Pi_{t+1}^{\epsilon} G_{t+1} \right] \tag{32}$$

$$F_t = U_{nd,ut} Y_{ndt} + \theta\beta E_t \left[\Pi_{t+1}^{\epsilon-1} F_{t+1} \right] \tag{33}$$

This system has the form of a New-Keynesian Phillips curve in the sense that current marginal cost and inflation are linked to future inflation.

Because of staggered price contracts, the aggregate price in non-durable sector evolves as:

$$P_{nd,t} = \left[(1-\theta) \left(p_{nd,t}^* \right)^{1-\varepsilon} + \theta P_{nd,t-1}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$$

So the price dispersion $\Delta_t = \int \left(\frac{p_{nd,t}(i)}{P_{nd,t}} \right)^{-\epsilon} di$ obeys (see Woodford (2003)):

$$\Delta_t = (1-\theta) \left(\frac{1-\theta\Pi_t^{\epsilon-1}}{1-\theta} \right)^{\frac{\epsilon}{\epsilon-1}} + \theta\Pi_t^{\epsilon}\Delta_{t-1} \tag{34}$$

2.3.2 Production of Durable Goods

In this section we derive the employment and price-setting behaviour for the durable sector. Each period new stocks of durable goods are produced. The relative stickiness in either sector is important, as discussed in Monacelli (2009). Durable prices, especially for houses, tend to be subject to individual bargaining processes so we follow the baseline framework of Monacelli (2007) with perfectly flexible durable prices. We use subscript 'd' for employment and output in this sector.

The profit maximisation problem takes the same form as in (21), and the same production technology:

$$y_{dt}(i) = Z_{dt} N_{dut}(i)^{\nu} N_{dct}(i)^{1-v} \tag{35}$$

where Z_{dt} is an exogenous technology shock in the durable goods sector, and the parameter ν captures the relative productivity of unconstrained labour compared to constrained labour, and which we assume is the same as in the non-durable goods sector.

Employment The nominal cost minimisation problem is:

$$\min_{N_{dut}(i), N_{dct}(i)} W_{ut} N_{dut}(i) + W_{ct} N_{dct}(i)$$

subject to the production constraint (35).

Write down the Lagrangian:

$$L = W_{ut} N_{dut}(i) + W_{ct} N_{dct}(i) - P_{ct}\zeta_t \left(Z_{dt} N_{dut}(i)^{\nu} N_{dct}(i)^{1-\nu} - y_{dt}(i) \right)$$

and the first order conditions are:

$$w_{ut} N_{dut}(i) = \nu \zeta_t y_{dt}(i) \tag{36}$$

$$w_{ct} N_{dct}(i) = (1-\nu) \zeta_t y_{dt}(i) \tag{37}$$

It also follows that marginal cost can be derived as:

$$\zeta_t = \frac{1}{Z_{dt}} \left(\frac{w_{ut}}{\nu}\right)^{\nu} \left(\frac{w_{ct}}{1-\nu}\right)^{1-\nu} \tag{38}$$

and we substitute (38) into (36) and (37):

$$\begin{aligned} N_{dut}(i) &= \frac{1}{Z_{dt}} y_{dt}(i) \frac{w_{ut}^{(\nu-1)}}{\nu^{(\nu-1)}} \frac{w_{ct}^{(1-\nu)}}{(1-\nu)^{(1-\nu)}} && (39) \\ N_{dct}(i) &= \frac{1}{Z_{dt}} y_{dt}(i) \frac{w_{ut}^{\nu}}{\nu^{\nu}} \frac{w_{ct}^{(-\nu)}}{(1-\nu)^{(-\nu)}} && (40) \end{aligned}$$

Then substitute real wages from (9) and (18) to obtain:

$$\begin{aligned} N_{dut}(i) &= \frac{1}{Z_{dt}} y_{dt}(i) \frac{N_{us}^{\phi(\nu-1)} C_{ut}^{\frac{1}{\eta}(\nu-1)} X_{us}^{\left(\sigma-\frac{1}{\eta}\right)(\nu-1)}}{\nu^{(\nu-1)}} \frac{N_{cs}^{\phi(1-\nu)} C_{ct}^{\frac{1}{\eta}(1-\nu)} X_{cs}^{\left(\sigma-\frac{1}{\eta}\right)(1-\nu)}}{(1-\nu)^{(1-\nu)}} \\ N_{dct}(i) &= \frac{1}{Z_{dt}} y_{dt}(i) \frac{N_{us}^{\phi\nu} C_{ut}^{\frac{1}{\eta}\nu} X_{us}^{\left(\sigma-\frac{1}{\eta}\right)\nu}}{\nu^{\nu}} \frac{N_{cs}^{\phi(-\nu)} C_{ct}^{\frac{1}{\eta}(-\nu)} X_{cs}^{\left(\sigma-\frac{1}{\eta}\right)(-\nu)}}{(1-\nu)^{(-\nu)}} \end{aligned}$$

And finally, with flexible prices, output in each firm is the same, and aggregation is straightforward:

$$\begin{aligned} N_{dut} &= \frac{1}{Z_{dt}} \left(\frac{N_{ct}^{\phi} C_{ct}^{\frac{1}{\eta}} X_{ct}^{\left(\sigma-\frac{1}{\eta}\right)}}{N_{ut}^{\phi} C_{ut}^{\frac{1}{\eta}} X_{ut}^{\left(\sigma-\frac{1}{\eta}\right)}} \right)^{(1-\nu)} \frac{\nu^{(1-\nu)}}{(1-\nu)^{(1-\nu)}} Y_{dt} \\ N_{dct} &= \frac{1}{Z_{dt}} \frac{(1-\nu)^{\nu}}{\nu^{\nu}} \left(\frac{N_{ut}^{\phi} C_{ut}^{\frac{1}{\eta}} X_{ut}^{\left(\sigma-\frac{1}{\eta}\right)}}{N_{ct}^{\phi} C_{ct}^{\frac{1}{\eta}} X_{ct}^{\left(\sigma-\frac{1}{\eta}\right)}} \right)^{\nu} Y_{dt} \end{aligned}$$

Relative Price Setting In the absence of the Calvo price rigidity constraint (24), the setting of durable prices relative to non-durables is more straight forward (see appendix (C.2.2). Firms choose prices to maximise expected profit:

$$\max_{\{p_s^*(i)\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} Q_{t,s} \left(y_{ds}(i) p_{ds}(i) - y_{ds}(i) MC_{ds}\right)$$

where $MC_{ds} = \xi_t P_{nd,t}$ is nominal marginal cost, subject to the demand constraint:

$$y_{dt}(i) = Y_{dt} \left(\frac{p_{dt}(i)}{P_{dt}}\right)^{-\epsilon}$$

The first order condition then is simply:

$$\frac{MC_{dt}}{P_{dt}} = \frac{\epsilon - 1}{\epsilon} = \mu$$

from where the price of durables relative to non-durables (q_t) is determined by:

$$q_t = \frac{1}{Z_{dt}} \frac{\varkappa\mu N_{ct}^{\phi(1-\nu)} C_{ct}^{\frac{1}{\eta}(1-\nu)} X_{ct}^{\left(\sigma-\frac{1}{\eta}\right)(1-\nu)} N_{ut}^{\phi\nu} C_{ut}^{\frac{1}{\eta}\nu} X_{ut}^{\left(\sigma-\frac{1}{\eta}\right)\nu}}{(1-\alpha)^{\frac{1}{\eta}} \nu^{\nu} (1-\nu)^{(1-\nu)}} \tag{41}$$

2.4 Market Clearing Conditions

Total output is equal to the sum of consumption:

$$Y_{ndt} = C_{ct} + C_{ut} \tag{42}$$

$$Y_{dt} = D_{ut} - (1-\delta) D_{ut-1} + D_{ct} - (1-\delta) D_{ct-1} \tag{43}$$

$$Y_t = Y_{ndt} + Y_{dt} \tag{44}$$

Similarly, total hours worked adds up to:

$$N_{ct} = N_{ndct} + N_{dct} \tag{45}$$

$$N_{ut} = N_{ndut} + N_{dut} \tag{46}$$

$$N_t = N_{ut} + N_{ct} \tag{47}$$

The net saving in the closed economy is zero:

$$A_{ut} - A_{ct} = 0$$

and we divide by the same price index, so the same condition holds in real terms:

$$a_{ut} - a_{ct} = 0 \tag{48}$$

2.5 Financial Intermediaries

In this section we incorporate the Graham and Wright (2007) model of financial sector behaviour into our framework, which assumes that mortgages are issued infrequently, and set in nominal terms. As in Graham and Wright (2007) (henceforth 'G&W') we assume that all loan contracts are reconsidered each period with a constant probability ρ. Decisions to renegotiate may come from either the borrower or the lender: in the case of mortgages, households may change jobs and move house. On the other hand, mortgage lenders in the UK for example, often provide contracts which are discounted or fixed for an agreed period, after which the nature of the contract changes. Conditions for reassessment of collateral at future dates may also be agreed. Whether the household or the bank trigger re-financing then, we assume that ρ captures the associated degree of debt contract stickiness.

Following G&W, regardless of the type of contract – fixed or variable rate – with probability ρ, we assume that the bank decides on the quantity of funds, Z_t to lend to a borrower who provides collateral K_t. When the amount of debt is adjusted, the fixed rate is also adjusted optimally, whereas variable rates are always set equal to the central bank rate.

In this section we derive the optimal amount of debt, Z_t, that banks will want to lend the constrained borrower, based on the value of their collateral, K_t. In the G&W model this is fixed, but here we follow Calza et al. (2009) and base it on the value of owned durable goods, as described by equation (14). We then describe the implications for the evolution of aggregate debt, before deriving the optimal fixed rate, and finally the implied aggregate average rate payable on constrained household debt. Each time a constrained household's debt contract is re-set, Ψ share of the new debt is set under a variable rate, and the remaining $1 - \Psi$ share is tied to a fixed rate. Based on the evidence that most people are locked in to one type of contract, we interpret Ψ as the proportion of the constrained household population that prefers variable rates. We assume Ψ is exogenous and time-invariant for simplicity and tractability, but we also investigate the implications of different choices for social welfare.[5]

2.5.1 Optimal Debt Offers and Aggregate Debt Evolution

At time t any loan contract is reconsidered with probability ρ. When the financial intermediary adjusts the contract, it changes the nominal value of debt it issues, issuing a new quantity, Z_{t+1}. Its optimisation problem will be to maximise the present discounted value of profit that will flow from this contract, so that at time t, it will discount future periods s only for future scenarios in which the contract is not readjusted, so using $(1-\rho)^{s-t}$. In addition, since unconstrained households own the financial intermediaries, their stochastic discount factor ($Q_{t,s}$ derived in equation (64)) is also used to discount the future flow of profit, so the problem is written as:

$$\max_{Z_{t+1}} W_t = \sum_{s=t+1}^{\infty} Q_{t,s} (1-\rho)^{s-t} \mathcal{F}_s \tag{49}$$

where $\mathcal{F}_s$ is a flow objective.

It is assumed that unconstrained households provide funds to financial intermediaries at the central bank rate, so the flow objective of profit can be written as:

$$\mathcal{J}_s = (R_s^z - R_s) Z_{t+1} - \mathcal{C}_s$$

where

$$\mathcal{C}_s = \frac{\varpi}{2} \left(\frac{Z_{t+1} - K_s}{K_s} \right)^2 K_s$$

is the quadratic adjustment cost associated with deviating from the nominal value of the collateral, K_s, and we denote R_s^z the rate that is offered on a new contract,

$$R_t^z = (1-\Psi) R_t + \Psi R_t^{zF} \tag{50}$$

[5] Many (see for example Maclennan et al. (1998)) have suggested that the different contract structures observed between countries have evolved arbitrarily, in the sense that they are more related to legal and institutional factors than economic. In this sense, we can think of Ψ as capturing 'custom' or habit with respect to mortgage choice.

Lending more than the value of the collateral is risky, see e.g. Kiyotaki and Moore (1997), while lending less than the collateral value does not extract the maximum profit. So, the deviation from K_s is costly and parameter ϖ measures how large these quadratic costs are.

The first order condition for the financial intermediary's optimisation problem (49) implies the following dynamic system for real debt (see appendix (C.3.1):

$$\begin{aligned}
\frac{Z_{t+1}}{P_{ndt}} &= \frac{K_t}{P_{ndt}}\frac{M_t}{B_t} \\
M_t &= \beta_u (1-\rho) \Pi_{t+1}^{-1} \left(M_{t+1} + U_{nd,ut+1}\left(R_{t+1}^z - R_{t+1} + \varpi\right)\right) \\
B_t &= \beta_u (1-\rho) \Pi_{t+1}^{-1} \left(B_{t+1} + \varpi U_{nd,ut+1}\right) K_t K_{t+1}^{-1} \qquad (51)
\end{aligned}$$

This system defines the dynamics of the profit maximising level of real debt that is issued by individual financial intermediaries $\left(\frac{Z_{t+1}}{P_{ndt}}\right)$, as a function of inflation (Π_{t+1}), collateral, K_t, the markup of the average rate on debt over the central bank rate $(R_{t+1}^z - R_{t+1})$, and the marginal utility of consumption of non-durables for unconstrained households $(U_{nd,ut+1})$. Real debt evolution is affected by the discount rate of unconstrained households β_u, the stickiness of debt contracts ρ, and the parameter ϖ (which captures the relative size of quadratic costs associated with deviating from the target level of debt).

Appendix (C.3.2) shows how this behaviour translates into the dynamics of aggregate debt. Since constrained households always take all debt offered to them, constrained debt a_{ct} as defined in equation (13) follows the same path, and we have our final real debt evolution equation:

$$a_{ct} = (1-\rho)\frac{a_{ct-1}}{\Pi_{ct}} + \rho\frac{M_t}{B_t}k_t \qquad (52)$$

2.5.2 Rates on Debt

Each financial intermediary adjusts the quantity of loan it offers with probability ρ. At the same time, we assume that the proportion of fixed and variable rates offered is always the same as in the whole population. We can think of each individual borrower as holding a portfolio contract containing fixed and variable rates, with the proportion determined by exogeneous factors related to legal structure and convention. Later we will be able to show what proportion will be optimal for welfare.

We assume that competition in prices between financial intermediaries forces variable rates to adjust with the central bank rate each period. Fixed rates are fixed until each new renegotiation, at which point the new fixed rates are reconsidered simultaneously with the quantity of debt. The interest rate on fixed rate contracts is then determined by a no arbitrage condition where the financial intermediary is indifferent between lending at fixed or variable rates:

$$\sum_{s=t+1}^{\infty} Q_{t,s} R_t^{zF} (1-\rho)^{t-s} Z_{t+1} = \sum_{s=t+1}^{\infty} Q_{t,s} (1-\rho)^{t-s} R_s Z_{t+1}$$

Appendix (C.3.3) shows how this condition can be used to derive the dynamic system determining the new fixed rate R_t^{zF}:

$$\begin{aligned} V_t &= \beta_u (1-\rho) \Pi_{t+1}^{-1} (V_{t+1} + U_{c,ut+1} R_{t+1}) && (53) \\ \mathcal{U}_t &= \beta_u (1-\rho) \Pi_{t+1}^{-1} (\mathcal{U}_{t+1} + U_{c,ut+1}) && (54) \\ R_t^{zF} &= \frac{V_t}{\mathcal{U}_t} && (55) \end{aligned}$$

The average fixed rate then evolves as a simple weighted average of the ρ proportion of financial intermediaries who have had the chance to re-set the contract at time t, and the other $(1-\rho)$ who have not:

$$R_t^F = \rho R_t^{zF} + (1-\rho) R_{t-1}^F \tag{56}$$

So the average interest rate paid by constrained borrowers is:

$$R_t^D = (1-\Psi) R_t + \Psi R_t^F \tag{57}$$

2.6 Private Sector Equilibrium

Appendix A provides the 20 equations in a reduced form that contains only the dynamic variables that were used for numerical analysis.

A private sector rational expectations equilibrium consists of a plan for allocating the sequence for $\{X_{ut}, X_{ct}, N_{ndt}, N_{dt}, D_{ut}, D_{ct}, C_{ut}, C_{ct}, a_{ct}, \Pi_t, q_t, \Xi_t\}$ given the policy $\{R_t\}$, the exogenous productivity shock process $\{Z_{ndt}\}$ and appropriate initial conditions. The system describing the private sector equilibrium can be simplified to leave only dynamic equations and important definitions, as summarised in Table 1.

3 Monetary Policy

We apply Ramsey optimal policy. Assuming feasible pre-committment, a plan that maximises household welfare is delivered, subject to the economy's resource constraints, and consistent with private sector equilibrium. Here we have heterogeneous households with different time preference rates, and collateral constraints which can be relaxed by purchases and price rises of durable goods. The impatience of the borrowers in our model makes it impossible to jointly 'satisfy' both types of household across time. We assume that the policymaking authority puts a relative weight on unconstrained and constrained household *intra-period* welfare, which in our baseline model the same as the labour share of these households. Specifically, the following welfare objective is minimised:

$$\min_{\{R_t\}_{t\geq 0}} \mathcal{W} = \frac{1}{2}\sum_{t=0}^{\infty} \left(\beta_u^{\nu}\beta_c^{1-\nu}\right)^t \left(\nu\left(\frac{X_u^{1-\sigma}}{1-\sigma} - \varkappa\frac{N_u^{1+\phi}}{1+\phi}\right) + (1-\nu)\left(\frac{X_c^{1-\sigma}}{1-\sigma} - \varkappa\frac{N_c^{1+\phi}}{1+\phi}\right)\right) \quad (58)$$

subject to constraints summarised in Table 1, (2), (3), (8)), (10), (13), (17), (19), (32), (33), (31), (34), (41), (52), (51), (51), (53), (54), (55), (50), (56), (57) and (57).

Note that unless the policymaker discounts intra-period welfare of both types using the same discount factor, consumption of those households whose welfare is heavier discounted rises unboundedly as the relative share of them will diminish in policymaker's objectives. Our setup guarantees unique well defined steady state under Ramsey policy, see Becker (1980), Becker and Foias (1987).

4 Calibration

We calibrate our model to a quarterly frequency, and follow Monacelli (2007) and Monacelli (2009) as closely as possible. We attempt to provide simulations of our model that are empirically relevant with respect to the US and the UK. Some parameterizations are based on associated country-specific evidence, whilst in others they are chosen to generate second moments that match US and UK data as closely as possible.

The individual discount factor for unconstrained households is set to 0.99, consistent with an annual steady state real rate of interest of 4%, whilst the constrained households' discount rate is 0.98. Since Hall (1988) estimated the intertemporal elasticity of substitution to be close to zero, other studies since then suggest it could be between 0.5 and 2. Engelhardt and Kumar (2009), for example, estimate a 95% confidence interval between 0.37 and 1.21. For the US model we assume 0.5, which translates into an inverse elasticity of $\sigma = 2$, and which helps us to match the strong correlation observed between consumption and durables spending in the US data. For the UK we assume 0.8, so that $\sigma = 1.2$. The elasticity of intertemporal substitution of labour is set equal to one in both models following Monacelli.

We allow some substitutability between non-durable and durable goods, by setting $\eta = 1.5$. In the non-durable goods sector, the stickiness of price contracts is determined by the Calvo parameter θ, which we set to 0.75, implying an average contract length of one year. The elasticity of substitution between different assortments of non-durable goods is 6, which works out as a mark-up of around 20%.

We follow Monacelli and set the down payment ratio χ equal to 0.25, with the deprecation rate δ for durables set at 0.01 per quarter, which is approximately a 4% deprecation in housing per year. In the UK however, the ONS assumes an annual depreciation rate for housing of 1.4% (see section 7.4.3.2 of the UK National Accounts Data available from the ONS), so we use $\delta = 0.005$

for the UK model.

In the Monacelli model, unconstrained households do not work, whereas we assume $\nu = 0.5$, so that unconstrained households receive 50% of the aggregate wage share. Iacoviello (2005) assumes $\nu = 0.64$ and claims that this estimate is within the range documented by various studies since Campbell and Mankiw (1989), which have estimated this proportion from aggregate consumption euler equations. Whilst Campbell and Mankiw (1989) interpret excess sensitivity to current income as 'rule-of thumb' behaviour, Mankiw (2000) is more careful to distinguish two interpretations. The first suggests that households deviate from rationality for psychological reasons, particularly as current income is far more salient than permanent income. On the other hand, such behaviour may be interpreted as stemming from binding credit constraints. It is empirically challenging to differentiate between rule of thumb and credit constrained behaviours, but Mankiw notes that the bottom two quintiles of the wealth distribution hold only 0.2% of household wealth, with a mean wealth of only $900. Such households, he argues, are unlikely to be able to use financial markets to smooth consumption. Carroll (2009) however, also show that excess sensitivity to current income can be explained by precautionary saving behaviour. According to this theory, such households are reluctant to borrow because of the risk of holding debt when future income is uncertain. In this sense, they can be thought of as self-imposing credit constraints. We use the Campbell and Mankiw (1989) estimate of 0.5, and our interpretation is that the 50% of US households that exhibit excess sensitivity to current income, do so because they are credit constrained.

For the UK, Jappelli and Pagano (1989) describe some general features of household access to credit markets. They note that in countries where there is excess consumption sensitivity to current income, there also tend to be less household debt levels, which in turn they argue are a result of capital market imperfections. They conclude that excess sensitivity is driven mainly by liquidity constraints. Their empirical estimate is that the proportion of credit-constrained households in the UK is between 0.4 and 0.51, whilst in other countries it ranges from as low as 0% in Sweden, to as high as 72% in Spain. More recently, Benito and Mumtaz (2006) estimate that 20-40% of UK households display excess sensitivity to current income using household panel data from 1992 to 2002. They show that these households tend to be associated with factors that exhibit credit constraints, such as having negative home equity, and being young, unmarried, non-white and degree educated. For our UK model, we assume this share is in the middle of their estimate of 30%, implying that $\nu = 0.7$.

Monacelli (2009) chooses α so that the steady state ratio of durable to total spending is 0.2 as in the US data. To match this ratio for our US model we set $\alpha = 0.07$, whilst in the UK model we choose $\alpha = 0.12$, implying a steady state durable to total spending ratio of 0.26. This matches the average ratio of spending on Durables, Semi-Durables and Dwellings, to Final Consumption Expenditure less Imputed Rents from our ONS quarterly dataset.

The interpretation of parameter ϖ is difficult, since it determines how large a financial institution's costs are when deviating from the target level of debt. We assumed quadratic adjustment costs, as in Graham and Wright (2007) for tractability, but the costs associated with endogenous default risk and missed profit opportunities relative to the size of the loan are hard to interpret, and indeed may not be symmetrical. We choose a value of 3 in our baseline calibration and focus our sensitivity analysis on ρ, the probability of readjustment, to see how the renegotiability of debt contracts affects results.

We select our baseline calibration for ρ from a range of sources, but inevitably this parameter is also hard to interpret. According to the Council of Mortgage Lenders, there are approximately 11.2 million mortgage loans outstanding in the UK [6], and in September 2012, there were 44,400 new mortgages issued. These figures imply that an average contract lasts 21 years, however since the financial crisis, new mortgage lending has fallen by at least 50%[7]. To the extent that the

[6] See footnote at http://www.cml.org.uk/cml/media/press/3338

[7] The Bank of England reports that Gross Mortgage Lending fell from £20.4bn

number of contracts has declined proportionately there could have been twice as many mortgages issued at the start of the crisis period, which suggests a turnover equivalent to contracts lasting 10 years on average. This could be a high estimate though, since it is also likely that gross lending per contract has fallen substantially during the crisis period. Miles (2004) provides a detailed survey of the UK mortgage market, and shows that households in the UK expect to move house '...every few years'. Since ρ captures all types of 'readjustment' (including those made by financial institutions to existing contracts), we choose an average contract length of 2 years, which implies $\rho = 0.125$. In the US, empirical estimates of the probability of households moving range between 0.1 and 0.2 (see Sinai and Souleles (2009) Schulhofer-Wohl (2011) and Chan (2001)), so again, noting that contracts can be adjusted for reasons other than moving house, we take the upper estimate and assume $\rho = 0.2$.

Campbell and Cocco (2003a) and Campbell and Cocco (2003b) show that the share of new lending at fixed rates (known as FRMs) has varied between 30% and 90% in the US since 1985. Many of the variable rate mortgages (ARMs) in the US are only adjusted every year, but some are adjusted as frequently as every month. We assume that $\Psi = 0.7$ so that the proportion of mortgages that are set at fixed rates is 70%. In the UK, variable rate mortgages are typically adjusted every month, and evidence from the Council of Mortgage Lenders suggests this proportion is around 30% in the UK (see Graham and Wright (2007)), so we take $\Psi = 0.3$ for the UK model.

For policy objectives in the UK model, we calibrate $a = \nu$, so the social planner uses the same 70% weight for unconstrained households based on their labour share in the economy. To help match the US model more closely to the data however, we assume that unconstrained households have more political influence and calibrate $a = 0.6$, higher than their assumed 50% labour share. This additional weight on unconstrained welfare helps to reduce the correlation

in January 2008 to £9.4bn in September 2010. See Chart 2 available at http://www.bankofengland.co.uk/publications/Pages/other/monetary/trendsinlending2010.aspx between consumption and durables spending, and increase the correlation between durables and house prices as observed in the US data set.

5 Results

5.1 Variable Rates

We study the propagation of a productivity shock in the non-durable goods sector by introducing an iid exogenous process to $Z_{nd,t}$. Examining the economy's responses to a simple one period shock then allows us to focus attention on the nature of the transmission mechanism, and how it differs between economies with predominantly variable rate contracts, and those with mainly fixed.

We first examine the effect of the presence of overlapping debt contracts. As one base line case we plot impulse responses to ND productivity shocks in an economy where the banking sector plays a very simple role of borrowing from unconstrained households and lending to constrained households under the collateral of durable goods. This setup is very similar to the one in Monacelli (2007) and corresponds to the case $\rho = 1$ and $\Psi = 0$. (All contracts are reset every period, the borrowing is determined by the value of collateral and all rates are variable. We denote it as 'no banking sector' case in Figure 1. We compare results to those in our economy where we assume that 95% of borrowers are on variable rate contracts ($\Psi = 0.05$), and that each contract is reconsidered every 2.5 years on average ($\rho = 0.1$). Although variable rates adjust automatically with the central bank rate, the stickiness of debt contracts restricts financial institutions' decisions on the quantity of debt to issue, which also depends on the amount of collateral of durable goods owned by constrained households. Although households are able to use collateral to expand consumption, they cannot smooth consumption as much as they would like.

Following a positive ND productivity shock real marginal cost in the ND sector falls and the maximisation of profit by monopolistically competitive firms implies fall in price and thus in ND inflation. The improvement in productivity allows firms to employ less workers but still produce the same level of output. (The output will be even higher because of increased demand.) For those workers that remain in the non-durable sector, wages go up, and because of perfect labour mobility between the two sectors, this wage increase spills over into the durable sector. Here, there is no increase in productivity, so employment falls in the durable sector as well. The rise in nominal wages in the durable sector increases firms' costs of production, which in turn causes the nominal price of durable goods to increase. This effect, combined with the fall in the price of non-durables, pushes the relative price of durables (q) up.

The response of monetary authorities to this productivity shock is to initially *increase* nominal interest rates, although by a very small amount. (The increase is much more pronounced in the model with 'no banking sector'.) Monetary policy only slightly changes interest rate in the first moment, that, together with expectations of high inflation in the future (when the productivity shock will disappear) allows the real rate to be negative in the first period.

The negative real interest rate is inversely related to the change in the marginal utility of consumption for unconstrained households, since a lower real interest rate requires marginal utility to fall in the future. The initial increase in consumption of non-durable goods, therefore, is explained by the increase in real wages.

With durables relatively expensive, constrained households are then able to expand debt levels to finance purchases. They also experience an initial increase in their real wage, allowing an increase in consumption of both goods, but mainly non-durable as they are relatively cheap. Also, with the stickiness of debt contracts, compared to the Monacelli model, there are only limited possibilities to use durable goods as collateral. In this sense, the financial accelerator mechanism that is fully operational in the Monacelli model, and switched off in the G&W model, is 'partly switched on' here. It is particularly apparent from Figure 1 that in the model with 'no banking sector' the constraint households try to accumulate far more durable goods than in the model with overlapping debt contracts.

The initial rise in interest rate in the model with 'no banking sector' essentially forbids high borrowing. This is not needed in the model with overlapping debt contracts as debt is relatively small (although more persistent) because of partly switched financial accelerator.

In the second period following the shock, the initial gain in productivity is lost, and marginal cost in the non-durable sector recovers to near previous levels. An increase in marginal cost then leads to an increase in the price of the non-durable goods, and real wages go down. The second period nominal interest rate level is well below the steady state, in order to drive the real rate down even further. The low nominal interest rate and high inflation rate at this stage makes borrowing relatively cheap, so the amount of debt stays relatively high. The financial accelerator mechanism becomes more apparent, as constrained households buy more durable goods, which can be used as collateral for more borrowing. As unconstrained consumers do not have the same collateral constraint they choose to reduce their consumption of the overpriced durables significantly, but continue consuming non-durable goods.

This optimal monetary policy response ensures that constrained agents do not face high deviations in their consumption path: the initial drop in interest rates keeps lending and therefore the demand for durables high. The relatively high price of the durable goods allows borrowers to secure against them, and non-durable consumption for the constrained agent remains above steady state.

Moving onto later periods, as the nominal interest rate returns closer to the steady state, real interest rates also begin to increase. Combined with the trend back towards the steady states for non-durable and durable consumption, this causes a slow reduction in levels of debt accrued by the constrained households. Nominal interest rates substantially overshoot the steady state level after several periods in order to stabilise inflation. As the future interest rate is expected to rise, the rate on fixed rate contracts rises immediately. However, this has practically no effect on the economy, since the majority of constrained households are on variable rates.

5.2 Fixed Rates

We now change the proportion of households that are on fixed rate contracts, to see how responses differ from predominately variable rate economies. Figures 2-5 demonstrate impulse responses of key variables, with the proportion of constrained households on fixed rate contracts at 5%, 35%, 65% and 95%. In general, the transmission mechanism of the non-durable productivity shock is similar to the one discussed above, except for the timing and strength of responses: with higher Ψ the response is more sluggish and less pronounced.

The first thing to notice in Figure 2 is that the amplitude of the drop in policy rate becomes considerably smaller as we move to a fixed rate economy. In anticipation of inflation-fighting hikes in the central bank rate, fixed rates rise early, since they are linked directly to the expected path of short rates. As a result, the average rate that constrained households pay on their debt is considerably higher for the fixed rate economy, and will remain high for a long time. With higher Ψ the average rate R^D increases following R^{zF} and it determines the flow of interest payments and the tightness of the budget constraint. Interest rate also determines the tilt of consumption towards the future consumption. Figure 3 demonstrates a substantial delay in adjustment of consumption.

With higher Ψ the average interest rate R^D increases in initial periods and so does the tightness of the collateral constraint. A tighter collateral constraint leads to less consumption of durable goods by the borrowers. The lenders reallocate their consumption towards durable goods as the relative price of durables does not rise as much as with small Ψ.

5.3 Frequency of Resetting

For an intermediate $\Psi = 0.5$ we demonstrate how longer debt contracts moderate financial accelerator mechanism even further. Figure 6 demonstrates that longer debt contracts imply smoother adjustment of durable goods stock accumulated by the borrowers, reduce the level of debt and require higher interest rate in future periods. The effect of longer contracts works in the same way as the effect of higher proportion of fixed rates.

However, unlike Ψ in this model the value of ρ affects steady state values as discussed above. Hence the welfare implications of changes in ρ may not be unambiguous.

5.4 Welfare

The system under optimal control (under Ramsey policy) can be linearised.[8] We can compute the second-order approximation to social welfare *along the optimal solution*, see Appendix A. As the steady state in this model does not depend on Ψ, we only look at the measure of welfare that is based on the measure of volatility of variables. We present these volatilities, as well as the welfare measure in Figure 7 (for any variable x_s we plot $\mathcal{E}_0 \sum_{s=0}^{\infty} \omega^s \left(x_s x_s'\right)$) based on iid ND productivity shocks.

Figure 7 demonstrates that the minimum volatility (and thus maximum of *stochastic component* of social welfare) is achieved in an intermediate point, for Ψ close to 0.5. Indeed, if Ψ rises then adjustment becomes slower. This immediately implies higher welfare losses with higher Ψ. However, higher Ψ also moderates the amplitude of immediate adjustment which carries most of weight in discounted flow of losses. So losses may also fall with Ψ. The minimum of losses is achieved when the first effect outweighs the second. For our calibration of the model the higher ρ (longer debt contracts) amplifies the first effect more than the second and the loss rises with ρ. The optimal Ψ in this model negatively depends on ρ.

The deterministic component of welfare, however, falls with longer debt contracts. As the financial multiplier is tempered, the households are not able to smooth consumption as they would like.

[8] We solve the model using Dynare's toolkit for Ramsey policy. The output produced by Dynare is the linearised model under the optimal policy. The optimal policy itself is given as a linear rule that includes feedback on Lagrange multipliers.

6 Conclusion

The aim of this paper was to integrate a quantity-optimising banking sector that lends under either fixed or variable rate with a model with borrowing-constrained households. This gives us a framework that can be used to investigate the relationships between the structure of debt contracts and monetary policy. In particular we study the propagation of productivity shock in the non-durable sector under Ramsey monetary policy. The introduction of overlapping debt contracts tempers the effect of financial multiplier. Although the implied steady state allocation of resources unambiguously reduces deterministic component of social welfare, an appropriate design of debt contracts, their length and the interest rate composition can reduce volatility of the key economic variables, so banks can play a stabilising role in the economy. In particular, we demonstrate that an intermediate ratio of fixed- and variable-rate debt contracts is socially optimal.

A Complete System in Reduced Form

Table 1 provides a summary of the whole dynamic system in reduced form.

B Steady State

Table (B) shows the recursive system to compute steady state (that follows from system in Table 1), given the steady state level of inflation Π. All interest rates in the steady state are equal to R. (Optimal policy then determines the steady state rate of inflation.)

Equations 1 and 12 from the table can be combined to obtain:

$$\Xi = \frac{\beta_u - \beta_c}{\Pi - \beta_c (1 - \rho)}$$

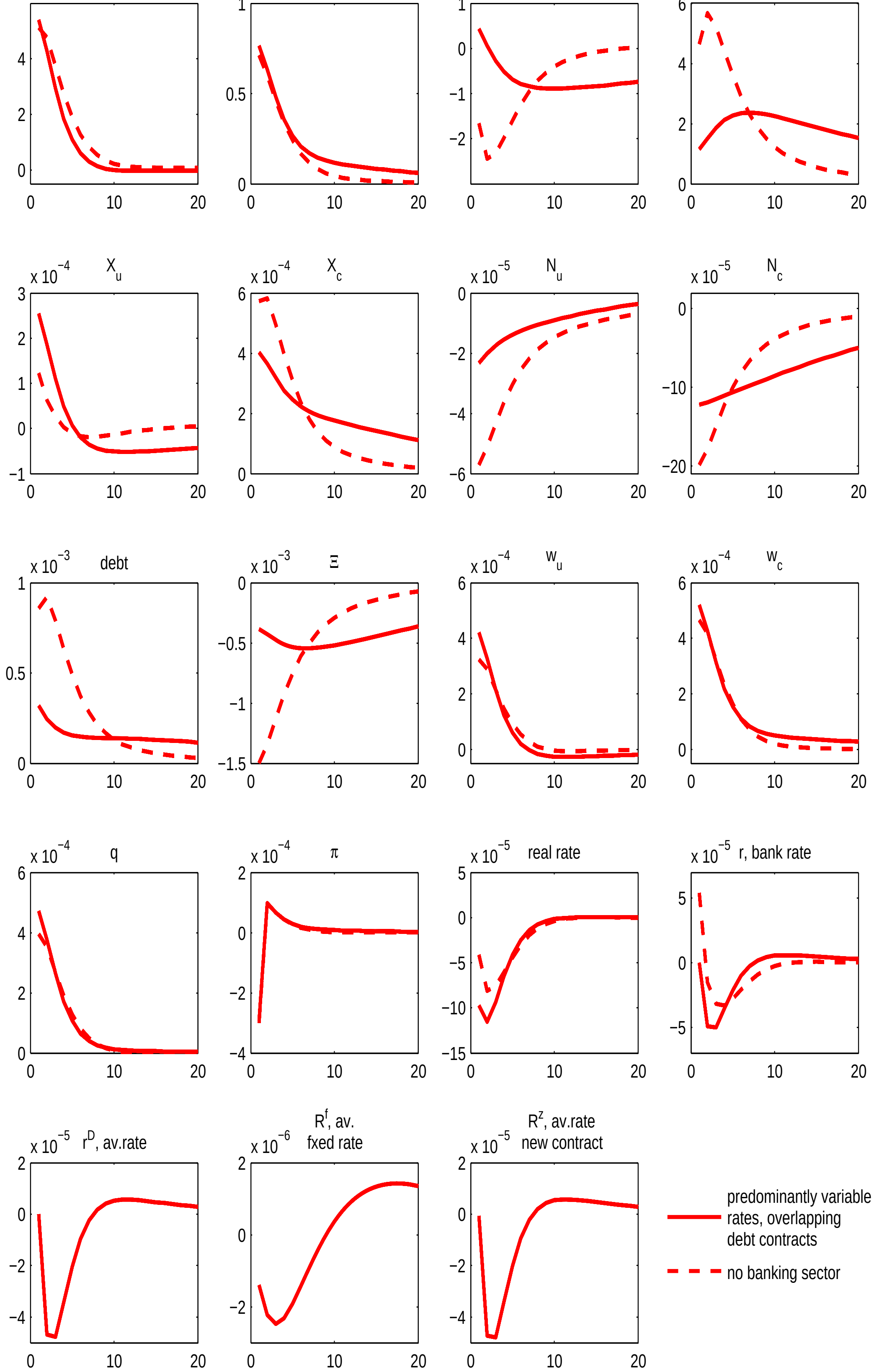

Figure 1: Impulse responses to a productivity shock in ND sector with s.d. 0.005. In the model with banking sector and overlapping debt contracts 95% of impatient households are on variable-rate contracts ($\Psi = 0.05$).

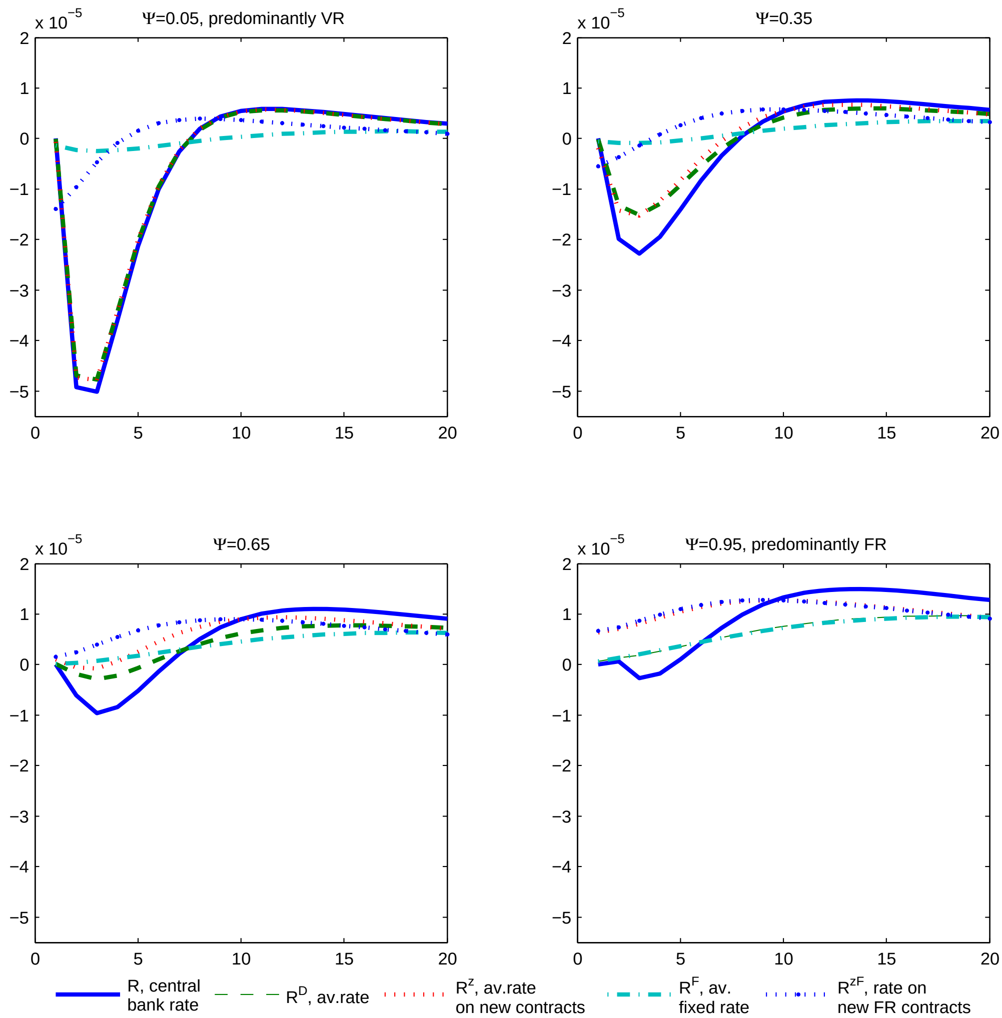

Figure 2: Impulse responses of interst rate to a productivity shock in ND sector with s.d. 0.005 for several values of Ψ.

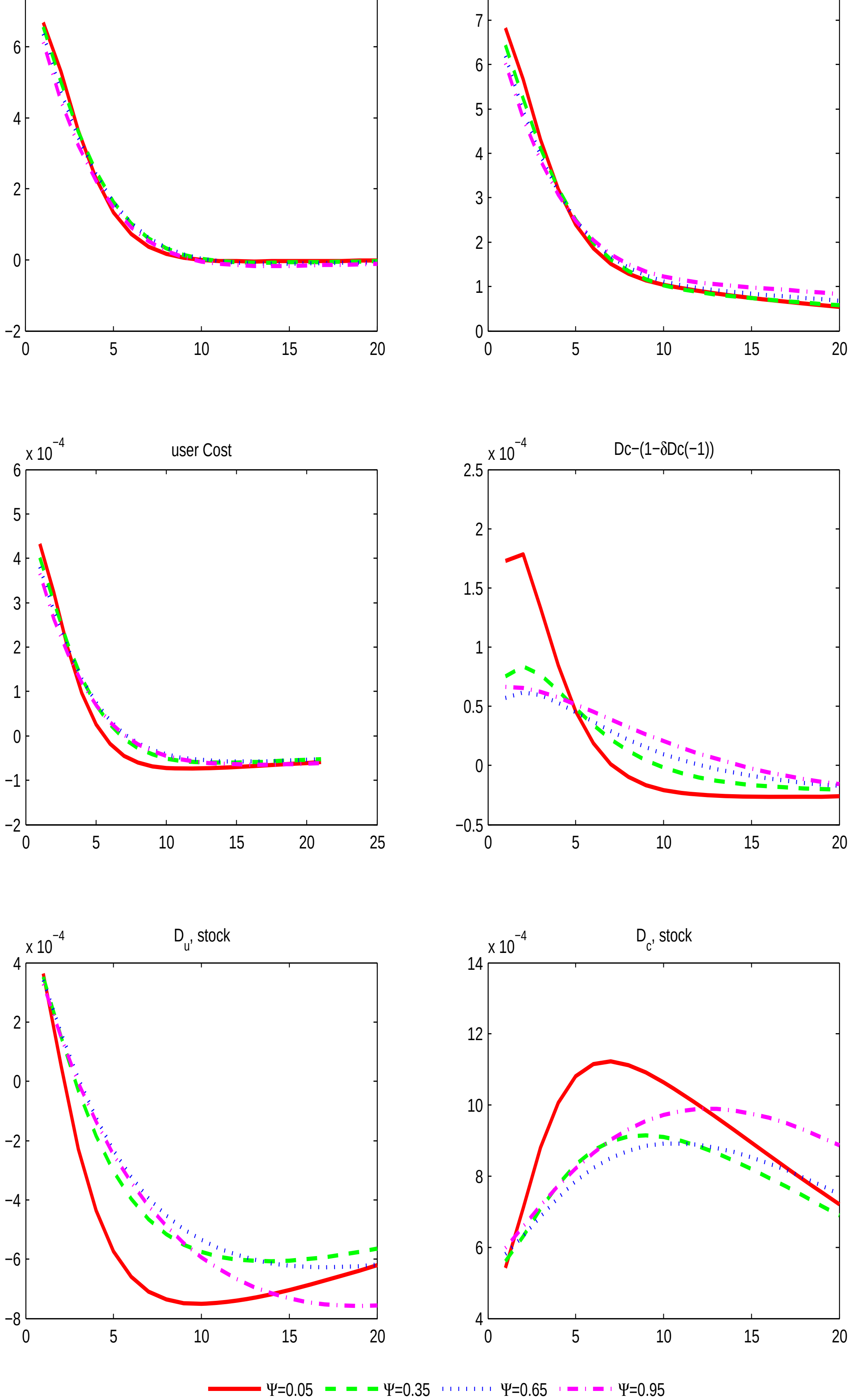

Figure 3: Impulse responses of consumption to a productivity shock in ND sector with s.d. 0.005 for several values of Ψ.

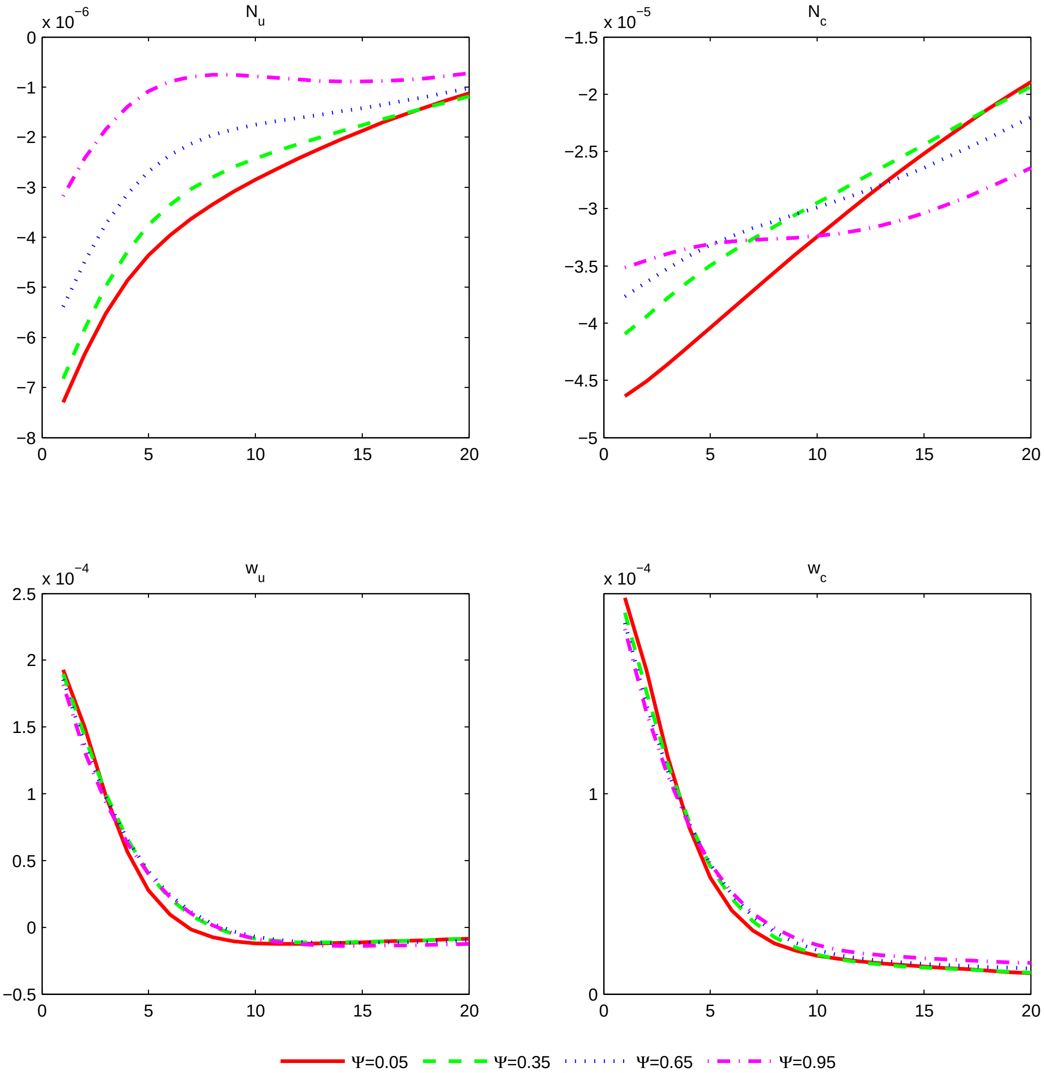

Figure 4: Impulse responses of labour supply and real wages to a productivity shock in ND sector with s.d. 0.005 for several values of Ψ.

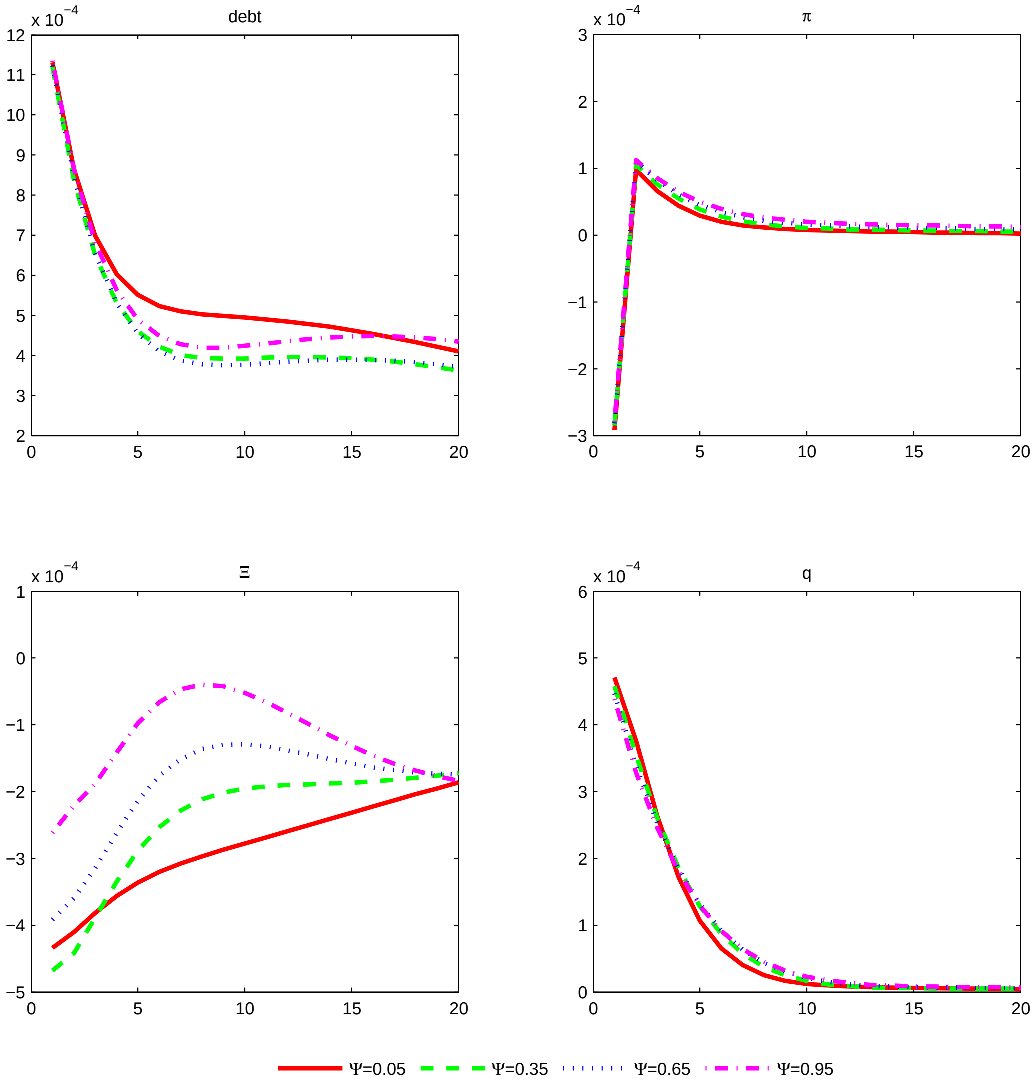

Figure 5: Impulse responses of debt, inflation, relative price and the strength of collateral constraintto a productivity shock in ND sector with s.d. 0.005 for several values of Ψ.

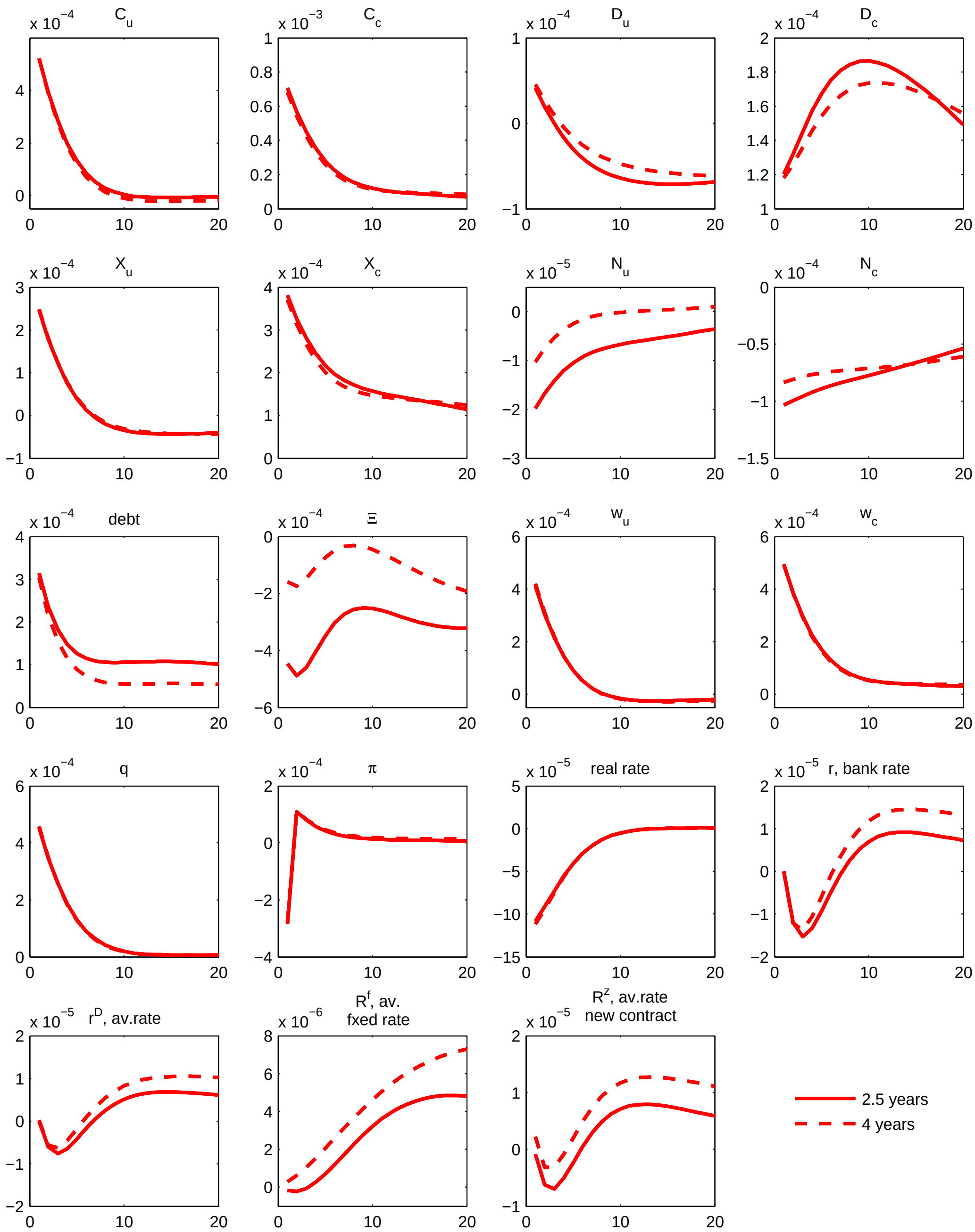

Figure 6: Impulse responses to an iid ND productivity shock with s.d. 0.005 for different average length of debt contracts

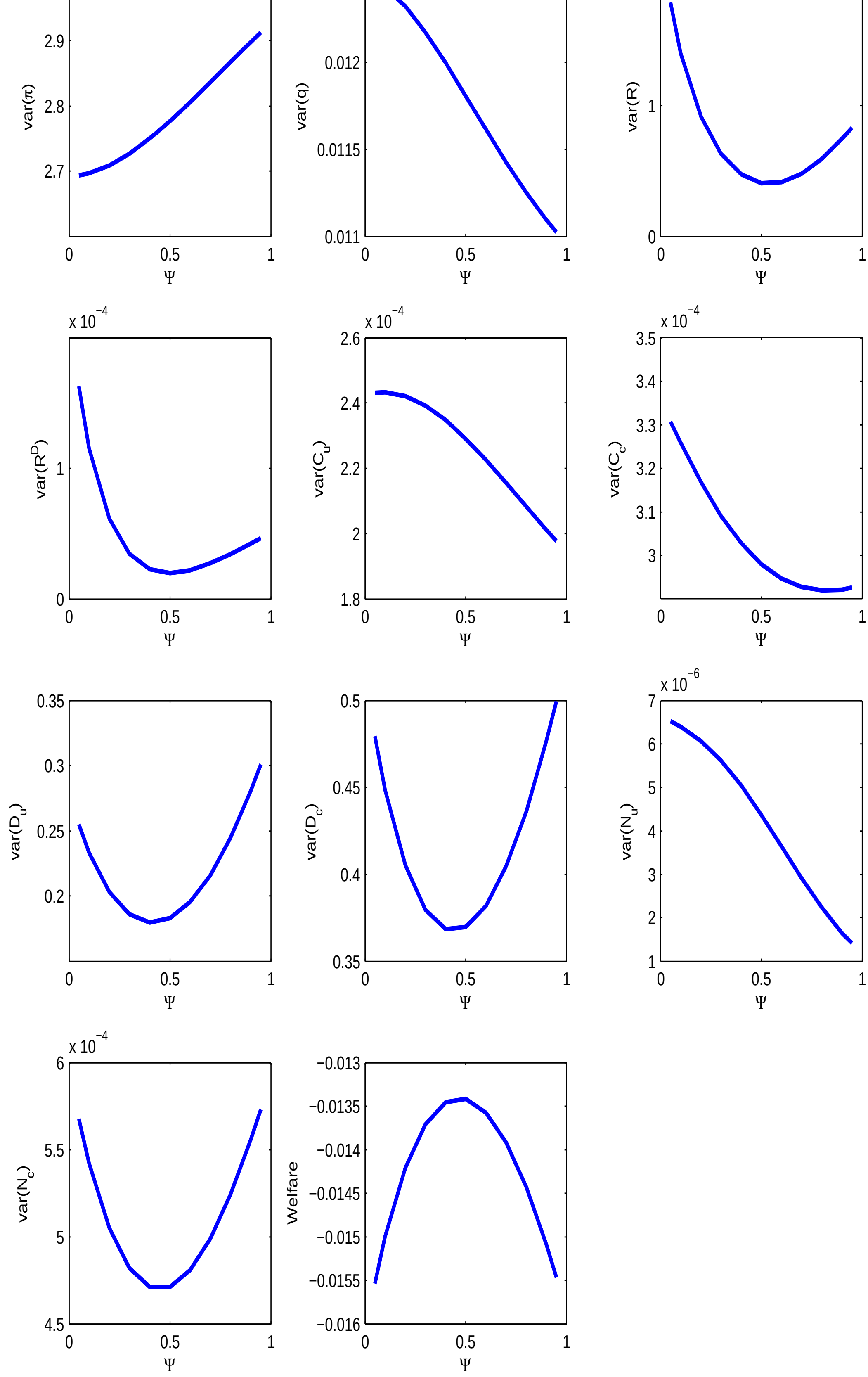

Figure 7: Variability of key variables as function of Ψ. Productivity shock in ND sector with s.d. 0.1

1 (eqn (2))	$X_{ut} = \left((1-\alpha)^{\frac{1}{\eta}} (C_{ut})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (D_{ut})^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$
2 (eqn (3))	$X_{ct} = \left((1-\alpha)^{\frac{1}{\eta}} (C_{ct})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (D_{ct})^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$
3 (eqn (8))	$X_{ut}^{\frac{1}{\eta}-\sigma} C_{ut}^{-\frac{1}{\eta}} q_t = \left(\frac{\alpha}{1-\alpha} \right)^{\frac{1}{\eta}} X_{ut}^{\frac{1}{\eta}-\sigma} D_{ut}^{-\frac{1}{\eta}} + \beta_u (1-\delta) E_t \left(X_{ut+1}^{\frac{1}{\eta}-\sigma} C_{ut+1}^{-\frac{1}{\eta}} q_{t+1} \right)$
4 (eqn (10))	$1 = \beta_u E_t \left(\frac{X_{ut+1}^{\frac{1}{\eta}-\sigma} C_{ut+1}^{-\frac{1}{\eta}}}{X_{ut}^{\frac{1}{\eta}-\sigma} C_{ut}^{-\frac{1}{\eta}}} \frac{(1+R_t)}{\Pi_{t+1}} \right)$
5 (eqn (13))	$a_{ct} = \left(1 + R_t^D\right) \begin{pmatrix} \frac{a_{ct-1}}{\Pi_t} + \nu \left(C_{ct} + q_t \left(D_{ct} - (1-\delta) D_{ct-1} \right) \right) \\ - (1-\nu) \left(C_{ut} + q_t \left(D_{ut} - (1-\delta) D_{ut-1} \right) \right) \end{pmatrix}$
6 (eqn (17))	$\left(\frac{\alpha}{(1-\alpha)} \frac{C_{ct}}{D_{ct}} \right)^{\frac{1}{\eta}} = q_t - \beta_c (1-\delta) \frac{X_{ct+1}^{\frac{1}{\eta}-\sigma} C_{ct+1}^{-\frac{1}{\eta}}}{X_{ct}^{\frac{1}{\eta}-\sigma} C_{ct}^{-\frac{1}{\eta}}} q_{s+1} - \rho \frac{\mathcal{M}_t}{\mathcal{B}_t} \Xi_t (1-\chi) q_{t+1} \Pi_{t+1}$
7 (eqn (19))	$\left(1 + R_t^D\right) \Xi_t = 1 - \beta_c E_t \left(\frac{\left(1+R_t^D\right)}{\Pi_{t+1}} \frac{X_{ct+1}^{\frac{1}{\eta}-\sigma} C_{ct+1}^{-\frac{1}{\eta}}}{X_{ct}^{\frac{1}{\eta}-\sigma} C_{ct}^{-\frac{1}{\eta}}} \left(1 - (1-\rho) \Xi_{t+1}\right) \right)$
8 (eqn (32))	$G_t = \frac{1}{Z_{ndt}} \frac{\mu\varkappa}{\nu^\nu (1-\nu)^{(1-\nu)}} C_{ut}^{\frac{\nu-1}{\eta}} X_{ut}^{\frac{(\sigma\eta-1)(\nu-1)}{\eta}} C_{ct}^{\frac{1}{\eta}(1-\nu)} X_{ct}^{\left(\sigma-\frac{1}{\eta}\right)(1-\nu)}$ $\times \left(\frac{(C_{ct}+C_{ut})}{Z_{ndt}} \Delta_{pt} + \frac{D_{ut}-(1-\delta)D_{ut-1}+D_{ct}-(1-\delta)D_{ct-1}}{Z_{dt}} \right)^{\phi} (C_{ct} + C_{ut}) + \theta\beta\Pi_{ct+1}^{\epsilon} G_{t+1}$
9 (eqn (33))	$F_t = (1-\alpha)^{\frac{1}{\eta}} X_{ut}^{\frac{1}{\eta}-\sigma} C_{ut}^{-\frac{1}{\eta}} (C_{ct} + C_{ut}) + \theta\beta\Pi_{ct+1}^{\epsilon-1} F_{t+1}$
10 (eqn (31))	$1 - \theta\Pi_t^{\epsilon-1} = (1-\theta) \left(\frac{G_t}{F_t} \right)^{1-\epsilon}$
11 (eqn (34))	$\Delta_t = (1-\theta) \left(\frac{1-\theta\Pi_t^{\epsilon-1}}{1-\theta} \right)^{\frac{\epsilon}{\epsilon-1}} + \theta\Pi_t^{\epsilon} \Delta_{t-1}$
12 (eqn (41))	$q_t = \frac{1}{Z_{dt}} \frac{\varkappa\mu}{\nu^\nu (1-\nu)^{(1-\nu)} (1-\alpha)^{\frac{1}{\eta}}} C_{ct}^{\frac{1}{\eta}(1-\nu)} X_{ct}^{\left(\sigma-\frac{1}{\eta}\right)(1-\nu)} C_{ut}^{\frac{1}{\eta}\nu} X_{ut}^{\left(\sigma-\frac{1}{\eta}\right)\nu}$ $\times \left(\frac{(C_{ct}+C_{ut})}{Z_{ndt}} \Delta_{pt} + \frac{D_{ut}-(1-\delta)D_{ut-1}+D_{ct}-(1-\delta)D_{ct-1}}{Z_{dt}} \right)^{\phi}$
13 (eqn (52))	$a_{ct} = (1-\rho) \frac{a_{ct-1}}{\Pi_{ct}} + \rho \frac{M_t}{B_t} (1-\chi) q_t D_{ct}$
14 (eqn (51))	$(1 + R_t) M_t = (1-\rho) E_t \left(M_{t+1} + \varpi + R_{t+1}^z - R_{t+1} \right)$
15 (eqn (51))	$(1 + R_t) B_t = (1-\rho) E_t \left(\frac{(B_{t+1}+\varpi)}{\Pi_{t+1}} \frac{q_t D_{ct}}{q_{t+1} D_{ct+1}} \right)$
16(eqn (53))	$(1 + R_t) V_t = (1-\rho) E_t (V_{t+1} + R_{t+1})$
17 (eqn (54))	$(1 + R_t) \mathcal{U}_t = (1-\rho) E_t (\mathcal{U}_{t+1} + 1)$
18 (eqn (55))	$R_t^{zF} = \frac{V_t}{\mathcal{U}_t}$
19 (eqn (50))	$R_t^z = (1-\Psi) R_t + \Psi R_t^{zF}$
20 (eqn (56))	$R_t^F = \rho R_t^{zF} + (1-\rho) R_{t-1}^F$
21 (eqn (57))	$R_t^D = (1-\Psi) R_t + \Psi R_t^F$

Table 1: Summary of dynamic system

which shows that the borrowing constraint Ξ does not bind without heterogeneity of time preference, as in Monacelli (2007). The reverse is also true: only a steady state in which there is a borrowing constraint can be consistent with our economy with households with different discount rates.

Combining equations 4 and 12 from the steady state system gives:

$$\frac{C_c}{D_c} = \frac{(1-\alpha)}{\alpha}\left((1-\beta_c(1-\delta)) - \Xi\frac{(1+R)\,\rho\,(1-\chi)\,(\beta_u-\beta_c)}{(\Pi-\beta_c(1+R))}\frac{\left(\Pi^2-\beta_u(1-\rho)\right)}{(\Pi-\beta_u(1-\rho))}\right)^{\eta} q^{\eta}$$

which shows that the steady state relative demand for durables is increasing in Ξ, consistent with the idea that households will want to hold more durables, the higher is the shadow value of the collateral constraint.

Equation 13-15 from the Table shows that in the steady state, constrained households will be more highly leveraged, defined by a high ratio of debt to durable goods owned, the higher is the loan to income ratio $(1-\chi)$:

$$\frac{a_c}{D_c} = (1-\chi)\,q\frac{\Pi\rho\left(\Pi^2-\beta_u(1-\rho)\right)}{(\Pi-(1-\rho))\,(\Pi-\beta_u(1-\rho))}$$

The presence of the banking sector with overlapping debt contracts affects steady state allocations of consumption. In our analogue of the Monacelli's model $\rho = 1$. Relative to that case, with $\rho \ll 1$ the collateral constraint binds less in the steady state, and X_u and X_c, N_c go down, while N_u goes up.

C Model Derivations

C.1 Households

C.1.1 Savers

Unconstrained households maximise their objective function:

$$\max U_{ut} = E_t\sum_{s=t}^{\infty}\beta_u^{s-t}u_{us}(C_{us}, D_{us}, N_{us})$$

1	$R = \frac{\Pi}{\beta_u} - 1$
2	$\Delta = \frac{(1-\theta)}{\left(1-\theta\Pi^{\epsilon}\right)} \left(\frac{1-\theta\Pi^{\epsilon-1}}{1-\theta}\right)^{\frac{\epsilon}{\epsilon-1}}$
3	$q = \frac{(1-\theta\beta_u\Pi^{\varepsilon})}{(1-\theta\beta_u\Pi^{\varepsilon-1})} \left(\frac{(1-\theta)}{(1-\theta\Pi^{\epsilon-1})}\right)^{\frac{1}{\epsilon-1}}$
4	$\frac{C_c}{D_c} = \frac{(1-\alpha)}{\alpha} \left((1-\beta_c(1-\delta)) - \frac{\rho(1-\chi)(\beta_u-\beta_c)}{(\Pi-\beta_c(1-\rho))} \frac{\left(\Pi^2-\beta_u(1-\rho)\right)}{(\Pi-\beta_u(1-\rho))} \right)^{\eta} q^{\eta}$
5	$\frac{C_u}{D_u} = \frac{(1-\alpha)}{\alpha} (1-\beta_u(1-\delta))^{\eta} q^{\eta}$
6	$\frac{X_u}{D_u} = \left((1-\alpha)^{\frac{1}{\eta}} (\frac{C_u}{D_u})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$
7	$\frac{X_c}{D_c} = \left[(1-\alpha)^{\frac{1}{\eta}} (\frac{C_c}{D_c})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$
8	$\frac{D_u}{D_c} = \frac{\nu\left(\frac{(1-\alpha)}{\alpha}\left((1-\beta_c(1-\delta)) - \frac{\rho(1-\chi)(\beta_u-\beta_c)}{(\Pi-\beta_c(1-\rho))}\frac{\left(\Pi^2-\beta_u(1-\rho)\right)}{(\Pi-\beta_u(1-\rho))}\right)^{\eta} q^{\eta-1}+\delta\right) + \frac{(1-\beta_u)\rho\left(\Pi^2-\beta_u(1-\rho)\right)(1-\chi)}{(\Pi-\beta_u(1-\rho))(\Pi-(1-\rho))}}{(1-\nu)\left(\frac{(1-\alpha)}{\alpha}(1-\beta_u(1-\delta))^{\eta} q^{\eta-1}+\delta\right)}$
9	$D_c = \left(\begin{array}{c} \frac{\nu^{\nu}(1-\nu)^{(1-\nu)}(1-\alpha)^{\frac{1}{\eta}}}{\mu\varkappa} \frac{(1-\theta\beta_u\Pi^{\varepsilon})}{(1-\theta\beta_u\Pi^{\varepsilon-1})} \left(\frac{(1-\theta)}{1-\theta\Pi^{\epsilon-1}}\right)^{\frac{1}{\epsilon-1}} \left(\frac{C_u}{D_u}\right)^{-\frac{\nu}{\eta}} \left(\frac{X_u}{D_u}\right)^{-\nu\left(\sigma-\frac{1}{\eta}\right)} \\ \times \left(\frac{C_c}{D_c}\right)^{-\frac{(1-\nu)}{\eta}} \left(\frac{X_c}{D_c}\right)^{-\left(\sigma-\frac{1}{\eta}\right)(1-\nu)} \left(\Delta\frac{C_c}{D_c} + \delta + \left(\Delta\frac{C_u}{D_u}+\delta\right)\frac{D_u}{D_c}\right)^{-\phi} \left(\frac{D_u}{D_c}\right)^{-\nu\sigma} \end{array} \right)^{\frac{1}{\sigma+\phi}}$
10	$G = \frac{\mu\varkappa C_u^{\frac{\nu-1}{\eta}} X_u^{\frac{(\sigma\eta-1)(\nu-1)}{\eta}} C_c^{\frac{1}{\eta}(1-\nu)} X_c^{\left(\sigma-\frac{1}{\eta}\right)(1-\nu)} ((C_c+C_u)\Delta+\delta D_u+\delta D_c)^{\phi}(C_c+C_u)}{\nu^{\nu}(1-\nu)^{(1-\nu)}(1-\theta\beta_u\Pi^{\varepsilon})}$
11	$F = \frac{(1-\alpha)^{\frac{1}{\eta}} X_u^{\frac{1}{\eta}-\sigma} C_u^{-\frac{1}{\eta}} (C_c+C_u)}{\left(1-\theta\beta_u\Pi_c^{\varepsilon-1}\right)}$
12	$\Xi = \frac{(\Pi-\beta_c(1+R))}{(1+R)(\Pi-\beta_c(1-\rho))}$
13	$M = \frac{\beta_u(1-\rho)(1-\alpha)^{\frac{1}{\eta}} X_u^{\frac{1}{\eta}-\sigma} C_u^{-\frac{1}{\eta}} \varpi}{(\Pi-\beta_u(1-\rho))}$
14	$B = \frac{\beta_u(1-\rho)\varpi(1-\alpha)^{\frac{1}{\eta}} X_u^{\frac{1}{\eta}-\sigma} C_u^{-\frac{1}{\eta}}}{(\Pi^2-\beta_u(1-\rho))}$
15	$a_c = \rho\frac{M\Pi(1-\chi)qD_c}{B(\Pi-(1-\rho))}$
16	$U = \frac{\beta_u(1-\rho)(1-\alpha)^{\frac{1}{\eta}} X_u^{\frac{1}{\eta}-\sigma} C_u^{-\frac{1}{\eta}}}{(\Pi-\beta_u(1-\rho))}$
17	$V = UR$

subject to the real budget constraint:

$$a_{ut} = (1 + R_t) \left(\frac{a_{ut-1}}{\Pi_t} + w_{ut} N_{ut} + t_{ut} - C_{ut} - q_t \left(D_{ut} - (1 - \delta) D_{ut-1} \right) \right) + \tilde{d}_t$$

The associated Lagrangian is:

$$L = \sum_{s=t}^{\infty} \beta_u^{s-t} \left(U_{us} + \Lambda_s \left(a_{us} - (1 + R_s) \begin{pmatrix} \frac{a_{us-1}}{\Pi_s} + w_{us} N_{us} + t_{us} \\ -C_{us} - q_s \left(D_{us} - (1 - \delta) D_{us-1} \right) \end{pmatrix} - \tilde{d}_s \right) \right)$$

For the partial derivative of the objective utility function U_{us} with respect to a choice variable x_s, we use notation $\frac{\delta U_s}{\delta x_s} = U_x$. FOCs are then:

$$\frac{\partial L}{\partial C_{us}} = \beta_u^{s-t} \left(U_{nd} + \Lambda_s (1 + R_s) \right) = 0 \tag{59}$$

$$\frac{\partial L}{\partial D_{us}} = \beta_u^{s-t} \left(U_d + \Lambda_s (1 + R_s) q_s \right) - \beta_u^{s-t+1} \Lambda_{s+1} (1 + R_{s+1}) q_{s+1} (1 - \delta) = 0 \tag{60}$$

$$\frac{\partial L}{\partial N_{us}} = \beta_u^{s-t} \left(U_n - \Lambda_s (1 + R_s) w_{us} \right) = 0 \tag{61}$$

$$\frac{\partial L}{\partial a_{us}} = \beta_u^{s-t} \Lambda_s - \beta_u^{s-t+1} \Lambda_{s+1} \frac{(1 + R_{s+1})}{\Pi_{s+1}} = 0 \tag{62}$$

$$\frac{\partial L}{\partial \Lambda_s} = a_{us} - (1 + R_s) \begin{pmatrix} \frac{a_{us-1}}{\Pi_{c,s}} + w_{us} N_{us} + t_{us} \\ -C_{us} - q_s \left(D_{us} - (1 - \delta) D_{us-1} \right) \end{pmatrix} - \tilde{d}_s = 0 \tag{63}$$

where the objective function partial derivatives are:

$$\begin{aligned} U_{nd} &= U_x \frac{\partial X}{\partial C_{us}} = (1 - \alpha)^{\frac{1}{\eta}} X_{us}^{\frac{1}{\eta} - \sigma} C_{us}^{-\frac{1}{\eta}} \\ U_d &= U_x \frac{\partial X}{\partial D_{us}} = \alpha^{\frac{1}{\eta}} X_{us}^{\frac{1}{\eta} - \sigma} D_{us}^{-\frac{1}{\eta}} \\ U_n &= -\varkappa N_{us}^{\phi} \end{aligned}$$

We can define the stochastic discount factor by combining FOCs (59) and (62):

$$Q_{t,t+1} = \beta_u \frac{U_{nd,ut+1}}{U_{nd,ut}} \frac{P_{nd,t}}{P_{nd,t+1}} \tag{64}$$

which implies that from period t iterating forward to period s we have:

$$Q_{t,s} = \beta_u^{s-t} \frac{U_{nd,us}}{U_{nd,ut}} \frac{P_{nd,t}}{P_{nd,s}} \tag{65}$$

Finally, after rearranging, we obtain system (7)-(10).

C.1.2 Borrowers

Constrained households maximise their objective function:

$$\max U_{ct} = E_t \sum_{s=t}^{\infty} \beta_c^{s-t} u_{cs}(C_{cs}, D_{cs}, N_{cs})$$

subject to the real budget constraint:

$$a_{ct} = \left(1 + R_t^D\right) \left(\frac{a_{ct-1}}{\Pi_t} + C_{ct} + q_t \left(D_{ct} - (1-\delta) D_{ct-1}\right) - w_{ct} N_{ct} + t_{ct} \right)$$

and the collateral constraint:

$$a_{ct} = (1-\rho) \frac{a_{ct-1}}{\Pi_t} + \rho \Omega \left[k_t\right]$$

The associated Lagrangian is:

$$L = \sum_{s=t}^{\infty} \beta_c^{s-t} \left(\begin{array}{c} U_{cs} + \Theta_s \left(a_{cs} - \left(1 + R_s^D\right) \left(\begin{array}{c} \frac{a_{cs-1}}{\Pi_s} + C_{cs} - w_{cs} N_{cs} + t_{cs} \\ + q_s \left(D_{cs} - (1-\delta) D_{cs-1}\right) \end{array} \right) - \omega_s^A \right) \\ -\Gamma_s \left(a_{cs} - (1-\rho) \frac{a_{cs-1}}{\Pi_s} - \rho \Omega \left[k_t\right] \right) \end{array} \right)$$

where the function $\Omega\left[k_t\right]$ is defined in section C.3.2 as $\frac{M_t}{B_t} k_t$, and real collateral is $k_t = (1-\chi) D_{ct} q_t$. In the following, we define the term $\Omega_k\left[k_s\right] = \frac{\partial \Omega[k_s]}{\partial k_s} = \frac{M_s}{B_s}$, so that using the chain rule, $\frac{\partial \Omega[k_s]}{\partial D_{cs}} = \frac{\partial \Omega[k_s]}{\partial k_s} \frac{\partial k_s}{\partial D_{cs}} = \Omega_k\left[k_s\right](1-\chi) q_s$.We also define $\Xi_s = \Gamma_s / U_{nd,cs}$ to derive the following FOCs:

$$\frac{\partial L}{\partial C_{cs}} = \beta_c^{s-t}[U_{nd,cs} - \Theta_s \left(1 + R_s^D\right)] = 0 \tag{66}$$

$$0 = \frac{\partial L}{\partial D_{cs}} = \beta_c^{s-t}[U_{d,cs} - \Theta_s \left(1 + R_s^D\right) q_s + \Gamma_s \rho \Omega_k [k_s](1-\chi) q_s] \tag{67}$$

$$+\beta_c^{s-t+1} \Theta_{s+1} \left(1 + R_{s+1}^D\right) q_{s+1} (1 - \delta) \tag{68}$$

$$\frac{\partial L}{\partial N_{cs}} = \beta_c^{s-t} \left(U_{n,cs} + \Theta_s \left(1 + R_s^D\right) w_{cs}\right) = 0 \tag{69}$$

$$\frac{\partial L}{\partial \Theta_s} = a_{cs} - \left(1 + R_s^D\right) \left(\frac{a_{cs-1}}{\Pi_s} + C_{cs} + q_s \left(D_{cs} - (1-\delta) D_{cs-1}\right) - w_{cs} N_{cs} - t_{cs}\right) = 0 \tag{70}$$

$$\frac{\partial L}{\partial \Gamma_s} = a_{cs} - (1-\rho) \frac{a_{cs-1}}{\Pi_s} - \rho \Omega [k_t] = 0 \tag{71}$$

$$\frac{\partial L}{\partial a_{cs}} = \beta_c^{s-t} (\Theta_s - \Xi_s) + \beta_c^{s-t+1} \left(-\Theta_{s+1} \frac{\left(1 + R_{s+1}^D\right)}{\Pi_{s+1}} + \Xi_{s+1} \frac{(1-\rho)}{\Pi_{s+1}}\right) = 0 \tag{72}$$

where the objective function partial derivatives are:

$$U_{nd,cs} = U_x \frac{\partial X_s}{\partial C_{cs}} = (1-\alpha)^{\frac{1}{\eta}} X_{cs}^{\frac{1}{\eta}-\sigma} C_{cs}^{-\frac{1}{\eta}}$$

$$U_{d,cs} = U_x \frac{\partial X_s}{\partial D_{cs}} = \alpha^{\frac{1}{\eta}} X_{cs}^{\frac{1}{\eta}-\sigma} D_{cs}^{-\frac{1}{\eta}}$$

$$U_{n,cs} = -\varkappa N_{cs}^{\phi}$$

Finally, after rearranging, we obtain system (16)-(19).

C.2 Price Setting

C.2.1 Non-Durable Prices

The FOC (30):

$$E_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} P_{nds} y_{nds}(i) \left(\frac{p_{ndt}^*(i)}{P_{nds}} - \mu m c_s\right) = 0$$

is rearranged as:

$$\left(\frac{p_{ndt}^*(i)}{P_{ndt}}\right) = \frac{E_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} Y_{nds} \mu m c_s \left(\frac{P_{ndt}}{P_{nds}}\right)^{-(\epsilon+1)}}{E_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} Y_{nds} \left(\frac{P_{ndt}}{P_{nds}}\right)^{-\epsilon}} \tag{73}$$

We substitute the unconstrained household discount factor from equation (12) and define the numerator in (73) as:

$$\begin{aligned}
G_t &= E_t \sum_{s=t}^{\infty} (\theta\beta)^{s-t} \mu U_{nd}\left(C_{us}, D_{us}, N_{us}\right) Y_{nds} mc_s \left(\frac{P_{nds}}{P_{ndt}}\right)^{\epsilon} \\
&= E_t \sum_{s=t}^{\infty} (\theta\beta)^{s-t} g_s \left(\frac{P_{nds}}{P_{ndt}}\right)^{\epsilon}
\end{aligned}$$

and the denominator as:

$$\begin{aligned}
F_t &= E_t \sum_{s=t}^{\infty} (\theta\beta)^{s-t} U_{nd}\left(C_{us}, D_{us}, N_{us}\right) Y_{nds} \left(\frac{P_{nds}}{P_{ndt}}\right)^{\epsilon-1} \\
&= E_t \sum_{s=t}^{\infty} (\theta\beta)^{s-t} f_s \left(\frac{P_{nds}}{P_{ndt}}\right)^{\epsilon-1}
\end{aligned}$$

where:

$$\begin{aligned}
g_s &= \mu U_{nd} Y_{nds} mc_s \\
&= \frac{1}{Z_{nds}} \frac{\varkappa \mu N_{us}^{\phi\nu} C_{us}^{\frac{\nu-1}{\eta}} X_{us}^{\left(\sigma-\frac{1}{\eta}\right)(\nu-1)} N_{cs}^{\phi(1-\nu)} C_{cs}^{\frac{1}{\eta}(1-\nu)} X_{cs}^{\left(\sigma-\frac{1}{\eta}\right)(1-\nu)} Y_{nds}}{\nu^{\nu}(1-\nu)^{(1-\nu)}} \\
f_s &= U_{nd}\left(C_{us}, D_{us}, N_{us}\right) Y_{nds} = (1-\alpha)^{\frac{1}{\eta}} X_{us}^{\frac{1}{\eta}-\sigma} C_{us}^{-\frac{1}{\eta}} Y_{nds}
\end{aligned}$$

It follows that:

$$\begin{aligned}
G_t &= g_t + \theta\beta E_t \Pi_{t+1}^{\epsilon} G_{t+1} \\
F_t &= f_t + \theta\beta E_t \Pi_{t+1}^{\epsilon-1} F_{t+1}
\end{aligned}$$

The price level in the non-durable sector is determined as:

$$P_{ndt} = \left[(1-\theta)\left(p_{ndt}^{*}\right)^{1-\epsilon} + \theta P_{ndt-1}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

From where:

$$\left(\frac{p_{ndt}^{*}}{P_{ndt}}\right)^{1-\epsilon} = \frac{\Pi_t^{1-\epsilon}-\theta}{(1-\theta)\Pi_t^{1-\epsilon}} = \frac{1-\theta\Pi_t^{\epsilon-1}}{(1-\theta)} = \left(\frac{p_{ct}^{*}}{P_{ct}}\right)^{1-\epsilon} = \left(\frac{G_t}{F_t}\right)^{1-\epsilon}$$

And finally we obtain our price-setting equation (New Keynesian Phillips curve) for the non-durable goods sector:

$$\frac{1-\theta\Pi_t^{\epsilon-1}}{(1-\theta)} = \left(\frac{G_t}{F_t}\right)^{1-\epsilon}$$

where

$$\begin{aligned} G_t &= \mu U_{nd,ut} Y_{ndt} mc_t + \theta\beta E_t\left[\Pi_{t+1}^{\epsilon} G_{t+1}\right] \\ F_t &= U_{nd,ut} Y_{ndt} + \theta\beta E_t\left[\Pi_{t+1}^{\epsilon-1} F_{t+1}\right] \end{aligned}$$

C.2.2 Durable Relative Prices

Durable goods firms choose prices to maximise expected their profit function:

$$\max_{\{p_s^*(i)\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} Q_{t,s}\left(y_{ds}(i)\, p_{ds}(i) - W_{us} N_{dus}(i) - W_{cs} N_{dcs}(i)\right)$$

Hours worked by unconstrained and constrained households from equations (39) and (40) can be written as:

$$\begin{aligned} N_{dut}(i) &= \frac{1}{Z_{dt}} y_{dt}(i) \frac{\left(\frac{W_{ut}}{P_{ct}}\right)^{(\nu-1)}}{\nu^{(\nu-1)}} \frac{\left(\frac{W_{ct}}{P_{ct}}\right)^{(1-\nu)}}{(1-\nu)^{(1-\nu)}} = \frac{1}{Z_{dt}} y_{dt}(i) \frac{W_{ut}^{(\nu-1)}}{\nu^{(\nu-1)}} \frac{W_{ct}^{(1-\nu)}}{(1-\nu)^{(1-\nu)}} \\ N_{dct}(i) &= \frac{1}{Z_{dt}} y_{dt}(i) \frac{w_{ut}^{\nu}}{\nu^{\nu}} \frac{w_{ct}^{(-\nu)}}{(1-\nu)^{(-\nu)}} = \frac{1}{Z_{dt}} y_{dt}(i) \frac{W_{ut}^{\nu}}{\nu^{\nu}} \frac{W_{ct}^{(-\nu)}}{(1-\nu)^{(-\nu)}} \end{aligned}$$

Substitute into the profit function to obtain:

$$\begin{aligned} & E_t \sum_{s=t}^{\infty} Q_{t,s} \begin{pmatrix} y_{ds}(i)\, p_{ds}(i) - W_{us} \frac{1}{Z_{dt}} y_{ds}(i) \frac{W_{us}^{(\nu-1)}}{\nu^{(\nu-1)}} \frac{W_{cs}^{(1-\nu)}}{(1-\nu)^{(1-\nu)}} \\ -W_{cs} \frac{1}{Z_{dt}} y_{ds}(i) \frac{W_{us}^{\nu}}{\nu^{\nu}} \frac{W_{cs}^{(-\nu)}}{(1-\nu)^{(-\nu)}} \end{pmatrix} \\ = \; & E_t \sum_{s=t}^{\infty} Q_{t,s}\left(y_{ds}(i)\, p_{ds}(i) - \frac{1}{Z_{dt}} y_{ds}(i) \frac{W_{us}^{\nu}}{\nu^{\nu}} \frac{W_{cs}^{(1-\nu)}}{(1-\nu)^{(1-\nu)}}\right) \\ = \; & E_t \sum_{s=t}^{\infty} Q_{t,s}\left(y_{ds}(i)\, p_{ds}(i) - y_{ds}(i) MC_{ds}\right) \end{aligned}$$

where $MC_{ds} = \zeta_t P_{dt}$. Note that wages here do not depend on index i, since labour of each type is assumed to be perfectly mobile and so wages for particular household type are equalised across all firms. So we come to the familiar formulation:

$$\max_{\{p_s^*(i)\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} Q_{t,s} \left(y_{ds}\left(i\right) p_{ds}\left(i\right) - y_{ds}(i) MC_{ds}\right)$$

subject to:

$$y_{dt}\left(i\right) = Y_{dt} \left(\frac{p_{dt}\left(i\right)}{P_{dt}}\right)^{-\epsilon}$$

The problem for the optimal price setting at time t can equivalently be written as:

$$\max_{\{p_{ds}(i)\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} Q_{t,s} \left(y_{ds}\left(i\right) p_{ds}\left(i\right) - y_{ds}(i) MC_{ds}\right)$$

Substitute demand:

$$\max_{\{p_{ds}(i)\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} P_{ds} Y_{ds} \left(\left(\frac{p_{ds}\left(i\right)}{P_{ds}}\right)^{1-\epsilon} - \left(\frac{p_{ds}\left(i\right)}{P_{ds}}\right)^{-\epsilon} \frac{MC_{ds}}{P_{ds}} \right)$$

So the FOC is:

$$\begin{aligned}
\frac{\partial}{\partial p_s\left(i\right)} Q_{t,s} P_{ds} Y_{ds} \left(\left(\frac{p_{ds}\left(i\right)}{P_{ds}}\right)^{1-\epsilon} - \left(\frac{p_{ds}\left(i\right)}{P_{ds}}\right)^{-\epsilon} mc_{ds} \right) &= 0 \\
Q_{t,s} Y_{ds} \left(\left(1-\epsilon\right) \left(\frac{p_{ds}\left(i\right)}{P_{ds}}\right)^{-\epsilon} + \epsilon \left(\frac{p_{ds}\left(i\right)}{P_{ds}}\right)^{-\epsilon-1} mc_{ds} \right) &= 0
\end{aligned}$$

$$\frac{p_{dt}\left(i\right)}{P_{ds}} - \mu mc_{ds} = 1 - \mu mc_{ds} = 0$$

where $\mu = -\frac{\epsilon}{(1-\epsilon)}$ and $mc_{ds} = \frac{MC_{ds}}{P_{ds}}$.

Real marginal cost will always be set equal to the constant $\frac{\epsilon-1}{\epsilon}$, and switching to time subscript t we have:

$$\begin{aligned}
\frac{MC_{dt}}{P_{dt}} &= \frac{1}{Z_{dt}} \frac{1}{\nu^{\nu}\left(1-\nu\right)^{(1-\nu)}} \frac{W_{ct}^{(1-\nu)} W_{ut}^{\nu}}{P_{dt}} = \frac{\epsilon - 1}{\epsilon} \\
\frac{1}{Z_{dt}} \frac{W_{ct}^{(1-\nu)} W_{ut}^{\nu}}{P_{dt}} &= \frac{\epsilon-1}{\epsilon} \nu^{\nu} \left(1-\nu\right)^{(1-\nu)} \\
\frac{1}{Z_{dt}} \left(\frac{W_{ct}}{P_{ndt}}\right)^{(1-\nu)} \left(\frac{W_{ut}}{P_{ndt}}\right)^{\nu} \frac{P_{ndt}}{P_{dt}} &= \frac{\epsilon-1}{\epsilon} \nu^{\nu} \left(1-\nu\right)^{(1-\nu)}
\end{aligned}$$

Substituting wages from the household FOCs (9) and (18) we then obtain:

$$\begin{aligned}
& \frac{1}{Z_{dt}}\left(\frac{\varkappa}{(1-\alpha)^{\frac{1}{\eta}}}N_{ct}^{\phi}C_{ct}^{\frac{1}{\eta}}X_{ct}^{\sigma-\frac{1}{\eta}}\right)^{(1-\nu)}\left(\frac{\varkappa}{(1-\alpha)^{\frac{1}{\eta}}}N_{ut}^{\phi}C_{ut}^{\frac{1}{\eta}}X_{ut}^{\sigma-\frac{1}{\eta}}\right)^{\nu} \\
= & \; q_t\frac{\epsilon-1}{\epsilon}\nu^{\nu}\left(1-\nu\right)^{(1-\nu)} \\
& \frac{1}{Z_{dt}}\frac{\varkappa}{(1-\alpha)^{\frac{1}{\eta}}}N_{ct}^{\phi(1-\nu)}C_{ct}^{\frac{1}{\eta}(1-\nu)}X_{ct}^{\left(\sigma-\frac{1}{\eta}\right)(1-\nu)}N_{ut}^{\phi\nu}C_{ut}^{\frac{1}{\eta}\nu}X_{ut}^{\left(\sigma-\frac{1}{\eta}\right)\nu} \\
= & \; q_t\frac{\epsilon-1}{\epsilon}\nu^{\nu}\left(1-\nu\right)^{(1-\nu)}
\end{aligned}$$

C.3 Financial Intermediaries

C.3.1 Optimal Debt Offers

The present discounted value of profits associated with lending amount Z_{t+1} is:

$$W_t=\sum_{s=t+1}^{\infty}Q_{t,s}\left(1-\rho\right)^{s-t}\left(\left(R_s^z-R_s\right)Z_{t+1}-\frac{\varpi}{2}\left(\frac{Z_{t+1}-K_s}{K_s}\right)^2K_s\right)$$

So the FOC for choosing Z_{t+1} to maximise profit is:

$$\frac{\partial W_t}{\partial Z_{t+1}}=\sum_{s=t+1}^{\infty}Q_{t,s}\left(1-\rho\right)^{s-t}\left(\left(R_s^z-R_s\right)-\varpi\left(\frac{Z_{t+1}-K_s}{K_s}\right)\right)=0$$

$$\sum_{s=t+1}^{\infty}Q_{t,s}\left(1-\rho\right)^{s-t}\left(R_s^z-R_s\right)=\varpi\sum_{s=t+1}^{\infty}Q_{t,s}\left(1-\rho\right)^{s-t}\left(\frac{Z_{t+1}-K_s}{K_s}\right) \tag{74}$$

$$\sum_{s=t+1}^{\infty}Q_{t,s}\left(1-\rho\right)^{s-t}\left(R_s^z-R_s+\varpi\right)=\varpi Z_{t+1}\sum_{s=t+1}^{\infty}Q_{t,s}\left(1-\rho\right)^{s-t}K_s^{-1} \tag{75}$$

We define the variables (replacing $Q_{t,s}$ with the unconstrained stochastic discount factor of equation (65):

$$\begin{aligned}
M_t &= \sum_{s=t+1}^{\infty}\frac{U_{nd,us}}{U_{nd,ut}}\frac{P_{nd,t}}{P_{nd,s}}\left(\beta_u(1-\rho)\right)^{s-t}\left(R_s^z-R_s+\varpi\right) \\
B_t &= \varpi\sum_{s=t+1}^{\infty}\frac{U_{nd,us}}{U_{nd,ut}}\frac{P_{nd,t}}{P_{nd,s}}\left(\beta_u(1-\rho)\right)^{s-t}\frac{K_t}{K_s}
\end{aligned}$$

so equation (75) becomes:

$$\frac{Z_{t+1}}{K_t} = \frac{M_t}{B_t} \tag{76}$$

and since variables with subscript t are constant across s, and defining a new variable $L_t = \frac{B_t}{K_t}$, we can write:

$$\begin{aligned}
M_t &= \sum_{s=t+1}^{\infty} U_{nd,us} \frac{P_{nd,t}}{P_{nd,s}} \left(\beta_u(1-\rho)\right)^{s-t} \left(R_s^z - R_s + \varpi\right) \\
L_t &= \varpi \sum_{s=t+1}^{\infty} U_{nd,us} \frac{P_{nd,t}}{P_{nd,s}} \left(\beta_u(1-\rho)\right)^{s-t} K_s^{-1}
\end{aligned}$$

Now, deal with M_t first. Define $m_s = U_{nd,us}\left(R_s^z - R_s + \varpi\right)$, rearrange, and use the fact that for $s = t$, $M_t = m_t$. Finally we define the variable $\tilde{M}_t = M_t + m_t$, so we have:

$$\begin{aligned}
M_t &= \sum_{s=t+1}^{\infty} m_s \left(\frac{P_{nd,s}}{P_{nd,t}}\right)^{-1} \left(\beta_u(1-\rho)\right)^{s-t} \\
M_t &= -m_t + \sum_{s=t}^{\infty} m_s \left(\frac{P_{nd,s}}{P_{nd,t}}\right)^{-1} \left(\beta_u(1-\rho)\right)^{s-t} \\
\tilde{M}_t &= M_t + m_t = \sum_{s=t}^{\infty} m_s \left(\frac{P_{nd,s}}{P_{nd,t}}\right)^{-1} \left(\beta_u(1-\rho)\right)^{s-t} \qquad (77)
\end{aligned}$$

Equation (77) is a direct analogue of the Calvo price setting formula derived in Woodford (2003), with appropriate timing.

It then follows that the relationship between $\tilde{M}_t$ and $\tilde{M}_{t+1}$ is defined by:

$$\tilde{M}_t = m_t + \beta_u (1-\rho) \Pi_{t+1}^{-1} \tilde{M}_{t+1}$$

so replacing our definitions $\tilde{M}_t = M_t + m_t$ and $m_t = U_{nd,ut}\left(R_t^z - R_t + \varpi\right)$ we have:

$$M_t = \beta_u (1-\rho) \Pi_{t+1}^{-1} \left(M_{t+1} + U_{nd,ut+1}\left(R_{t+1}^z - R_{t+1} + \varpi\right)\right) \tag{78}$$

We can follow the same procedure for L_t to derive:

$$L_t = \beta_u (1-\rho) \Pi_{t+1}^{-1} \left(L_{t+1} + \varpi U_{nd,ut+1} K_{t+1}^{-1}\right)$$

so that given our definition $L_t = \frac{B_t}{K_t}$, B_t is:

$$B_t = \beta_u (1-\rho) \Pi_{t+1}^{-1} \left(B_{t+1} + \varpi U_{nd,ut+1}) K_t K_{t+1}^{-1} \right) \tag{79}$$

and we can rearrange the equation (76) for Z_{t+1}:

$$\frac{Z_{t+1}}{P_{ndt}} = \frac{K_t}{P_{ndt}} \frac{M_t}{B_t} \tag{80}$$

$$\begin{aligned}
\frac{Z_{t+1}}{P_{ndt}} &= \frac{K_t}{P_{ndt}} \frac{M_t}{B_t} \\
M_t &= \beta_u (1-\rho) \Pi_{t+1}^{-1} \left(M_{t+1} + U_{nd,ut+1} \left(R_{t+1}^z - R_{t+1} + \varpi \right) \right) \\
B_t &= \beta_u (1-\rho) \Pi_{t+1}^{-1} \left(B_{t+1} + \varpi U_{nd,ut+1} \right) K_t K_{t+1}^{-1}
\end{aligned} \tag{81}$$

Equations (78), (79) and (80) define the dynamics of the profit maximising level of real debt that is issued by individual financial intermediaries $\left(\frac{Z_{t+1}}{P_{ndt}}\right)$, as a function of inflation $\left(\Pi_{t+1}^{-1}\right)$, collateral, $\left(\frac{K_t}{K_{t+1}}\right)$, the markup of the average rate on debt over the central bank rate $(R_{t+1}^z - R_{t+1})$, and the marginal utility of consumption of non-durables by unconstrained households $(U_{nd,ut+1})$. These equations also show how real debt evolution is affected by the discount rate of unconstrained households β_u, the stickiness of debt contracts ρ, and the parameter ϖ (which captures the relative size of quadratic costs associated with deviating from the optimal level of debt).

C.3.2 Evolution of Aggregate Debt

Aggregate nominal debt $\mathcal{Z}$ at time t is the sum of all debt contracts written prior to and at time t:

$$\mathcal{Z}_t = \rho \sum_{s=-\infty}^{t} (1-\rho)^{t-s} Z_s = \rho \left(Z_t + (1-\rho) Z_{t-1} + (1-\rho)^2 Z_{t-2} + \ldots \right)$$

It follows (since $|1-\rho|<1$ we have the sum of a geometric series that converges and is summable) at $t+1$:

$$\begin{aligned}
\mathcal{Z}_{t+1} &= \rho \sum_{s=-\infty}^{t+1} (1-\rho)^{t+1-s} Z_s = \rho \left(Z_{t+1} + (1-\rho) Z_t + (1-\rho)^2 Z_{t-1} + ... \right) \\
&= \rho Z_{t+1} + \rho (1-\rho) Z_t + \rho (1-\rho)^2 Z_{t-1} + ... \\
&= \rho Z_{t+1} + (1-\rho) \rho (Z_t + (1-\rho) Z_{t-1} + ...) \\
&= \rho Z_{t+1} + (1-\rho) \mathcal{Z}_t
\end{aligned}$$

so:

$$\mathcal{Z}_t = \frac{1}{(1-\rho)} \mathcal{Z}_{t+1} - \frac{\rho}{(1-\rho)} Z_{t+1}$$

Divide by the non-durable good price index, and substitute the individual quantity of debt issued, Z_{t+1}, as determined by equation (80) (we define real collateral with lower case k_t):

$$\begin{aligned}
\frac{\mathcal{Z}_t}{P_{ndt}} &= \frac{1}{(1-\rho)} \frac{\mathcal{Z}_{t+1}}{P_{ndt}} - \frac{\rho}{(1-\rho)} \frac{Z_{t+1}}{P_{ndt}} \\
\frac{\mathcal{Z}_t}{P_{ndt-1}} \frac{P_{ndt-1}}{P_{ndt}} &= \frac{1}{(1-\rho)} \frac{\mathcal{Z}_{t+1}}{P_{ndt}} - \frac{\rho}{(1-\rho)} \frac{M_t}{B_t} k_t \qquad (82)
\end{aligned}$$

Since constrained households always take all debt offered to them by financial intermediaries, the constrained debt a_{ct} defined in equation (13) follows the dynamics of $\frac{\mathcal{Z}_{t+1}}{P_{nd,t}}$. We can therefore re-write equation (82) as:

$$\begin{aligned}
a_{ct-1} &= \frac{\Pi_t}{(1-\rho)} \left(a_{ct} - \rho \frac{M_t}{B_t} k_t \right) \\
a_{ct} &= (1-\rho) \frac{a_{ct-1}}{\Pi_{ct}} + \rho \frac{M_t}{B_t} k_t \qquad (83)
\end{aligned}$$

C.3.3 Rates on Debt

Its optimisation problem will be to maximise the present discounted value of profit that will flow from this contract, so that at time t, it will discount future periods s only for future scenarios in which the contract is not readjusted, so using $(1-\rho)^{s-t}$. In addition, since unconstrained households own the financial intermediaries, their stochastic discount factor ($Q_{t,s}$ derived in equation (64)) is also used to discount the future flow of profit, so the problem is written as:

The optimal rate set on new fixed rate contracts, R_t^{zF}, is determined by a no arbitrage condition that equates the present value of future expected revenues from the variable and fixed rate contracts. As for the determination of the optimal quantity of debt, at time t the financial intermediary will discount future periods s only for future scenarios in which the contract is not readjusted, so using $(1-\rho)^{s-t}$. As before, the unconstrained households' stochastic discount factor ($Q_{t,s}$ derived in equation (64)) is also used to discount the future flow of profit, so the problem is written as:

$$\sum_{s=t+1}^{\infty} Q_{t,s} R_t^{zF} (1-\rho)^{t-s} Z_{t+1} = \sum_{s=t+1}^{\infty} Q_{t,s} (1-\rho)^{t-s} R_s Z_{t+1}$$

$$R_t^{zF} = \frac{\sum_{s=t+1}^{\infty} Q_{t,s} (1-\rho)^{t-s} R_s}{\sum_{s=t+1}^{\infty} Q_{t,s} (1-\rho)^{t-s}} \tag{84}$$

We define the variables (replacing $Q_{t,s}$ with the unconstrained stochastic discount factor of equation (65)):

$$\begin{aligned} V_t &= \sum_{s=t+1}^{\infty} \beta_u^{s-t} \frac{U_{nd,us}}{U_{nd,ut}} \frac{P_{nd,t}}{P_{nd,s}} (1-\rho)^{t-s} R_s \\ \mathcal{U}_t &= \sum_{s=t+1}^{\infty} \beta_u^{s-t} \frac{U_{nd,us}}{U_{nd,ut}} \frac{P_{nd,t}}{P_{nd,s}} (1-\rho)^{s-t} \\ R_t^{zF} &= \frac{V_t}{\mathcal{U}_t} \end{aligned} \tag{85}$$

and since variables with subscript t are constant across s we can write:

$$\begin{aligned} V_t &= \sum_{s=t+1}^{\infty} (\beta_u (1-\rho))^{s-t} U_{nd,us} R_s \left(\frac{P_{nds}}{P_{ndt}}\right)^{-1} \\ \mathcal{U}_t &= \sum_{s=t+1}^{\infty} (\beta_u (1-\rho))^{s-t} U_{nd,us} \left(\frac{P_{nds}}{P_{ndt}}\right)^{-1} \end{aligned}$$

As for the determination of debt, we organise V_t and $\mathcal{U}_t$ into the same form as for Calvo pricing in Woodford (2003). Dealing with V_t first we define $v_s = U_{nd,us}R_s$, then use the fact that for $s = t$, $V_t = v_t$, and define $\tilde{V}_t = V_t + v_t$ so we have:

$$
\begin{aligned}
V_t &= \sum_{s=t+1}^{\infty} \left(\beta_u \left(1-\rho\right)\right)^{s-t} v_s \left(\frac{P_{nds}}{P_{ndt}}\right)^{-1} \\
V_t &= -v_t + \sum_{s=t}^{\infty} \left(\beta_u \left(1-\rho\right)\right)^{s-t} v_s \left(\frac{P_{nds}}{P_{ndt}}\right)^{-1} \\
\tilde{V}_t &= V_t + v_t = \sum_{s=t}^{\infty} \left(\beta_u \left(1-\rho\right)\right)^{s-t} v_s \left(\frac{P_{nds}}{P_{ndt}}\right)^{-1} \qquad (86)
\end{aligned}
$$

It then follows that the relationship between $\tilde{V}_t$ and $\tilde{V}_{t+1}$ is defined by:

$$\tilde{V}_t = v_s + \beta_u \left(1-\rho\right) \Pi_{t+1}^{-1} \tilde{V}_{t+1}$$

so replacing our definitions $\tilde{V}_t = V_t + v_t$ and $v_t = U_{nd,ut}R_t$ we have:

$$V_t = \beta_u \left(1-\rho\right) \Pi_{t+1}^{-1} \left(V_{t+1} + U_{nd,ut+1}R_{t+1}\right) \qquad (87)$$

We can follow the same procedure for $\mathcal{U}_t$ to derive:

$$\mathcal{U}_t = \beta_u \left(1-\rho\right) \Pi_{t+1}^{-1} \left(\mathcal{U}_{t+1} + U_{nd,ut+1}\right) \qquad (88)$$

Equation (85) combined with (87) and (88) describe the dynamics of the new fixed rates R_t^{zF} set each period by those financial intermediaries who have the opportunity to re-set the contract.

C.4 Profits of Firms and Financial intermediaries

Aggregate intra-period nominal profit in non-durable goods sector is

$$
\begin{aligned}
\Pi_{ndt} &= P_{ndt}Y_{ndt} - W_{ut}N_{nd,ut} - W_{ct}N_{nd,ct} \\
&= P_{ndt}Y_{ndt} - P_{ndt}w_{ut}N_{ndut} - P_{ndt}w_{ct}N_{ndct}
\end{aligned}
$$

And in the durable goods sector it is

$$\begin{aligned}
\Pi_{dt} &= P_{ndt}Y_{dt} - W_{ut}N_{dut} - W_{ct}N_{dct} \\
&= P_{ndt}q_tY_{dt} - P_{ndt}w_{ut}N_{dut} - P_{ndt}w_{ct}N_{dct}
\end{aligned}$$

Total nominal profit is

$$\begin{aligned}
\tilde{\Pi}_t &= \Pi_{pt} + \Pi_{dt} = P_{ndt}Y_{pt} - P_{ndt}w_{ut}N_{put} - P_{ndt}w_{ct}N_{pct} \\
&\quad + P_{ndt}q_tY_{dt} - P_{ndt}w_{ut}N_{dut} - P_{ndt}w_{ct}N_{dct} \\
&= P_{ndt}Y_{pt} + P_{ndt}q_tY_{dt} - P_{ndt}w_{ut}\left(N_{put} + N_{dut}\right) - P_{ndt}w_{ct}\left(N_{pct} + N_{dct}\right) \\
&= P_{ndt}Y_{pt} + P_{ndt}q_tY_{dt} - P_{ndt}w_{ut}N_{ut} - P_{ndt}w_{ct}N_{ct} \qquad (89)
\end{aligned}$$

We assume that the profit is 100% taxed by the government and redistributed according to the following rule:

$$\begin{aligned}
t_{ct} &= (1-x)\frac{\tilde{\Pi}_t}{P_{ndt}} \\
t_{ut} &= x\frac{\tilde{\Pi}_t}{P_{ndt}}
\end{aligned}$$

where $x = \nu$ in the simplest case, but we can also look at more general case, so I leave it with x for now. We can keep at as a parameter for now. From (89) it follows that

$$P_{ndt}Y_{pt} + P_{ndt}q_tY_{dt} - W_{ut}N_{ut} - W_{ct}N_{ct} = \tilde{\Pi}_t = P_{ndt}t_{ct} + P_{ndt}t_{ut} \qquad (90)$$

We can substitute it in budget constraint and will do it later.

$$\begin{aligned}
t_{ct} &= (1-x)\left(Y_{pt} + q_tY_{dt} - w_{ut}N_{ut} - w_{ct}N_{ct}\right) \\
t_{ut} &= x\left(Y_{pt} + q_tY_{dt} - w_{ut}N_{ut} - w_{ct}N_{ct}\right)
\end{aligned}$$

The two budget constraints:

$$a_{ut} = (1+R_t)\left(\frac{a_{ut-1}}{\Pi_{c,t}} + w_{ut}N_{ut} + t_{ut} - C_{ut} - q_t\left(D_{ut} - (1-\delta)D_{ut-1}\right)\right) + \tilde{d}_t, \qquad (91)$$

$$a_{ct} = \left(1 + R_t^D\right)\left(\frac{a_{ct-1}}{\Pi_{c,t}} + C_{ct} + q_t\left(D_{ct} - (1-\delta)\,D_{ct-1}\right) - w_{ct}N_{ct} - t_{ct}\right)$$

imply that aggregate real profits of financial intermediaries that is distributed as dividends:

$$\tilde{d}_t = \left(R_t^D - R_t\right)\left(\frac{a_{ct-1}}{\Pi_{c,t}} + C_{ct} + q_t\left(D_{ct} - (1-\delta)\,D_{ct-1}\right) - w_{ct}N_{ct} - t_{ct}\right)$$

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