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Abstract

Microfoundations for the environmental Kuznets curve: Invoking by-production, normality, and inferiority of emissions.

A by-production-cum-preference based approach is adopted to study the relation between national income and environmental quality under non-cooperative behaviour. While emission is an inferior good for richly endowed economies, it is a normal good for the developing economies. With increases in endowments, the marginal willingness to pay declines (respectively, increases) in the set of poorly (respectively, richly) endowed economies. Hence, for emissions, the income and substitution effects work in opposite directions. Abatement strategies include cleaning-up; decreases in and substitution between fuels of varying costs, emission, and energy intensities; and the diversion of capital from fuel-intensive to non-fuel intensive uses. Poorly (respectively, richly) endowed economies are characterized by weak (respectively, strong) environmental policies. Consequently, deteriorating abatement practices are adopted by the developing economies. The shape of the income-environmental quality graph depends on the relative strengths of income and substitution effects and the set of available abatement strategies. Both *inverted U* and *N*-shaped environmental Kuznets curves are possible. The latter arises due to stronger substitution effects and lower opportunity costs of fuel-intensive capital in the more richer of the richest economies.

JEL classification codes: Q50, Q56, O12, O13, D62, H23

Keywords: environmental Kuznets curve, income and substitution effects, inferior good, normal good, by-production of emissions, returns to scale, abatement, inter-fuel substitution.

1. Introduction.

Starting with the seminal works of Grossman and Krueger [1993, 1995], there is a vast and proliferating literature that has confirmed, empirically, the existence of an *inverted U*-shaped relation between national income and several measures of environmental quality, such as emissions of CO_2, several measures of water pollution, chloroflorocarbons, *etc.*[1] Typically, in the literature, such graphs are called environmental Kuznets curves (EKCs). The interpretation of these graphs is that countries, which are in the initial phases of their development (*e.g.,* those on the verge of subsistence), place a higher value on consumption relative to the environment, and so are not responsive to the environmental damage created by increases in their consumption levels. Environmental quality hence deteriorates with growth at lower levels of development. However, after a critical level of development has been reached, the relative valuation of the environment increases, and increasing amounts of resources are deployed towards emission-mitigation activities. Hence, at this stage, emission levels start falling with growth. Additionally, there are also indicators of environmental quality, such as levels of SO_2 emission, where empirical papers find *N*-shaped graphs.[2] Explanations have been sought for why some of these graphs start turning-up again after displaying an initial *inverted U*-shaped relation.[3] We will, hence, distinguish between *inverted U* and *N*-shaped EKCs.

Most of the empirical literature estimates reduced form relations between national income and various measures of environmental quality, where the latter are separately regressed on the former. Copeland and Taylor [2003] argue that this approach is problematic, because both national income and environmental quality are themselves endogenous variables, which are determined by more fundamental factors – the sources of growth. Economies differ with respect to these sources of growth – *e.g.,* they may differ with respect to their endowments and the composition of their resources – and, ceteris paribus, these may explain the cross-country differences in emission and income levels. There is

[1] See, *e.g.,* Grossman and Krueger [1995], Selden and Song [1994], Holz-Eakin and Selden [1995], Mason and Swanson [2001] and many references therein.

[2] See, *e.g.,* Grossman and Krueger [1995].

[3] See, *e.g.,* Jones and Manuelli [2001] and Andreoni and Levinson [2004].

a growing but relatively small theoretical literature which seeks to explain the EKC phenomenon on this basis and with the help of familiar economic models. This paper seeks to contribute to this literature.

The literature spans across technological, preference-based, international trade-based, and political and other institutional considerations for explaining the EKC.[4] Though the model developed in this paper provides a rich framework to study all such considerations, in this work, we will focus only on the interactions between preferences and technological factors in a static framework to understand the basic relation between national income and environmental quality, when countries differ with respect to their endowments. We believe that a systematic analysis and a thorough understanding of these basic interactions is a key to extending the analysis further to incorporate more complex and realistic factors. Moreover, we show that many models in the existing literature can be derived as special cases of the more general model we present in this paper.

In most papers, emission is regarded as an inferior good.[5] The income effects brought about by increased resources hence contribute in explaining the declining part of the EKC in most of these papers. The challenge is in identifying factors that explain the upward sloping part(s) of the EKC. Typically, papers adopt extensions of the partial equilibrium framework in Weitzman [1974]. Countries differ with respect to their endowments of a numeraire commodity, which each country can allocate between direct consumption or for creating an abatement resource that is an input into cleaning-up.[6] Stokey [1998] argues that, in countries with poor endowments of the numeraire, the technologically feasible choices are restricted, resulting in corner solutions – these economies choose to consume at the brink of their endowments and do no cleaning-up. Thus, in this set of economies, emission and consumption levels rise with endowment. Beyond a threshold level of endowment, the set of technologically feasible options expands sufficiently to ensure interior solutions to a country-level planner's problem, and then it is possible that income effects contribute to richer countries emitting less and consuming more. In a model, which focuses on *consumption* externalities, Andreoni and Levinson [2001] show that EKC phenomenon

[4] *Some* prominent works include John and Pecchenino [1994], Stokey [1998], Jones and Manuelli [2001], Andreoni and Levinson [2001], Israel and Levinson [2004], and Copeland and Taylor [2003].

[5] Equivalently, environmental quality is considered to be a normal good.

[6] In a static framework, direct consumption is synonymous to national income.

arises if the cleaning-up technology exhibits increasing returns in levels of both the abatement resource and consumption. As consumption increases, gross emission (externality) increases. However, an EKC becomes possible if increasing returns to cleaning-up kicks in strongly enough after a threshold level of consumption so as to decrease the net emission level. This, combined with income effects, motivates richer economies to abate more without compromising on their consumption levels.

The more general model developed in this paper has two key innovations: (1) Preferences are modeled in a way that permits an emission to behave as a normal good in some region of the commodity space and as an inferior good in the other.[7] The income effects associated with an emission are linked to the responsiveness of the marginal damage (the marginal willingness to pay (MWTP)) to changes in the level of the consumption good. Emission is a normal (respectively, an inferior) good in the region where this responsiveness is negative (respectively, positive). This captures the intuition above that marginal valuation of consumption relative to emission is high and increasing in consumption in subsistence economies, while the reverse is true for richer economies. (2) Technology is modeled by employing a by-production approach developed in some of our recent papers Murty [2011, 2012] and Murty, Russell and Levkoff [2012], which gives due importance to goods (such as fossil fuels) which cause emissions in nature.[8] These goods form a part of both an "intended production technology," which is a standard neo-classical production technology, and "nature's emission generating mechanism" that relates emissions to goods that cause emissions in nature.

We show that, by abstracting from explicit modeling of the emission-generating goods, most of the existing papers implicitly assume that each country is born with a fixed level of gross emission (which varies across countries) and that the only available abatement strategy is cleaning-up. The by-production approach, on the other hand, permits a country to *choose* its level of gross emission, by permitting it to choose its usage of emission-causing

[7] That this is not ruled out by economic theory, is observed also by Israel and Levinson [2004].

[8] See also Chambers and Melkonyan [2012] who employ an alternative "material balance approach" in a production framework to study the EKC phenomenon. In contrast, a general-equilibrium framework is adopted in this paper.

inputs. A plethora of abatement strategies, which include cleaning-up, decreases in fuel inputs, and inter-fuel substitution between various energy inputs become open to it.[9]

Countries are alike in all aspects but one in our analysis. They differ only due to differences in their endowment of a resource, which we call capital, but which we interpret in a very broad sense.[10] Primarily, we have in mind a resource, possession of which gives a country an advantage. As our analysis is conducted in a static framework, we do not discuss how the endowment of this resource was accumulated/acquired over time by any country. We assume that this resource could have two types of uses in intended production: (1) the use of capital could be energy intensive, *e.g.,* energy generating fuel inputs are often required to run machines, tools, equipment, *etc.* (energy inputs are complementary to the use of capital) and (2) there may also be forms of capital, such as buildings, human capital, *etc.,* whose use is not energy/fuel intensive. This provides another abatement strategy for a country – diversion of capital from its fuel intensive to its non-fuel intensive use.

Increases in endowments imply not only increases in the availability of some inputs and hence greater productivity and higher shadow income levels but also changes in the MWTP (which reflects both the shadow price of emission and the Pigou tax level). Hence, both preference-based and technological factors explain changes in emission levels due to changes in the endowment levels. From the point of view of preferences, both substitution and income effects are important in determining the differences in emission levels that are *permitted* by aggregate consumers of different economies – in particular, income and substitution effects work in opposite directions. Typically, we find that, if the technology exhibits diminishing returns to inputs, the Pigou tax level is decreasing (respectively, increasing) with capital in the set of poorly (respectively, richly) endowed economies – *i.e,* poorly (respectively, richly) endowed economies are characterized by weak (respectively, strong) environmental policy. Hence, poorly endowed economies are characterized by deteriorating abatement practices. Nevertheless, the *N*-shaped EKC phenomenon can arise even as the Pigou tax level continues to rise with capital in the set of richly endowed economies. We show that this phenomenon can be explained by stronger substitution

9 Cleaning-up includes activities such as end-of-pipe treatment plants, scrubbers, afforestation, *etc.*

10 This is in keeping with most of the literature. Addition of more sources of realistic heterogeneity into the model is definitely possible, and will perhaps amplify our results.

effects and lower opportunity costs of fuel-intensive capital in the more richer of the richest economies. Perhaps this sheds some light on the differences in emission levels between the European block and countries such as the U.S. Under constant returns, MWTP (and hence the Pigou tax levels) are equalized across all countries, and cross-country differences in emission levels are explained purely by the income effects.

Our analysis assumes non-cooperative behaviour of individual countries and looks at the relation between emission and consumption levels at a Nash equilibrium of a model where emissions result in both local and global externalities. If global emission exhibits a property of strategic substitutability, then local emissions are shown to be declining in the global emission level.[11]

A general model is laid out in Section 2. In Section 3, some influential models in the literature are derived/interpreted as special cases of the general model. In Section 4, a working model for this paper is derived from the general model . In Section 5, we derive the optimum of a country-level planner's problem and define its decentralization by two price mechanisms. In Section 6, we turn to a global analysis where we define a Nash equilibrium emission outcome. In Section 7, we derive the income and substitution effects of a change in capital endowment. The tools developed in all the previous sections are used in Section 8 to study three important special cases, each of which isolates a particular feature of our working model and studies its implications for the EKC: (i) there is substitutability but no complementarity between capital and fuel inputs, (ii) capital and fuel are complementary inputs, and (iii) there exists inter-fuel substitutability possibilities between fuels of varying extraction costs, emission intensities, and energy efficiencies. We conclude in Section 9. The analysis is supported by several examples and diagrams, and all proofs are relegated to the appendix.

[11] A distinction between local and global pollutants is made in Mason and Swanson [2001], who focus on the ozone depleting emission of chloroflorocarbons. They find that the critical income level in the EKC of this emission is higher than for many local pollutants. In this paper, we assume that a country ignores the effect its decision to emit has on the global emission level.

2. A general model.

The global economy comprises of S closed economies.[12] There is one consumption good and one type of emission, whose levels in a typical economy are denoted by $y \in \mathbf{R}_+$ and $z \in \mathbf{R}_+$, respectively. The level of global emission, *i.e.,* the sum of local emissions from all countries, is denoted by $g \in \mathbf{R}_+$. There are I emission-causing inputs (*e.g,* various types of fuels), indexed by $i = 1, \dots, I$. An input vector of emission-causing inputs is denoted by $x = \langle x_1, \dots, x_I \rangle \in \mathbf{R}^I_+$. The endowment of i^{th} emission-causing input is denoted by $\omega_i \in \mathbf{R}_+ \cup \{\infty\}$. The vector of endowments of emission-causing inputs is the I dimensional non-negative vector ω.[13] Labour and capital are the non-emission causing inputs in our analysis. We assume that these are inelastically supplied in any economy. Their endowments are denoted by $\bar{l} \in \mathbf{R}_+$ and $k \in \mathbf{R}_+$, respectively. All inputs are used to produce the consumption good and an abatement resource. The latter, whose level is denoted by $a \in \mathbf{R}_+$, is an input into an activity that can clean-up/mitigate the gross emission generated. The extent of mitigation of emission made possible by a amount of the abatement resources is denoted by $v \in \mathbf{R}_+$.

In every economy, there is an aggregate consumer. Preferences of the economy depend on the consumption by the economy of the consumption good, the emission generated locally by the country, and the global emission level. Preferences are represented by a utility function $u : \mathbf{R}^3_+ \longrightarrow \mathbf{R}_+$ with image $u(y, z, g)$. We assume that u is increasing in y and decreasing in z and g. Except in Example 2, throughout this paper, u will be assumed to be strictly quasi-concave and twice continuously differentiable in the interior of its domain.

The by-production approach in Murty, Russell, and Levkoff [2012] and Murty [2012] is used to specify the technology of the economy. In this approach, the overall technology is obtained as an intersection of an intended production technology and a nature's emission generation set. We discuss these two sets below.

[12] Except when we study the Nash equilibrium of the global economy, it will prove notationally convenient to avoid indexing the economies and study a typical economy.

[13] Note, at a given time point, ω could be finite (reflecting scarcity of these resources) or unbounded. As we will see later, the latter case need not imply that extraction of these inputs is costless.

The intended production technology is a standard neo-classical production technology that shows how inputs are transformed into intended outputs. As discussed in the introductory section, we assume that the aggregate capital resource k available to a country can be put to two uses: k_1 denotes the amount of aggregate capital that is put to a fuel intensive use and $k_2 = k - k_1$ denotes the amount put to a non-fuel intensive use. Let x be the vector of total amounts of fuel inputs extracted. These are partly used to run the fuel intensive component of capital. We call this an indirect use of these inputs in intended production. Suppose $x^1 \in \mathbf{R}_+^I$ denotes the vector of quantities of fuel inputs put to this use. We also leave open the possibility that there is also a direct use of fuel inputs in intended production, *i.e.,* there could be some degree of substitutability between these inputs and the other inputs in intended production. Suppose $x^2 \in \mathbf{R}_+^I$ denotes the vector of amounts of fuel inputs that is put directly to use in intended production.

We assume that total energy generated by x^1 amount of fuel inputs is given by the continuous function $\psi : \mathbf{R}_+^I \longrightarrow \mathbf{R}_+$ with image $\psi(x^1)$, which is differentiable in the interior of its domain with $\psi_{x_i^1} > 0$ for all $i = 1, \ldots, I$. The energy requirement of k_1 amount of fuel intensive capital is given by the continuous function $\phi : \mathbf{R}_+ \longrightarrow \mathbf{R}_+$ with image $\phi(k_1)$, which is differentiable in the interior of its domain with $\phi_1 > 0$. Thus x^1 amount of fuel inputs can meet the energy requirements to run k_1 amount of fuel intensive capital if and only if $\phi(k_1) \leq \psi(x^1)$.

An implicit production function $h(y, a) = f(k_1, k_2, l, x^2)$ is used to define a standard neo-classical production technology. h and f are two real valued functions that are twice continuously differentiable on $\mathbf{R}_{++}^2$ and $\mathbf{R}_{++}^{I+3}$, respectively. The function f reflects the production of an aggregate output (sometimes denoted by $Y = f(k_1, k_2, l, x^2) \in \mathbf{R}_+$) by using all the inputs. This aggregate output can be transformed either into the consumption good y or the abatement resource a. The function h reflects the way in which the abatement resource gets transformed into the consumption good as more and more of the aggregate output is diverted from abatement resource production into consumption good production. We assume that f is concave with $f_1 \equiv \frac{\partial f}{\partial k_1} \geq 0$, $f_l \geq 0$, $f_2 \equiv \frac{\partial f}{\partial k_2} \geq 0$, and $f_{x_i^2} \leq 0$ for all $i = 1, \ldots, I$. h is assumed to be quasiconvex with $h_a \geq 0$ and $h_y \geq 0$. These sign conventions yield the standard neo-classical trade-offs between various goods in intended production. The intended production technology in this economy is the set

$$T_1 = \{\langle y, a, k_1, k_2, l, x^1, x^2, z, v\rangle \in \mathbf{R}_+^{2I+7} \mid h(y,a) \leq f(k_1, k_2, l, x^2) \text{ and } \phi(k_1) \leq \psi(x^1)\}.$$

Nature's emission generating technology links the level of emission to the levels of the emission-causing goods and the cleaning-up activity. In most of this paper, we will assume that emission is generated by emission-causing inputs. Thus, we mainly focus on production externalities. However, our model can be extended to include consumption externalities as in Andreoni and Levinson [2001]. In nature, the emission-causing goods determine the level of gross emission. The abatement resource produced in intended production is used to clean-up the gross emission. In general, the extent of mitigation of gross emission will depend not only on the level of the abatement resource produced but also on the scale of gross emission, which is in turn determined by the amounts of the emission-causing goods in the economy. Suppose a continuous function $\mathcal{C} : \mathbf{R}^{I+2} \longrightarrow \mathbf{R}_+$ with image $v = \mathcal{C}(x, y, a)$ defines the level of cleaning-up. We assume that $\mathcal{C}$ is non-decreasing in a. Net emission depends on the extent of emission-causing goods used or produced in the economy and the level of cleaning-up effort. It is given by the continuous function $\zeta : \mathbf{R}_+^{I+2} \longrightarrow \mathbf{R}_+$ with image $z = \zeta(x, y, v)$. To capture emission generation in nature and its cleaning-up by human abatement efforts, we assume that ζ is non-decreasing in x and y, while it is non-increasing in v. The emission generation set is specified functionally by the set

$$T_2 = \{\langle y, a, k_1, k_2, l, x^1, x^2, z, v\rangle \in \mathbf{R}_+^{2I+7} \mid z \geq \zeta(x^1 + x^2, y, v) \text{ and } v \leq \mathcal{C}(x, y, a)\}.$$

The overall by-production technology is defined as $T = T_1 \cap T_2$. We assume that T is the common global technology. Different economies may choose to produce at different points on T. We also assume that the utility function u is also common for all countries, albeit countries can choose to consume different levels of the consumption good and local emission.

3. Some models in the literature as special cases.

In this section, we show how some popular models in the literature can be derived or interpreted as special cases of our model. The literature considered in this section focusses mainly on the local aspect of emission (*i.e,* it does not model global emission) and, more importantly, it abstracts from modeling emission-causing goods in nature.

- Weitzman [1974] is a model where capital and labour are not modeled, quasi-linear preferences are defined over a consumption good and clean environment, and there is an initial endowment of a numeraire good that can be allocated between direct consumption and cleaning-up of the environment. One can interpret this as a special case of our general model, where each country has a bounded endowment of an emission-generating input ($I = 1$ and ω is finite) and the numeraire good is the aggregate output produced in the economy from its endowment of the emission-causing input. In particular, the transformation of the emission-causing input into aggregate output is one for one, *i.e.,* $Y = \omega$. The aggregate output is partly converted into an abatement resource a and partly converted into the consumption good y. Thus, the neo-classical intended production function is

$$h(y, a) = f(k_1, k_2, l, \omega) \iff y + a = \omega. \tag{3.1}$$

  The use of its endowment of the emission-causing input results also in a gross amount of local emission. In nature, one unit of the emission-causing input results in one unit of gross emission. Hence, gross emission level is ω. Cleaning-up depends only on the level of the abatement resource used, so that we redefine the function $\mathcal{C}$ as $v = \mathcal{C}(\omega, y, a) \equiv \hat{C}(a)$. The net emission produced is $z = \omega - v$. Thus, the amount of clean environment available is given by the extent of mitigation of gross emission: $v = \omega - z$. Preferences are redefine as $u(y, z, g) \equiv y - \Psi(z) = y - \Psi(\omega - v) = \omega - a - \Psi(\omega - v) = \omega - \hat{C}^{-1}(v) - \Psi(\omega - v) \equiv B(v) - C(v)$ where, as in Weitzman [1974], $B(v) \equiv -\Psi(\omega - v)$ shows the benefits from cleaning-up, while $C(v) \equiv \omega - \hat{C}^{-1}(v)$ shows the costs of cleaning-up. This is also the model in Guesnerie [2008].

- We interpret Stokey [1998] as our general model in Section 2 where, in a typical economy, $I = 1$, and capital and labour are not employed. Like in Weitzman [1974],

ω is bounded and is employed to produce an aggregate output that can be allocated between the consumption good and an abatement resource, *i.e.,* (3.1) holds. In particular, $y = \lambda\omega$ and $a = (1-\lambda)\omega$ with $\lambda \in [0,1]$. ω results in an equivalent amount of gross emission. There is no consumption externality, and hence we can redefine $\mathcal{C}$ as $\mathcal{C}(\omega, y, a) \equiv \hat{C}(\omega, a)$. $\hat{C}$ is assumed to be linearly homogenous in its arguments, so that $\zeta(\omega, a) \equiv \omega - \mathcal{C}(\omega, a) = \omega(1 - \mathcal{C}(1, 1-\lambda)) \equiv \omega\ \chi(\lambda)$. Preferences are additively separable: $u(y, z, g) = \Psi^1(y) + \Psi^2(z)$.

- Andreoni and Levinson [2001] focus on a consumption externality, *i.e.,* the consumption good is emission generating in nature. In terms of our general model in Section 2, in a typical economy, there are no emission-causing inputs ($I = 0$ and $\omega = 0$) and capital and labour are not employed. A country is born with an endowment Y of a numeraire good and $h(y, a) = f(k_1, k_2, l, x^2) \iff y + a = Y$. Gross emission depends only on the extent to which the consumption good is consumed by the aggregate consumer, so that we redefine $\mathcal{C}$ and ζ as $\mathcal{C}(y, \omega, a) \equiv \hat{C}(y, a)$ and $\zeta(\omega, y, a) \equiv y - \hat{C}(y, a)$, respectively. $\hat{C}$ is assumed to exhibit increasing returns to scale.
- Copeland and Taylor [2003] model of a small open economy can also be interpreted as a special case of our model. This model is an extension of Stokey [1998], with preferences redefined as $u(y, z, g) \equiv \Psi(y, z)$. We interpret this model as one where a typical country employs inelastic amounts of labour ($\bar{l}$) and capital (k) along with an emission generating input (*i.e.,* $I = 1$) to produce the consumption good, and where there is perfect complementarity between emission-causing and non-emission causing inputs in production. In particular, it is as if capital is used only in its fuel-intensive form (*i.e.,* $k_1 = k$ and $x^1 = x$ with $\psi(x) = x$ and $\phi(k) = k$) and there is no scarcity of the emission-causing input ($\omega = \infty$), which is also extracted costlessly. The fixed endowments of labour and capital define its maximum potential intended output Y, which can be allocated to consumption y or to produce an abatement resource a. Precisely, these assumptions boil down to $Y = \min\{G(k, l), x\}$, $y = \lambda Y$, and $a = (1-\lambda)Y$, where $\lambda \in [0,1]$. Hence, $h(y, a) = f(k, 0, l, 0) \iff y + a = G(k, l)$ and $x = \psi^{-1}(\phi(k))$. There is no consumption externality and $\mathcal{C}$ is redefined as in our version of the Stokey [1998] model above.

To relate models such as Weitzman [1974] and Stokey [1998] to our by-production approach, we assumed above that every economy has a *fixed* endowment of an emission-causing input which, under the by-production approach, is used to produce its endowment of the numeraire good as well as its endowment of gross emission (a by-product) via technologies T_1 and T_2, respectively. Two conclusions follow: (i) The gross emission of each economy is *fixed*. Hence, net emission depends only on its choice of cleaning-up effort. Thus, cleaning-up is the *only* abatement strategy available to a country. (ii) Differences in the endowments of the numeraire good across countries follow from differences in the endowments of the emission-causing input. This implies countries are born with differences in their initial endowments of gross emissions.

In our interpretation of Copeland and Taylor's [2003] model, though emission is linked to emission-causing inputs, it is as if these inputs are costlessly available and their use is complementary to the use of non-emission causing inputs. Hence, once again, for any country, the gross emission is fixed; in particular, in this model, it is fixed by the extent of its labour and capital endowments.

Thus, in all these models, environmental policy of each country is constrained not just by the resources it has available for cleaning-up but also by the level of gross emission it was born with. In particular, these models imply that countries with more resources are also born with greater endowments of gross emissions. The reality of this implication is questionable.

4. A working model.

We now derive a working model for this paper from the general model in Section 2, which lends itself to a tractable analysis.[14] In this model, any economy has a non-trivial choice problem concerning the levels of its usage of emission-generating inputs. However, unlike in our interpretations of some popular models above, no country has an advantage over any other in terms of endowments of these resources: In particular, we assume there is no scarcity of these resources in any country. Nevertheless, usage of these resources are subject to extraction costs. By choosing the levels of the emission-generating inputs,

[14] In doing so, many simplifying assumptions are made in the interest of tractability of the rather complex problem at hand.

implicitly, each economy chooses also its gross emission level. Net emission depends not only on its choice of the cleaning-up level, but also on its choice of the gross emission level. Hence, abatement strategies for an economy include increases in its cleaning-up effort, decreases in and inter-fuel substitution between its usage of various fuel inputs, and reallocation of its aggregate capital endowment from fuel intensive to non-fuel intensive uses.

4.1. Production.

Assume $I > 0$. The labour endowment is common for all countries, and its level is denoted by $\bar{l}$. Assume also that, for any country, the endowments of emission-causing inputs are unbounded ($\omega = \infty$). Thus, countries differ only with respect to the capital input. We assume that the function f is concave, linear in l, and additively separable in k_1, k_2, l, and x^2, while the function h is quasiconvex, linear in y (in particular, $h_y = 1$), and additively separable.

Labour is used to extract fuel inputs, in addition to its use as an input in intended production. The amount of labour used in intended production is denoted by l. For any fuel $i = 1, \ldots, I$, the marginal extraction cost is constant and is denoted by $c_i > 0$. Thus, the restriction $l + \sum_{i=1}^{I} c_i x_i \leq \bar{l}$ captures the resource constraint for the labour input.

We also assume that the function $\mathcal{C}$ takes the form $v = \mathcal{C}(y, x, a) \equiv \theta a$, while the function ζ takes the form

$$z = \zeta(y, x, v) \equiv \sum_{i=1}^{I} \alpha_i x_i - v = \sum_{i=1}^{I} \alpha_i x_i - \theta a,$$

where $\theta \geq 0$ and $\alpha_i \geq 0$ for all $i = 1, \ldots, I$. θ denotes the effectiveness (productivity) of the abatement resource in mitigating emission, while α_i denotes the emission intensity (emission generated per unit of input) of the i^{th} emission-generating input.[15]

[15] Hence, the gross emission level is given by $\sum_{i=1}^{I} \alpha_i x_i$.

4.2. Preferences:

Each country takes the level of the global emission g as given and chooses its levels of the consumption good y and the local emission z. Define the determinant of the bordered Hessian of the utility function u, conditional on g, and evaluated at $\langle y, z, g\rangle \in \mathbf{R}^3_{++}$ as

$$Q(y,z,g) = Det \begin{bmatrix} 0 & u_y(y,z,g) & u_z(y,z,g) \\ u_y(y,z,g) & u_{yy}(y,z,g) & u_{yz}(y,z,g) \\ u_z(y,z,g) & u_{yz}(y,z,g) & u_{zz}(y,z,g) \end{bmatrix}. \tag{4.1}$$

The consumption marginal rate of substitution between the local emission and the consumption good is denoted by $r = r(y,z,g) \equiv -\frac{u_z}{u_i}(y,z,g) > 0$. This reflects the marginal willingness to pay (in terms of the consumption good) for an incremental decrease in the local emission level evaluated at the point $\langle y, z, g\rangle$. In the literature (see, *e.g.,* Copeland and Taylor [2003]) this is also called the marginal damage of local emission evaluated at that point.

Later, we will be interested in splitting the changes in emission and consumption levels as we move from capital poor to capital rich countries into income and substitution effects. For this purpose, it is helpful to define hypothetical utility maximization and expenditure minimization exercises in any economy, where it is as if the aggregate consumer has the rights over the environment and can choose levels of both the consumption good and local emission. He receives a price $-p_z$ for bearing the emission generated in the production side of the economy, where $p_z < 0$. The consumption good is assumed to be the numeraire, so that its price is normalized to be one. Suppose the consumer's hypothetical income is denoted by $m \in \mathbf{R}$. Consumer's expenditure minimization exercise for fixed level of utility is defined as

$$E(p_z, g, u) \equiv \min_{y,z}\{y + p_z z \mid u(y,z,g) \geq u\}. \tag{4.2}$$

Suppose the resulting Hicksian demand functions are denoted by $\langle z^H(p_z,g,u), y^H(p_z,g,u)\rangle$. The substitution effects of a change in the price of the emission on the consumption good or on the local emission are given by $\partial y^H(p_z,g,u)/\partial p_z$ and $\partial z^H(p_z,g,u)/\partial p_z$, respectively. Consumer's utility maximization problem is defined as

$$\max_{y,z}\ \{u(y,z,g) \mid p_z z + y \leq m\}. \tag{4.3}$$

Suppose the resulting Marshallian demands are denoted by $\langle z(p_z, m, g),\ y(p_z, m, g)\rangle$. The effects of a change in income on the consumption good or on emission (the income effects) are given by $\partial y(p_z, m, g)/\partial m$ and $\partial z(p_z, m, g)/\partial m$, respectively.

We will also be interested in noting how the local emission level in any country responds to changes in the global emission level . This will depend on how marginal utilities respond to changes in the global emission level. The following definition signs these responses.

Definition. Preferences exhibit *strategic substitutability with respect to global emission* if $u_{zg}(y, z, g) \leq 0$ and $u_{yg}(y, z, g) \leq 0$ for all $\langle y, z, g\rangle \in \mathbf{R}^3_{++}$.

Strategic substitutability with respect to global emission influences the responsiveness of marginal damage r to changes in the global emission level. There is also a link between the income effects and the responsiveness of r to changes in consumption and local emission levels. Lemma ΔMRS characterizes these links.[16] Before stating it, we define two functions $\mathcal{A} : \mathbf{R}^3_{++} \longrightarrow \mathbf{R}$ and $\mathcal{B} : \mathbf{R}^3_{++} \longrightarrow \mathbf{R}$ with images

$$\mathcal{A}(y, z, g) \equiv u_y^2 \frac{\partial r(y, z, g)}{\partial y} \quad \text{and} \quad \mathcal{B}(y, z, g) \equiv u_y^2 \frac{\partial r(y, z, g)}{\partial z},$$

which reflect the responsiveness of r to changes in y and z, respectively.

Lemma ΔMRS: *Suppose $\langle y, z\rangle \in \mathbf{R}^2_+$ solve (4.3) for some $\langle p_z, m, g\rangle \in \mathbf{R}_- \times \mathbf{R} \times \mathbf{R}_+$. Then the following are true:*

(i) $Q \equiv Q(y, z, g) > 0$

(ii) $\frac{\partial z}{\partial m} = -\frac{u_y^3}{Q}\frac{\partial r}{\partial y} = -\frac{u_y \mathcal{A}}{Q}$ *and* $\frac{\partial y}{\partial m} = \frac{u_y^3}{Q}\frac{\partial r}{\partial z} = \frac{u_y \mathcal{B}}{Q}$.

(iii) $\frac{\partial z^H}{\partial p_z} = -\frac{u_y^3}{Q} \leq 0$ *and* $\frac{\partial y^H}{\partial p_z} = \frac{u_y^2 u_z}{Q} \leq 0.$

(iv) $Q = -u_z \mathcal{A}(y, z, g) + u_y \mathcal{B}(y, z, g)$.

(v) $\frac{\partial r}{\partial g} \geq 0$, $\frac{\partial z}{\partial g} = -\frac{u_y^3}{Q}\frac{\partial r}{\partial g} \leq 0$, *and* $\frac{\partial y}{\partial g} = \frac{u_z u_y^2}{Q}\frac{\partial r}{\partial g} \leq 0$ *if u exhibits strategic substitutability with respect to global emission.*

16 The proof follows from standard comparative statics of utility maximization.

It is clear from Lemma ΔMRS that the income effect with respect to the consumption good is positive (*i.e.,* consumption good is a normal good) if and only if $\mathcal{B} > 0$ at all points in the consumption space. The income effect with respect to local emission is negative or positive depending on whether $\mathcal{A}$ is positive or negative, respectively.

Definition. Local emission exhibits *normality at* $\langle y, z, g\rangle \in \mathbf{R}^3_{++}$ if $\mathcal{A}(y, z, g) < 0$ and it exhibits *inferiority at* $\langle y, z, g\rangle \in \mathbf{R}^3_{++}$ if $\mathcal{A}(y, z, g) \geq 0$. If local emission exhibits normality (respectively, inferiority) on $\mathbf{R}^3_{++}$, then it is a *normal* (respectively, *inferior*) good.

Throughout this paper, we will assume that the consumption good is a normal good. To the best of our knowledge, at least thus far, all of the literature treats emission as an inferior good.

Assumption (1): For every $g \in \mathbf{R}_{++}$, there exists a set $\mathcal{N}(g) \subset \mathbf{R}^2_{++}$ such that local emission

(i) exhibits normality at all $\langle y, z, g\rangle \in \mathbf{R}^3_{++}$ such that $\langle y, z\rangle \in \mathcal{N}(g)$ and

(ii) exhibits inferiority at all $\langle y, z, g\rangle \in \mathbf{R}^3_{++}$ such that $\langle y, z\rangle \notin \mathcal{N}(g)$.

Our analysis distinguishes between the concepts of a "bad" commodity and inferiority or normality properties of a commodity. A bad commodity yields disutility, while normality or inferiority properties of a commodity depend on the income effects associated with it. In our analysis, local emission is a bad commodity. However, Examples 1 and 2, below, which support Assumption (1), clearly show that this phenomenon can be consistent with local emission exhibiting normality in some region of the consumption space and inferiority in the other. At points in the consumption space where emission exhibits normality, *i.e.,* when $\mathcal{A} < 0$, marginal damage decreases (or equivalently, marginal valuation of consumption relative to emission increases) with increase in consumption. The reverse is true when emission exhibits inferiority. It is plausible that poorly (respectively, richly) endowed economies operate in the region of the consumption space where emission exhibits normality (respectively, inferiority). This hints at an EKC based purely on income effects.[17]

[17] However, the special cases studied later will demonstrate that, in general, the relation between emission and consumption levels depends on interactions between preference-based and technological factors.

Example 1: Suppose u is additively separable in global emission, so that ranking over the consumption good and emission bundles is independent of the level of the global emission. Suppose u is such that, for any $g \in \mathcal{G}$, the locus of consumption good and emission combinations where the marginal rate of substitution is held fixed, say at 0.033, is the set (see Figure 1)

$$ICC(-0.033, g) := \{\langle y, z\rangle \in \mathbf{R}^2_+ \mid z = -y^2 + y + 1 \text{ and } y \in [0, 1.61803]\}. \tag{4.4}$$

This set corresponds to the income-consumption curve (ICC) when the price of emission (p_z) is held fixed at -0.033 and the income (m) is allowed to vary. Note, that our additive separability assumption implies that $ICC(-0.033, g)$ is independent of g. As seen in Figure 1, the ICC is backward bending: along this curve, z increases from 1 to 1.25 as y increases from 0 to 0.5 (*i.e.,* as m increases from -0.033 to 0.459), and z falls from 1.25 to 0 as y increases from 0.5 to 1.618 (*i.e.,* as m increases from 0.459 to 1.618). Thus, in the upward sloping part of the ICC, local emission exhibits normality, while in the backward bending part it exhibits inferiority.

Although Figure 1 clearly indicate that a backward bending ICC is not at all pathological, it is analytically difficult to write down functional forms for a differentiable utility function that exhibits such a property.[18] Example 2 studies a case of non-smooth preferences.

Example 2: Let us abstract from global emission and consider preferences in the space of of local emission and the consumption good. For convenience, we restrict the consumption space by bounding the local emission and consumption good levels to lie in the interval $[0, 100]$. The indifference curves in this space are shown in Figures 2 a and b. They are obtained as follows: Define the set

$$\mathcal{N} = \Big\{\langle z, y\rangle \in [0, 100] \times [0, 100] \;\Big|\; (300 - z)(y + 200) < 60000 \text{ and } y < 44.949\Big\},$$

and suppose that, the restriction of u in the region $\mathcal{N}$ is

$$u(y, z) = 60000\ \ln(300 - z) + 80000\ \ln(y + 200) - (300 - z)(y + 200) \quad \forall\ \langle y, z\rangle \in \mathcal{N}.$$

[18] See, *e.g.,* Doi, Iwasa, and Shimomura [2009] and references therein which discuss these difficulties in the context of the Giffen phenomenon.

For any $\langle y, z\rangle \in \mathcal{N}$, the trade-off between local emission and the consumption good, under our parametric restriction, is positive (emission is a bad good):

$$-\frac{u_z}{u_y} = \left(\frac{60000-(300-z)(y+200)}{80000-(300-z)(y+200)}\right)\left(\frac{y+200}{300-z}\right) > 0. \tag{4.5}$$

Denote $(300-z)(y+200)$ as χ. Then, for all $\langle y, z\rangle \in \mathcal{N}$, we have $40000 \leq \chi < 60000$.[19] This implies that for all $\langle y, z\rangle \in \mathcal{N}$, we have

$$\begin{aligned}\frac{\mathcal{A}}{u_y^2} &= \frac{\partial\frac{-u_z}{u_y}}{\partial y} = \left(\frac{1}{(y+200)u_y^2}\right)\left(\frac{80000(60000-\chi)-\chi(80000-\chi)}{\chi}\right) \leq 0 \text{ and}\\ \frac{\mathcal{B}}{u_y^2} &= \frac{\partial\frac{-u_z}{u_y}}{\partial z} = \frac{60000(80000-\chi)-\chi(60000-\chi)}{\chi} \geq 0.\end{aligned} \tag{4.6}$$

Hence, both local emission and the consumption good are normal goods in the region $\mathcal{N}$. The marginal damage or the slope of the indifference curves in the region $\mathcal{N}2$ is zero, while it is 0.4348 in the region $\mathcal{N}1$. Figures 2 (a) and (b) indicate that emission exhibits inferiority in the region $\mathcal{N}1 \cup \mathcal{N}2$.[20] The consumption good continues to be normal in this region.

5. A country-level analysis.

In this section, we consider the problem of a social planner of a typical economy, and discuss decentralization of a country-level social optimum.

5.1. Country-level planner's problem.

A social planner of a typical economy with capital endowment k and facing global emission g solves the following problem:

$$\begin{gathered}\mathcal{V}(k,g) \equiv \max_{x,x^1,l,a,k_1,y,z} u(y,z,g)\\ \text{subject to}\\ h(y,a) \leq f(k_1, k-k_1, l, x-x^1), \quad \phi(k_1) \leq \psi(x^1),\\ l + \sum_{i=1}^{I} c_i x_i \leq \bar{l}, \quad \text{and} \quad z \geq \sum_{i=1}^{I} \alpha_i x_i - \theta a.\end{gathered} \tag{5.1}$$

[19] Note χ is decreasing in z and increasing in y. Hence, the minimum value of χ occurs when $z = 100$ and $y = 0$.

[20] *E.g.,* holding z fixed, the marginal damage jumps discontinuously to a higher value as y increases so that we move from region $\mathcal{N}$ to $\mathcal{N}1$ or from region $\mathcal{N}2$ to $\mathcal{N}1$.

The Lagrangian of (5.1) is

$$\begin{aligned} L = u(y, z, g) \ &- \lambda[h(y, a) - f(k_1, k - k_1, l, x - x^1)] - \mu[\phi(k_1) - \psi(x^1)] \\ &- \gamma[l + \sum_{i=1}^{I} c_i x_i - \bar{l}] - \delta[\sum_{i=1}^{I} \alpha_i x_i - \theta a - z]. \end{aligned} \tag{5.2}$$

At an interior optimum, all constraints are binding, *i.e.*,[21]

$$-[h(y, a) - f(k_1, k - k_1, l, x - x^1)] = 0, \tag{5.3}$$

$$-[\phi(k_1) - \psi(x^1)] = 0, \tag{5.4}$$

$$-[\sum_{i=1}^{I} \alpha_i x_i - \theta a - z] = 0, \text{ and} \tag{5.5}$$

$$-[l + \sum_{i=1}^{I} c_i x_i - \bar{l}] = 0, \tag{5.6}$$

and the first-order conditions (FOCs) of (5.1) can be used to show that the trade-offs in consumption and production between the consumption good and local emission are equalized, *i.e.*,

$$-\frac{u_z}{u_y} = \frac{h_a/h_y}{\theta} = \frac{f_{x_i^2} - f_l c_i}{\alpha_i} = \frac{[f_1 - f_2] - \frac{\phi_1}{\psi_{x_i^1}} f_l c_i}{\frac{\phi_1}{\psi_{x_i^1}} \alpha_i}, \qquad \forall\ i = 1, \ldots, I. \tag{5.7}$$

With respect to production, every abatement strategy yields its own trade-off between intended production and local emission generation: this is reflected in the net increase in the intended output per unit increase in local emission that results when a given abatement strategy is relaxed. (5.7) implies that, at an optimum of (5.1), the trade-offs resulting from all abatement strategies are equal:

- A decrease in the level of cleaning-up by one unit implies an increase in the level of local emission by an amount θ and an increase in the level of the intended output by an amount h_a/h_y, so that h_a/θ is the increase in intended output per unit increase in local emission that results from a unit decrease in the level of the cleaning-up activity.[22] As less and less resources are diverted to cleaning up, more and more resources are

[21] The rest of the first-order conditions are listed in the appendix. Note, $f_{k_1} = f_1 - f_2$, $\frac{\partial f}{\partial k} \equiv f_k = f_2$, and $f_{x_i^1} = -f_{x_i^2}$, $\forall\ i = 1, \ldots, I$.

[22] Recall, $h_y = 1$ under our maintained assumptions.

available to produce the consumption good. Under non-increasing returns in the transformation of the abatement resource into the consumption good ($h_{aa} \geq 0$), the increases in the level of the intended output decrease. Hence, the graph of h_a/θ has a non-negative slope. Under our maintained assumptions, this slope is $h_{aa}/\theta \geq 0$. See Figure 3 (a).

- An increase in the level of direct usage of the i^{th} fuel in intended production (*i.e.,* an increase in x_i^2) by one unit implies an increase in the level of local emission by an amount α_i and an increase in the (net of extraction cost) level of the intended output by an amount $f_{x_i^2} - f_l c_i$, so that $[f_{x_i^2} - f_l c_i]/\alpha_i$ is the change in intended output per unit change in local emission that results from a unit increase in the level of the direct usage of the i^{th} fuel. Under our maintained assumptions and under non-increasing returns to fuel inputs (*i.e.,* $f_{x_i x_i} \leq 0$), this graph has a non-positive slope $f_{x_i x_i}/\alpha_i$. See Figure 3 (b).
- Consider an increase in the the allocation of a given amount of capital to its fuel-intensive use (*i.e.,* an increase in k_1) by one unit that is fuelled by the i^{th} fuel input. This implies a decrease in the non-fuel intensive use of capital (*i.e.,* a decrease in k_2) by one unit and an increase in the level of usage of the i^{th} fuel (*i.e.,* an increase in x_i^1) by an amount $\phi_1/\psi_{x_i^1}$.[23] Hence, there is an increase in local emission by an amount $[\phi_1/\psi_{x_i^1}]\alpha_i$ and an increase in the (net of extraction cost) level of the intended output by an amount $[f_1 - f_2] - [\phi_1/\psi_{x_i^1}]f_l c_i$, where f_1 is the marginal productivity of fuel intensive capital, while $-f_2$ is the decline in output because of diverting one unit of capital from its non-fuel intensive to fuel intensive use. Hence, we can interpret f_2 as the opportunity cost of using capital in its fuel-intensive form. Thus, $\Big[[f_1 - f_2] - [\phi_1/\psi_{x_i^1}]f_l c_i\Big] \,/\, \Big[[\phi_1/\psi_{x_i^1}]\alpha_i\Big]$ is the change in intended output per unit change in local emission that results from a unit increase in the allocation of a given amount of capital to its fuel-intensive use.

Let the mapping of interior optima of (5.1) be denoted by $\varphi : \mathbf{R}_+^2 \longmapsto \mathbf{R}_+^{5+2I}$ with image $\langle y, a, k_1, l, x, x^1, z\rangle \in \varphi(k, g)$. If the mapping φ is a function, then $\varphi(k, g) \equiv \Big\langle\ \hat{y}(k, g),\ \hat{a}(k, g),\ \hat{k}_1(k, g),\ \hat{l}(k, g),\ \hat{x}(k, g),\ \hat{x}^1(k, g),\ \hat{z}(k, g)\ \Big\rangle$.

[23] Recall, ϕ_1 is the increase in energy requirement because of a unit increase in k_1 and $\psi_{x_i^1}$ is the energy produced by a unit increase in x_i^1.

5.2. Decentralization of the planner's solution.

We present two institutional structures based on price instruments for decentralizing an interior optimum of (5.1). In both structures, we assume that there are two production sectors in the economy – the consumption good sector and the extraction/mining sector that extracts the energy/fuel inputs. In each sector, without loss of generality, let us assume that the production activities of firms are aggregated to obtain an aggregated firm. Emission is generated by the consumption good producing sector, which owns the capital endowment of the economy. The technology of the mining sector is constant returns to scale and is given by:

$$\sum_{i=1}^{I} c_i \tilde{\tilde{x}}_i \leq \tilde{\tilde{l}}, \tag{5.8}$$

where, for $i = 1, \ldots, I$, $\tilde{\tilde{x}}_i$ is the amount of the i^{th} fuel input extracted and $\tilde{\tilde{l}}$ is the total labour input used for extraction. The aggregate consumer owns all firms in the economy and inelastically supplies the labour resource, so that he derives profit and labour incomes.

5.2.1. Decentralization with a market for rights to emit.

Suppose the right over the environment is held by the aggregate consumer and the consumption good producing sector needs to buy them to be able to generate emission during intended production.[24] We normalize the price of the consumption good to one. Suppose the price of labour and fuel inputs are denoted by $p_l > 0$ and $p_x > 0$, respectively, while the price received by consumer for selling a right to emit is $-p_z > 0$. The restricted profit maximization problem of the consumption good producing sector is

$$\begin{gathered} \Pi(p_l, p_x, p_z, k) \equiv \max_{y,a,l,x^1,x^2,k_1,z} \; y + p_z z - p_l l - p_x x \\ \text{subject to} \\ h(y,a) \leq f(k_1, k-k_1, l, x^2), \quad \phi(k_1) \leq \psi(x^1), \quad \text{and} \quad z \geq \sum_{i=1}^{I} \alpha_i [x_i^1 + x_i^2] - \theta a. \end{gathered} \tag{5.9}$$

[24] Note, since in our model local and global emissions do no impose externalities on producers, Starret [1972] type fundamental non-convexities in production are absent. See also Murty [2010].

The mining sector maximizes $p_x\tilde{\tilde{x}} - p_l\tilde{\tilde{l}}$ subject to (5.8). Constant returns to scale implies that its maximum profit is zero. The problem of the consumer is (4.3), where $m = p_l\bar{l} + \Pi(p_l, p_x, p_z, k)$.

Definition. A *competitive equilibrium with a market for rights to emit* is a vector of prices and income $\rho = \langle p_l, p_x, p_z, m\rangle \in \mathbf{R}_+^{I+1} \times \mathbf{R}_- \times \mathbf{R}$ and an allocation $\langle y, a, k_1, l, x^1, x^2, z, \tilde{\tilde{l}}, \tilde{\tilde{x}}\rangle \in \mathbf{R}_+^{3I+6}$ such that, given ρ, (i) $\langle y, z\rangle$ solves (4.3) with $m = p_l\bar{l} + \Pi(p_l, p_x, p_z, k)$, (ii) $\langle y, a, k_1, l, x^1, x^2, z\rangle$ solves (5.9), (iii) $\langle\tilde{\tilde{l}}, \tilde{\tilde{x}}\rangle$ satisfies (5.8), and (iv) $\tilde{\tilde{l}} + l = \bar{l}$ and $\tilde{\tilde{x}} = x^1 + x^2$.

Theorem DECENTRALIZATION1 (D1): *Suppose $\langle y, a, k_1, l, x, x^1, z\rangle \in \varphi(k, g)$. Then the vector of prices and income $\rho = \langle p_l, p_x, p_z, m\rangle \in \mathbf{R}_+^{I+1} \times \mathbf{R}_- \times \mathbf{R}$ and an allocation $\langle y, a, k_1, l, x^1, x^2, z, \tilde{\tilde{l}}, \tilde{\tilde{x}}\rangle \in \mathbf{R}_+^{3I+6}$ is a competitive equilibrium with a market for rights to emit, where $x^2 = x - x^1$, $\tilde{\tilde{x}} = x$, and $\tilde{\tilde{l}} = \sum_{i=1}^I c_i x_i$ and, evaluated at $\varphi(k, g)$, we have*

$$p_z = \frac{u_z}{u_y}; \quad p_{x_i} = c_i f_l \ \forall\ i = 1, \ldots, I; \quad \textit{and} \quad p_l = f_l. \tag{5.10}$$

5.2.2. Decentralization with a Pigouvian tax.

Suppose a Pigouvian tax is imposed on the consumption good producing sector at the rate $\tau > 0$. The Pigouvian tax revenue accrues to the government and is paid out as a lump-sum transfer to the aggregate consumer. The restricted profit maximization problem of the consumption good producing sector is

$$\begin{gathered} \Pi(p_l, p_x, \tau, k) \equiv \max_{y,a,l,x^1,x^2,k_1,z} \ y - \tau z - p_l l - p_x x \\ \text{subject to} \\ h(y, a) \leq f(k_1, k - k_1, l, x^2), \quad \phi(k_1) \leq \psi(x^1), \quad \text{and} \quad z \geq \sum_{i=1}^I \alpha_i[x_i^1 + x_i^2] - \theta a. \end{gathered} \tag{5.11}$$

The utility maximization problem of the consumer is:

$$\max_{\tilde{y} \geq 0}\{u(\tilde{y}, z, g) \mid \tilde{y} \leq p_l\bar{l} + \tau z + \Pi(p_l, p_x, \tau, k)\}. \tag{5.12}$$

Definition. A *competitive equilibrium with a Pigouvian tax* is a vector of prices and Pigou tax level $\rho = \langle p_l, p_x, \tau\rangle \in \mathbf{R}_+^{I+2}$ and an allocation $\langle y, a, k_1, l, x^1, x^2, z, \tilde{y}, \tilde{\tilde{l}}, \tilde{\tilde{x}}\rangle \in \mathbf{R}_+^{3I+7}$ such that, given ρ, (i) $\tilde{y}$ solves (5.12), (ii) $\langle y, a, k_1, l, x^1, x^2, z\rangle$ solves (5.11), (iii) $\langle\tilde{\tilde{l}}, \tilde{\tilde{x}}\rangle$ satisfies (5.8), and (iv) $\tilde{y} = y$, $\tilde{\tilde{l}} + l = \bar{l}$, and $\tilde{\tilde{x}} = x^1 + x^2$.

Theorem DECENTRALIZATION2 (D2): *Suppose* $\langle y, a, k_1, l, x, x^1, z\rangle \in \varphi(k, g)$. *Then the vector of prices and Pigou tax level* $\rho = \langle p_l, p_x, \tau\rangle \in \mathbf{R}_+^{I+2}$ *and the allocation* $\langle y, a, k_1, l, x^1, x^2, z, \tilde{\tilde{y}}, \tilde{\tilde{l}}, \tilde{\tilde{x}}\rangle \in \mathbf{R}_+^{3I+7}$ *is a competitive equilibrium with a Pigouvian tax, where* $\tilde{\tilde{y}} = y$, $x^2 = x - x^1$, $\tilde{\tilde{x}} = x$, *and* $\tilde{\tilde{l}} = \sum_{i=1}^{I} c_i x_i$ *and, evaluated at* $\varphi(k, g)$, *we have*

$$\tau = -\frac{u_z}{u_y}; \quad p_{x_i} = c_i f_l \ \forall\ i = 1, \ldots, I; \quad \textit{and} \quad p_l = f_l. \tag{5.13}$$

In particular,

$$\tau = \frac{h_a/h_y}{\theta} = \frac{f_{x_i^1} - f_l c_i}{\alpha_i} = \frac{[f_1 - f_2] - \frac{\phi_1}{\psi_{x_i^1}} f_l c_i}{\frac{\phi_1}{\psi_{x_i^1}}\alpha_i}, \qquad \forall\ i = 1, \ldots, I. \tag{5.14}$$

6. A global analysis.

Problem (5.1) summarizes the non-cooperative behavior of individual economies. Each economy, takes the global emission level as given, while determining its local emission level. In particular, it ignores the effect that its own emission decision has on the global emission level. In this section, we study the emission outcome that results in the global economy due to non-cooperative behaviour of individual economies.

In this context, the proof of existence of a Nash equilibrium requires that all the feasible local and global emission levels lie in bounded intervals. This will be true if the feasible extraction levels of fuel inputs are bounded. The latter are bounded by the availability $\bar{l}$ of the labour input, as extraction of emission-generating inputs is costly in labour input. For each $i = 1, \ldots, I$, the maximum amount of the i^{th} emission-causing input a country can use is $\frac{\bar{l}}{c_i}$. Hence the upper bound on local emission levels for any country is given by $\boldsymbol{z} \equiv \max\{\alpha_1 \frac{\bar{l}}{c_1}, \ldots, \alpha_I \frac{\bar{l}}{c_I}\}$. Thus, emission level of each country lies in the bounded interval $[0, \boldsymbol{z}] \equiv \mathcal{Z}$. g denotes the sum of emissions from all S countries. If emission level of each country is bounded by $\boldsymbol{z}$, then the sum of emissions generated by all countries is bounded by $S\boldsymbol{z}$. Hence $g \in [0, S\boldsymbol{z}] \equiv \mathcal{G}$. Redefine the mapping $\hat{z} : \mathbf{R}_+^2 \longmapsto \mathbf{R}_+$ as $\hat{z} : \mathbf{R}_+ \times \mathcal{G} \longmapsto \mathcal{Z}$ with image $z \in \hat{z}(k, g) \subset \mathcal{Z}$. We index the S countries by s. Suppose the capital endowment of each country $s = 1, \ldots, S$ is fixed equal to k^s.

Definition. $\langle \overset{*}{z}{}^1, \ldots, \overset{*}{z}{}^S, \overset{*}{g}\rangle$ is a *Nash equilibrium emission outcome* of the global economy if, for all $s = 1, \ldots, S$, we have: $\overset{*}{z}{}^s = \hat{z}(k^s, \overset{*}{g})$, $\forall\ s = 1, \ldots, S$ and $\overset{*}{g} = \sum_{s=1}^{S} \overset{*}{z}{}^s$.

Theorem NASH EQUILIBRIUM (NE): *Suppose $\hat{z} : \mathbf{R}_+ \times \mathcal{G} \longmapsto \mathcal{Z}$ is a continuous function. Then there exists a Nash equilibrium emission outcome of the global economy.*

The following theorem examines the relation between the levels of capital endowment and local emission at a non-cooperative Nash equilibrium. At a Nash equilibrium emission outcome, $\langle \overset{*}{z}^1, \ldots, \overset{*}{z}^S, \overset{*}{g} \rangle$, all countries face the same aggregate (global) level of emissions, $\overset{*}{g}$. The relation between individual (local) emission levels and capital endowments, hence, crucially depends on the monotonicity property of function $\hat{z}(\cdot,\ \overset{*}{g})$ in k, *i.e.,* on the sign of $\frac{\partial \hat{z}(k,\ \overset{*}{g})}{\partial k}$, when $\hat{z}$ is differentiable.

Theorem NASH kz RELATION (NkzR): *Suppose $k^1 \leq k^2 \leq \ldots \leq k^S$. Let $\langle \overset{*}{z}^1, \ldots, \overset{*}{z}^S, \overset{*}{g} \rangle$ be a Nash equilibrium emission outcome of the global economy.*

(i) *<u>Monotonic $\hat{z}(\cdot,\ \overset{*}{g})$:</u> If the function $\hat{z}(\cdot,\ \overset{*}{g})$ is non-increasing (resp., non-decreasing) in k then $\overset{*}{z}^1 \geq \overset{*}{z}^2 \geq \ldots \geq \overset{*}{z}^S$ (resp., $\overset{*}{z}^1 \leq \overset{*}{z}^2 \leq \ldots \leq \overset{*}{z}^S$).*

(ii) *<u>Inverted U-shaped $\hat{z}(\cdot,\ \overset{*}{g})$:</u> If there exists a unique $\overset{*}{k} > 0$ such that $k^1 < \overset{*}{k} < k^S$ and the function $\hat{z}(\cdot,\ \overset{*}{g})$ has a maximum at $\overset{*}{k}$, then there exists a non-negative integer $\overset{*}{s}$ such that $1 \leq \overset{*}{s} \leq S$ and $\overset{*}{z}^1 \leq \overset{*}{z}^2 \leq \ldots \leq \overset{*}{z}^{\overset{*}{s}-1} \leq \hat{z}(\overset{*}{k}, \overset{*}{g})$ and $\hat{z}(\overset{*}{k}, \overset{*}{g}) \geq \overset{*}{z}^{\overset{*}{s}} \geq \overset{*}{z}^{\overset{*}{s}+1} \geq \ldots \geq \overset{*}{z}^S$.*

(iii) *<u>N-shaped $\hat{z}(\cdot,\ \overset{*}{g})$:</u> If there exist unique $\overset{*}{k} > 0$ and $\hat{k} > 0$ such that $k^1 < \overset{*}{k} < \hat{k} < k^S$ and the function $\hat{z}(\cdot,\ \overset{*}{g})$ has a maximum at $\overset{*}{k}$ and a minimum at $\hat{k}$, then there exist two non-negative integers $\overset{*}{s}$ and $\hat{s}$ such that $1 \leq \overset{*}{s} < \hat{s} \leq S$, $\overset{*}{z}^1 \leq \overset{*}{z}^2 \leq \ldots \leq \overset{*}{z}^{\overset{*}{s}-1} \leq \hat{z}(\overset{*}{k}, \overset{*}{g})$, $\hat{z}(\overset{*}{k}, \overset{*}{g}) \geq \overset{*}{z}^{\overset{*}{s}} \geq \overset{*}{z}^{\overset{*}{s}+1} \geq \ldots \geq \overset{*}{z}^{\hat{s}-1} \geq \hat{z}(\hat{k}, \overset{*}{g})$, and $\hat{z}(\hat{k}, \overset{*}{g}) \leq \overset{*}{z}^{\hat{s}} \leq \overset{*}{z}^{\hat{s}+1} \leq \ldots \leq \overset{*}{z}^S$.*

The EKC phenomenon, as defined in the literature, is essentially a relationship between two endogenous variables, namely, local emission and the consumption good at a Nash equilibrium. If $\hat{y}$ is positive monotonic in k, then depending on whether function $\hat{z}$ is monotonic, *inverted U*, or *N*-shaped function of k, we obtain, respectively, a monotonic, *inverted U*, or a *N*-shaped relationship between local emission and consumption at a Nash equilibrium. Hence, in the subsequent analysis, we focus on the monotonicity properties of functions $\hat{z}$ and $\hat{y}$ in the level of capital.

7. Income and substitution effects of changes in the capital input.

To understand the shape of the cross-country consumption-emission graph, it will prove helpful to decompose the effects of a change in k on consumption and emission levels into income and substitution effects. In this section, we derive such a decomposition. For any $\langle k, g\rangle \in \mathbf{R}_+ \times \mathcal{G}$, assuming that the consumption good is the numeraire, it follows from Theorem D1 that there exist a shadow price of emission $p_z(k, g)$ and an income level $m(k, g)$ such that $\hat{y}(k, g)$ and $\hat{z}(k, g)$ solve the utility maximization problem (4.3). Precisely,

$$\begin{aligned} p_z(k,g) &\equiv \frac{u_z}{u_y} \text{ and} \\ m(k,g) &= p_z(k,g)\hat{z}(k,g) + \hat{y}(k,g), \end{aligned} \tag{7.1}$$

where u_z and u_y are evaluated at the optimal quantity vector $\varphi(k, g)$. Thus, the resulting Marshallian demands are equal to $\langle \hat{z}(k, g),\ \hat{y}(k, g)\rangle$, *i.e.*,

$$\Big\langle\ y\left(p_z(k,g), m(k,g), g\right),\ z\left(p_z(k,g), m(k,g), g\right)\Big\rangle \quad = \quad \langle\ \hat{z}(k,g),\ \hat{y}(k,g)\ \rangle. \tag{7.2}$$

Define an index $\hat{t}$ that can take values $\hat{z}$ or $\hat{y}$.[25] With the help of the Slutsky decomposition, from (7.2) we obtain

$$\begin{aligned} \frac{\partial \hat{t}}{\partial k} \equiv &= \frac{\partial t}{\partial p_z}\frac{\partial p_z}{\partial k} + \frac{\partial t}{\partial m}\frac{\partial m}{\partial k} \\ &= \left[\frac{\partial t^H}{\partial p_z} - z\frac{\partial t}{\partial m}\right]\frac{\partial p_z}{\partial k} + \frac{\partial t}{\partial m}\left[z\frac{\partial p_z}{\partial k} + p_z\frac{\partial \hat{z}}{\partial k} + \frac{\partial \hat{y}}{\partial k}\right] \\ &= \frac{\partial t^H}{\partial p_z}\frac{\partial p_z}{\partial k} + \frac{\partial t}{\partial m}\left[p_z\frac{\partial \hat{z}}{\partial k} + \frac{\partial \hat{y}}{\partial k}\right]. \end{aligned} \tag{7.3}$$

Define the substitution effects of changes in the levels of the capital input as the income compensated changes in demands due to a change in price brought about by a change in the level of the capital input:

$$SE_t(k) \equiv \frac{\partial t^H}{\partial p_z}\frac{\partial p_z}{\partial k}. \tag{7.4}$$

Similarly, we define the income effects of change in the level of the capital input:

$$IE_t(k) \equiv \frac{\partial t}{\partial m}\left[p_z\frac{\partial \hat{z}}{\partial k} + \frac{\partial \hat{y}}{\partial k}\right], \tag{7.5}$$

[25] Or, equivalently, define index $t\ = z, y$.

so that (7.3) implies

$$\frac{\partial \hat{t}}{\partial k} \equiv \frac{dt}{dk} = SE_t(k) + IE_t(k). \tag{7.6}$$

8. Some special cases of the working model.

In order to understand factors that determine cross-country differences in emission levels, and hence the shape of the cross-country consumption-emission graph, it is illuminating to study some special cases of the working model. Each special case, below, focusses on the role of some features of the model by abstracting from the others.

8.1. The case of zero fuel intensity of the capital input.

Consider a simple case where capital is productive only when it is used in its non-fuel intensive role. Thus, $f_1 = 0$ and $f_2 > 0$. In that case, it is clear that at an optimum of problem (5.1), $k_1 = 0$ and $x^1 = 0$ so that $x^2 = x$ and the second constraint of (5.1) will be irrelevant/non-binding ($\mu = 0$ in (5.2)). Redefine the restriction of function f to $k_1 = 0$, $k_2 = k$, and $x^2 = x$ as $\hat{f}(k,l,x) \equiv f(0,k,l,x)$. This implies $\hat{f}_k = f_2$. For notational convenience, we assume $I = 1$. The lemma, below, summarizes the effects of a change in capital and global emission on consumption and emission levels. Implicit in the detailed proof of the lemma, is a standard comparative static exercise for the problem (5.1).

Lemma NO FUEL COST OF k (NFCk): *Suppose $I = 1$, $f_1 = 0$, $f_2 > 0$, $\hat{f}(k,l,x) \equiv f(0,k,l,x)$, and the solution mapping φ of (5.1) is a differentiable function.*

(1) If $\hat{f}_{xx} \neq 0$ and $h_{aa} \neq 0$, then the following hold:

$$(i)\quad \frac{\partial \hat{a}}{\partial k} = \frac{-\mathcal{A}\hat{f}_k u_y \hat{f}_{xx}\theta}{Q\left[h_{aa}\alpha^2 - \hat{f}_{xx}\theta^2\right] - h_{aa}\hat{f}_{xx}u_y^3} \quad \text{and} \quad \frac{\partial \hat{a}}{\partial g} = -\frac{\partial r}{\partial g}\left[\frac{u_y^3 \hat{f}_{xx}\theta}{Q\left[h_{aa}\alpha^2 - \hat{f}_{xx}\theta^2\right] - h_{aa}\hat{f}_{xx}u_y^3}\right].$$

$$(ii)\quad \frac{\partial \hat{x}}{\partial k} = \frac{-\mathcal{A}\hat{f}_k u_y \alpha h_{aa}}{Q\left[h_{aa}\alpha^2 - \hat{f}_{xx}\theta^2\right] - h_{aa}\hat{f}_{xx}u_y^3} \quad \text{and} \quad \frac{\partial \hat{x}}{\partial g} = \left[\frac{h_{aa}\alpha}{\hat{f}_{xx}\theta}\right]\frac{\partial \hat{a}}{\partial g}.$$

$$(iii)\quad \frac{\partial \hat{z}}{\partial k} = -\mathcal{A}\hat{f}_k u_y\left[\frac{h_{aa}\alpha^2 - \hat{f}_{xx}\theta^2}{Q\left[h_{aa}\alpha^2 - \hat{f}_{xx}\theta^2\right] - h_{aa}\hat{f}_{xx}u_y^3}\right] \quad \text{and} \quad \frac{\partial \hat{z}}{\partial g} = \alpha\frac{\partial \hat{x}}{\partial g} - \theta\frac{\partial \hat{a}}{\partial g}.$$

$$(iv)\quad \frac{\partial \hat{y}}{\partial k} = \frac{\hat{f}_k\left[\mathcal{B}u_y\left[h_{aa}\alpha^2 - \hat{f}_{xx}\theta^2\right] - h_{aa}\hat{f}_{xx}u_y^3\right]}{Q\left[h_{aa}\alpha^2 - \hat{f}_{xx}\theta^2\right] - h_{aa}\hat{f}_{xx}u_y^3} \quad \text{and} \quad \frac{\partial \hat{y}}{\partial g} = \left[\hat{f}_x - \hat{f}_l c\right]\frac{\partial \hat{x}}{\partial g} - h_a\frac{\partial \hat{a}}{\partial g}.$$

(2) If $\hat{f}_{xx} = 0$ and $h_{aa} \neq 0$, then $\frac{\partial \hat{a}}{\partial k} = 0$, $\frac{\partial \hat{x}}{\partial k} = \frac{-\mathcal{A}\hat{f}_k u_y}{\alpha Q}$, $\frac{\partial \hat{z}}{\partial k} = \frac{-\mathcal{A}\hat{f}_k u_y}{Q}$, and $\frac{\partial \hat{y}}{\partial k} = \frac{\mathcal{B}\hat{f}_k u_y}{Q}$.

(3) If $h_{aa} = 0$ and $\hat{f}_{xx} \neq 0$, then $\frac{\partial \hat{a}}{\partial k} = \frac{\mathcal{A}\hat{f}_k u_y}{\theta Q}$, $\frac{\partial \hat{x}}{\partial k} = 0$, $\frac{\partial \hat{z}}{\partial k} = \frac{-\mathcal{A}\hat{f}_k u_y}{Q}$.

(4) $\frac{\partial \tau}{\partial k} = \frac{h_{aa}}{\theta}\frac{\partial \hat{a}}{\partial k} = \frac{\hat{f}_{xx}}{\alpha}\frac{\partial \hat{x}}{\partial k}$ and $\frac{\partial \tau}{\partial g} = \frac{h_{aa}}{\theta}\frac{\partial \hat{a}}{\partial g} = \frac{\hat{f}_{xx}}{\alpha}\frac{\partial \hat{x}}{\partial g}$.

(5) $SE_z(k) = \frac{u_y^3}{Q}\frac{\partial \tau}{\partial k}$, $IE_z(k) = -\frac{u_y \mathcal{A}}{Q}\hat{f}_k$, $SE_y(k) = -\frac{u_y^2 u_z}{Q}\frac{\partial \tau}{\partial k}$, and $IE_y(k) = \frac{u_y \mathcal{B}}{Q}\hat{f}_k$.[26]

The theorem below follows directly from Lemma NFCk, when we recall that $\mathcal{A}$ is negative (respectively, non-negative) whenever local emission exhibits normality (respectively, inferiority) and that, under the maintained assumptions of the working model, we have $f_{xx} \leq 0$ and $h_{aa} \geq 0$.

Theorem NO FUEL COST OF k (NFCk): *Suppose $I = 1$, $f_1 = 0$, $f_2 > 0$, $\hat{f}(k,l,x) \equiv f(0,k,l,x)$, u exhibits strategic substitutability with respect to the global emission, and the solution mapping φ of (5.1) is a differentiable function. Suppose Assumption 1 holds and, for any $g \in \mathcal{G}$, there exists a $\overset{*}{k}(g) > 0$ such that $\langle \hat{y}(k,g), \hat{z}(k,g)\rangle \in \mathcal{N}(g)$ for all $k < \overset{*}{k}(g)$ and $\langle \hat{y}(k,g), \hat{z}(k,g)\rangle \notin \mathcal{N}(g)$ for all $k \geq \overset{*}{k}(g)$. Then for any $g \in \mathcal{G}$, the following are true:*

(i) $\frac{\partial \hat{z}}{\partial k} > 0$ if and only if $\langle \hat{z}(k,g), \hat{y}(k,g)\rangle \in \mathcal{N}(g)$ and $\frac{\partial \hat{y}}{\partial k} > 0 \ \forall\ k$.

(ii) $\frac{\partial \hat{z}}{\partial g} \leq 0$ and $\frac{\partial \hat{y}}{\partial g} \leq 0$.

(iii) $\frac{\partial r}{\partial k} = -\frac{\partial p_z}{\partial k} = \frac{\partial \tau}{\partial k} \leq 0$ whenever $\langle \hat{z}(k,g), \hat{y}(k,g)\rangle \in \mathcal{N}(g)$,
$\frac{\partial r}{\partial k} = -\frac{\partial p_z}{\partial k} = \frac{\partial \tau}{\partial k} \geq 0$ whenever $\langle \hat{z}(k,g), \hat{y}(k,g)\rangle \notin \mathcal{N}(g)$, and
$\frac{\partial r}{\partial k} = -\frac{\partial p_z}{\partial k} = \frac{\partial \tau}{\partial k} = 0$ if $\hat{f}_{xx} = 0$ or $h_{aa} = 0$.

(iv) $SE_z(k) = 0$ if $\hat{f}_{xx} = 0$ or $h_{aa} = 0$ and

$$SE_z(k) \leq 0 \text{ and } IE_z(k) > 0 \text{ whenever } \langle \hat{z}(k,g), \hat{y}(k,g)\rangle \in \mathcal{N}(g), \text{ and}$$

$$SE_z(k) \geq 0 \text{ and } IE_z(k) \leq 0 \text{ whenever } \langle \hat{z}(k,g), \hat{y}(k,g)\rangle \notin \mathcal{N}(g).$$

(v) If $\langle \overset{*}{z}{}^1, \ldots, \overset{*}{z}{}^S, \overset{*}{g}\rangle$ is a Nash equilibrium emission outcome then $\overset{*}{k}(\overset{*}{g})$ is a maximum of the function $\hat{z}(\cdot, \overset{*}{g})$ and (ii) of Theorem Nk, zR holds.

[26] Note, we have dropped the subscript $i = 1$ because we are considering the case when $I = 1$.

For any $g \in \mathcal{G}$, suppose we define poorly (respectively, richly) endowed economies as those for which capital endowments are less than (respectively, more than) $\overset{*}{k}(g)$, as defined in Theorem NFCk. Then this theorem shows that, in the set of poorly endowed economies, the level of local emission z increases as capital endowment increases, while the opposite is true in the set of richly endowed economies. However, in both types of economies, the consumption good level y increases as the capital increases. Thus, in this special case, the EKC phenomenon emerges.

Conclusion (iii) of this theorem shows that, under diminishing returns to fuel inputs and in the transformation of abatement resource into the consumption good (*i.e.,* $f_{xx} < 0$ and $h_{aa} > 0$), the marginal rate of substitution between the consumption good and local emission is decreasing (respectively, increasing) in capital in the case of poorly (respectively, richly) endowed economies.[27] The consequences of this for the differences in the emission levels as we move from low capital to high capital economies can be explained with respect to both the supply and demand side factors.

<u>Demand for rights to emit.</u> The Pigou tax level is the marginal cost per unit marginal local emission because of relaxing any abatement strategy. (5.14) shows that, at a decentralized optimum of problem (5.1), this should be equal to the marginal returns per unit marginal emissions from relaxing any available abatement strategy. In the special case under study, there are two types of abatement strategies – increases in the level of cleaning up and decreases in the level of the direct use of fuel. Consider the set of poorly endowed economies. Conclusion (iii) of Theorem NFCk implies that the Pigou tax level is non-increasing in capital in this set of economies. Thus, suppose Pigou tax is τ_1 in a low capital economy and τ_2 in a high capital economy with $\tau_2 < \tau_1$. Figures 3 (a) and (b) show that, for each country, the marginal return per unit marginal local emission from both abatement strategies are equalized to its Pigou tax level when the low capital country chooses a_1 and x_1 levels of cleaning-up and fuel input, respectively, and when the higher capital country chooses a_2 and x_2 levels, respectively. The figures assume diminishing returns. Hence, $a_1 > a_2$ and $x_1 < x_2$. Thus, as we move from a low to a high capital economy in the set of poorly endowed economies, changes in both abatement strategies work in the direction of increasing the local emission level – cleaning-up level falls and fuel

[27] It is constant under constant returns, *i.e.,* when $f_{xx} = 0$ or $h_{aa} = 0$.

usage rises. Similarly, one can argue that exactly the opposite holds in the set of richly endowed economies – as we move from low to high capital economies, Pigou tax level rises, cleaning-up level rises, fuel usage falls, and so local emission level falls. Thus, in both sets of countries, in production, the conventional negative relationship between Pigou tax and the local emission levels emerges – countries where Pigou taxes are high produce less emissions relative to countries where Pigou taxes are low.

<u>Supply of rights to emit.</u> Consider the decentralization of the optima of (5.1) through a market for rights to emit. Differences in capital level across countries imply differences in the levels of both the shadow income and the shadow price of emission faced by the aggregate consumer. In both groups of poorly and richly endowed economies, shadow income increases as capital increases: higher amounts of capital imply greater production and hence higher incomes to consumers in private ownership economies. However, as conclusion (iii) of Theorem NFCk shows, the shadow price $(-p_z)$ received by the aggregate consumer from selling rights to emit decreases (respectively, increases) in the set of poorly (respectively, richly) endowed economies. The change in the supply of rights to emit is a net result of two opposing effects. Take, *e.g.,* the case of an increase in capital in the set of poorly endowed economies. A decrease in $-p_z$ implies that the aggregate consumer in the higher capital economy receives a lower price from selling emission rights. Hence, under a welfare preserving income compensation, he sells less emission rights, *i.e.,* he prefers to consume less emission – this is the substitution effect of an increase in capital. In Figure 4, this is marked by a movement from D to S. However, emission is a normal good in these economies. Hence, higher income to the aggregate consumer in the higher capital economy implies that the income effect of an increase in capital on emission is positive. In Figure 4, this is marked by a movement from S to N. Since, according to Theorem NFCk, in this group of economies the local emission rises as capital increases, it must be the case that the income effect dominates the substitution effect – emission level increases in a movement from D to N. Exactly the reverse arguments hold for the group of richly endowed economies.

Theorem NFCk also shows that, if u exhibits strategic substitutability with respect to global emission, then local emission and consumption good levels in any economy fall as global emission increases. The fall in consumption can be interpreted as the externality effect of an increase in global emission, while the fall in local emission reflects the strategic response of any individual economy to increase in the aggregate level of emission.

Example 1 continued: Suppose $h(y,a) = y + a^2$, $\hat{f}(k,l,x) = k^{\frac{1}{2}} + .001l + .102x$, $\alpha = 3$, $\theta = 1$, $c = 2$, and $\bar{l} = 100$. Thus, the intended production technology, nature's emission generating technology, and the resource constraint on the labour input are:

$$y \leq k^{\frac{1}{2}} + .001l + .102x - a^2, \qquad z \geq 3x - a, \qquad \text{and} \qquad l + 2x \leq 100. \tag{8.1}$$

The technological specification in (8.1) implies that there is constant returns to the emission-causing input ($\hat{f}_{xx} = 0$). Hence, the trade-off in production between emission and the consumption good is a constant. It is given by $\frac{\hat{f}_x - \hat{f}_l c}{\alpha} = \frac{.102 - .001 \times 2}{3} = .0333$. From (5.7) it follows that, at the solution to (5.1), we have $\frac{h_a}{\theta} = .0333$. This implies $a = 0.0167$, *i.e,* the level of cleaning-up is independent of the endowment level.

Furthermore, (5.7) implies that, at a solution to (5.1), $\frac{-u_z}{u_y}$ should also be equal to 0.033, and hence it does not vary with capital. Hence, in this example, there are no substitution effects of a change in capital, and the solution to (5.1) lies on $ICC(-0.0333, g)$. Additive separability of u in g in this example implies that $\overset{*}{k}(g)$, as defined in Theorem NFCk, is independent of g. It can be computed to be equal to 0.1282. As k varies from 0 to 2.3036, the local emission level first increases from 1.119 reaching a maximum of 1.25 at $k = 0.1282$. This is because of an increase in the usage of emission-causing inputs in the poorly endowed economies from 0.378 to 0.422. The local emission level then decreases to zero, and this is because of a decrease in the usage of emission-causing input in the richly endowed economies from 0.422 to .006. On the other hand, the level of the consumption good increases monotonically from 0.1375 to 1.61803. Hence, we obtain an *inverted U*-shaped EKC. See Figures 5 (a) to (c). Poorly endowed economies are characterized by deteriorating abatement practices, while in the richly endowed economies, increases in capital substitute out the emission-causing input without compromising on the production of the consumption good.

Example 2 continued:

Suppose cleaning-up option is not employed, so that mitigation of emission can only be done by consuming less of the emission-causing input. Suppose $\alpha = 10$, $c = 1$, and

$\bar{l} = 10$. The by-production technology and the labour resource constraint are:

$$y \leq k^{\frac{1}{2}} + l + 4.496x, \qquad z \geq 10x, \qquad \text{and} \qquad l + x \leq 10. \tag{8.2}$$

The technological specification in (8.2) implies that there is constant returns to the emission-causing input. Hence, the trade-off in production between emission and the consumption good in production is fixed as $\frac{\hat{f}_x - c\hat{f}_l}{\alpha} = \frac{4.496-1}{10} = 0.3496$. (8.2) implies that

$$\frac{\partial y}{\partial k} = \frac{1}{2\sqrt{k}} + 10 + 3.496\frac{\partial x}{\partial k} \quad \text{and} \quad \frac{\partial z}{\partial k} = 10\frac{\partial x}{\partial k}. \tag{8.3}$$

In this example, $\overset{*}{k}(g)$ can be computed to be equal to 12.132.[28] Since, in the region $\mathcal{N}$, u is twice continuously differentiable and (5.7) is true at a solution to (5.1), we find that for all k such that $0 \leq k < 12.132$, the consumption marginal rate of substitution should be $-\frac{u_z}{u_y} = \frac{\hat{f}_x - c\hat{f}_l}{\alpha} = 0.3496$, which is a constant. Hence, in this range of k, substitution effects are zero. Hence, (8.3) implies that, for all k such that $0 \leq k < 12.132$, we have

$$-\frac{\partial \frac{u_z}{u_y}}{\partial k} = -\left[\frac{\partial \frac{u_z}{u_y}}{\partial y}\frac{\partial y}{\partial k} + \frac{\partial \frac{u_z}{u_y}}{\partial z}\frac{\partial z}{\partial k}\right] = 0$$

$$\Longrightarrow \frac{\partial \hat{x}}{\partial k} = -\frac{u_y\mathcal{A}}{20Q\sqrt{k}} > 0, \; \frac{\partial \hat{y}}{\partial k} = \frac{1}{2\sqrt{k}} + (3.496)\left[-\frac{u_y\mathcal{A}}{20Q\sqrt{k}}\right] > 0, \text{ and } \frac{\partial \hat{z}}{\partial k} = -\frac{u_y\mathcal{A}}{2Q\sqrt{k}} > 0. \tag{8.4}$$

We can compute that, as k increases from 0 to 12.132, optimal local emission increases from 87.75 to 90 and the optimal consumption good output increases from 40.678 to 44.949. In Figure 2 (c), this corresponds to movements along the locus bd on the ICC corresponding to consumption marginal rate of substitution equal to 0.3496. In this region, local emission increases because of increase in the usage of the emission-causing input.

For $k \geq 12.132$, the equalization of the production and consumption trade-offs between emission and the consumption good lead us to points where u is not differentiable. In particular, for $k \in [12.132, 246.553]$, the solution to (5.1) lies on the path de in Figure 2 (c), where y is fixed at 44.949, while z decreases from 90 to 55.05. For $k \geq 246.553$, the solution to (5.1) lies along the path en in Figure 2 (c), where y increases steadily from 44.949 to 100 and z continues to decrease from 55.05 to 0. The decrease in the local emission level for $k \geq 12.132$ comes about due to a decrease in the use of the emission-causing input. The resulting z - k relation and y - k relations are shown in Figures 6 (a) and (b).

[28] Recall, we are abstracting from global emission in this example.

8.2. The case of fuel intensity of the capital input.

The examples in the previous special case, where we abstracted from fuel costs of running capital, showed that, in the group of richly endowed economies, capital tends to substitute out the emission-causing input, so that countries with higher capital will have lower emissions with no compromise in their level of production of the consumption good. Let us now turn to an opposite case of fuel intensive capital. For notational convenience, we continue assuming that $I = 1$. But suppose $f_{x^2} = 0$, *i.e.*, there is no direct role of the fuel input in intended production. Rather, it is only required to operate the fuel-intensive form of capital. In that case, at an optimum of (5.1), it is clear that $x^2 \equiv x - x^1 = 0$. Redefine the restriction of function f to $x^2 = 0$ as $\hat{f}(k_1, k - k_1, l) \equiv f(k_1, k - k_1, l, 0)$. For computational tractability, we assume that the functions ψ and ϕ are linear with $\psi_1 = 1$. The lemma, below, summarizes the effects of change in capital on emission and consumption levels.

Lemma FUEL INTENSIVE k (FIk): *Suppose $I = 1$, $f_{x^2} = 0$, $\hat{f}(k_1, k - k_1, l) \equiv f(k_1, k - k_1, l, 0)$, ψ and ϕ are linear with $\psi_1 = 1$, and the solution mapping φ of (5.1) is a differentiable function.*

(1) If $\hat{f}_{11} < 0$, $\hat{f}_{22} < 0$, and $h_{aa} > 0$, then the following hold:

$$(i) \quad \frac{\partial \hat{a}}{\partial k} = \frac{-\theta \left[\mathcal{A}\hat{f}_2 u_y [\hat{f}_{11} + \hat{f}_{22}] + \hat{f}_{22}\alpha\phi_1 Q\right]}{Q\left[\alpha^2\phi_1^2 h_{aa} - \theta^2[\hat{f}_{11} + \hat{f}_{22}]\right] - h_{aa}u_y^3[\hat{f}_{11} + \hat{f}_{22}]},$$

$$(ii) \quad \frac{\partial \hat{k}_1}{\partial k} = \frac{-\left[\mathcal{A}\hat{f}_2 u_y \alpha\phi_1 h_{aa} + \hat{f}_{22}[\theta^2 Q + h_{aa}u_y^3]\right]}{Q\left[\alpha^2\phi_1^2 h_{aa} - \theta^2[\hat{f}_{11} + \hat{f}_{22}]\right] - h_{aa}u_y^3[\hat{f}_{11} + \hat{f}_{22}]},$$

$$(iii) \quad \frac{\partial \hat{k}_2}{\partial k} = 1 - \frac{\partial \hat{k}_1}{\partial k} = \frac{\mathcal{A}\hat{f}_2 u_y \alpha\phi_1 h_{aa} + Q[\alpha^2\phi_1^2 h_{aa} - \theta^2 \hat{f}_{11}] - h_{aa}u_y^3 \hat{f}_{11}}{Q\left[\alpha^2\phi_1^2 h_{aa} - \theta^2[\hat{f}_{11} + \hat{f}_{22}]\right] - h_{aa}u_y^3[\hat{f}_{11} + \hat{f}_{22}]},$$

$$(iv) \quad \frac{\partial \hat{z}}{\partial k} = \frac{-\left[\mathcal{A}\hat{f}_2 u_y[\alpha^2\phi_1^2 h_{aa} - \theta^2[\hat{f}_{11} + \hat{f}_{22}]] + \hat{f}_{22}\alpha\phi_1 h_{aa}u_y^3\right]}{Q\left[\alpha^2\phi_1^2 h_{aa} - \theta^2[\hat{f}_{11} + \hat{f}_{22}]\right] - h_{aa}u_y^3[\hat{f}_{11} + \hat{f}_{22}]}, \text{ and}$$

$$(v) \quad \frac{\partial \hat{y}}{\partial k} = \frac{\mathcal{B}\hat{f}_2 u_y \left[\alpha^2\phi_1^2 h_{aa} - \theta^2[\hat{f}_{11} + \hat{f}_{22}]\right] - h_{aa}u_y^3\left[\hat{f}_{22}[\hat{f}_1 - \hat{f}_2 - \hat{f}_l\phi_1 c] + \hat{f}_2[\hat{f}_{11} + \hat{f}_{22}]\right]}{Q\left[\alpha^2\phi_1^2 h_{aa} - \theta^2[\hat{f}_{11} + \hat{f}_{22}]\right] - h_{aa}u_y^3[\hat{f}_{11} + \hat{f}_{22}]}.$$

(2) $\frac{\partial \tau}{\partial k} = \frac{h_{aa}}{\theta}\frac{\partial \hat{a}}{\partial k} = \frac{1}{\alpha\phi_1}\left[\frac{\partial \hat{k}_1}{\partial k}[\hat{f}_{11} + \hat{f}_{22}] - \hat{f}_{22}\right]$ *and*

$$SE_z(k) = \frac{u_y^3}{Q}\frac{\partial \tau}{\partial k}, \quad IE_z(k) = -\frac{u_y\mathcal{A}}{Q}\hat{f}_2, \quad SE_y(k) = -\frac{u_y^2 u_z}{Q}\frac{\partial \tau}{\partial k}, \quad \textit{and} \quad IE_y(k) = \frac{u_y\mathcal{B}}{Q}\hat{f}_2.$$

The following theorem follows as a direct consequence of Lemma FIk, when one notes that, under the maintained assumptions, $f_{11} \le 0$, $f_{22} \le 0$, and $h_{aa} \ge 0$.

Theorem FUEL INTENSIVE k (FIk): *Suppose $I = 1$, $f_{x^2} = 0$, $\hat{f}(k_1, k-k_1, l) \equiv f(k_1, k-k_1, l, 0)$, $f_{11}, 0$, $f_{22} < 0$, ψ and ϕ are linear with $\psi_1 = 1$, and the solution mapping φ of (5.1) is a differentiable function. Suppose Assumption 1 holds and, for any $g \in \mathcal{G}$, there exists $\overset{*}{k}(g) > 0$ such that $\langle \hat{y}(k,g), \hat{z}(k,g)\rangle \in \mathcal{N}(g)$ for all $k < \overset{*}{k}(g)$ and $\langle \hat{y}(k,g), \hat{z}(k,g)\rangle \notin \mathcal{N}(g)$ for all $k \ge \overset{*}{k}(g)$. Then for any $g \in \mathcal{G}$, the following are true:*

(1) $\frac{\partial \hat{y}}{\partial k} > 0 \ \forall\ k$.

(2) $\frac{\partial \hat{z}}{\partial k} < 0$, $\frac{\partial \hat{k}_1}{\partial k} > 0$, *and* $IE_z(k) > 0$ *whenever* $\langle \hat{z}(k), \hat{y}(k)\rangle \in \mathcal{N}(g)$.

(3) $1 - \frac{\partial \hat{k}_1}{\partial k} \ge 0$, $\frac{\partial \hat{a}}{\partial k} \ge 0$, *and* $\frac{\partial r}{\partial k} = -\frac{\partial p_z}{\partial k} = \frac{\partial \tau}{\partial k} \ge 0$ *whenever* $\langle \hat{z}(k), \hat{y}(k)\rangle \notin \mathcal{N}(g)$.

(4) *If* $\langle \hat{z}(k), \hat{y}(k)\rangle \notin \mathcal{N}(g)$, *then* $SE_z(k) \ge 0$, $IE_z(k) \le 0$, *and*

  (a) $\frac{\partial \hat{z}}{\partial k} \le 0$ *if* $|IE_z(k)| \ge |SE_z(k)|$,

  (b) $\frac{\partial \hat{z}}{\partial k} \ge 0$ *if* $|IE_z(k)| \le |SE_z(k)|$, *and*

  (c) $\frac{\partial \hat{k}_1}{\partial k} \le 0 \implies \frac{\partial \hat{z}}{\partial k} \le 0 \iff \frac{\partial \hat{z}}{\partial k} > 0 \implies \frac{\partial \hat{k}_1}{\partial k} > 0$.

(5) *If* $h_{aa} = 0$, *then* $\frac{\partial \tau}{\partial k} = 0$ *and* $SE_z(k) = 0$.

(6) *Suppose* $\langle \overset{*}{z}{}^1, \ldots, \overset{*}{z}{}^S, \overset{*}{g}\rangle$ *is a Nash equilibrium emission outcome.*

  (i) *If* $|SE_z(k)| \le |IE_z(k)|$ *for all* $k \ge \overset{*}{k}(\overset{*}{g})$, *then* $\overset{*}{k}(\overset{*}{g})$ *is a maximum of the function* $\hat{z}(\cdot,\ \overset{*}{g})$ *and (ii) of Theorem Nk, zR holds.*

  (ii) *If there exists* $\hat{k}(\overset{*}{g}) > \overset{*}{k}(\overset{*}{g})$ *such that* $|SE_z(k)| \le |IE_z(k)|$ *for all* $k \in [\overset{*}{k}(\overset{*}{g}), \hat{k}(\overset{*}{g})]$ *and* $|IE_z(k)| \le |SE_z(k)|$ *for all* $k \ge \hat{k}(\overset{*}{g})$, *then* $\overset{*}{k}(\overset{*}{g})$ *is a maximum of the function* $\hat{z}(\cdot,\ \overset{*}{g})$, $\hat{k}(\overset{*}{g})$ *is a minimum of the function* $\hat{z}(\cdot,\ \overset{*}{g})$, *and (iii) of Theorem Nk, zR holds.*

As before, for all $g \in \mathcal{G}$, the poorly (respectively, richly) endowed economies are those with capital endowments below (respectively, above) $\overset{*}{k}(g)$. Theorem FIk shows that, as capital increases, the consumption good level increases in both groups of economies. The emission level increases with capital in the poorly endowed economies, while this relation is ambiguous in the set of richly endowed economies. Thus, in this special case, both *inverted U* and *N*-shaped EKCs are plausible. To understand this, let us focus on the set of richly endowed economies.[29] Theorem FIk shows that, in these economies, the marginal rate of substitution between emission and the consumption good is non-decreasing in capital. Once again, demand and supply-side factors can be invoked to explain differences in emission levels across these countries.

Demand for rights to emit. With diminishing returns ($f_{11} < 0$, $f_{22} < 0$, and $h_{aa} > 0$), Lemma and Theorem (FIk) show that the Pigou tax level is increasing in capital. Hence, profit maximization will imply that the marginal return per unit marginal local emission from relaxing any abatement strategy must also increase to match the increase in the Pigou tax. In this special case, there are two types of abatement strategies available to a country – (1) diverting more resources to cleaning-up activities (*i.e.,* increasing a) and (2) reallocating the total capital endowment from the fuel intensive form to the non-fuel intensive form (*i.e.,* increasing k_2 and decreasing k_1).

(1) Recall, diminishing returns in the transformation of the abatement resource into the consumption good implies $h_{aa} > 0$. Hence, the marginal return per unit marginal local emission increases when cleaning-up level increases (see Figure 3 (a)). Hence, as in conclusion 3 of Theorem (FIk), increases in Pigou tax accompanying increases in the capital endowment result in higher levels of cleaning-up.

(2) Under our maintained assumptions, f is additively separable in k_1 and k_2 and is linear in l and ϕ. With an abuse of notation, define the function

$$F(k_1, k) \equiv \frac{f_1(k_1) - f_2(k - k_1) - f_l \phi_1 c}{\phi_1 \alpha}.$$

Thus, $F(k_1, k)$ is the net increase in output per unit increase in emission because of an increase in the allocation of capital to its fuel-intensive use. It is decreasing in k_1: As k_1 increases, $k_2 = k - k_1$ decreases. Diminishing returns to both fuel intensive and non-fuel intensive uses of capital hence imply that f_1 decreases and f_2 (the opportunity cost of fuel intensive capital[30]) increases with increase in k_1.[31] See also Figures 7 (a) and (b) where, for a fixed k, the graph of F is downward sloping. Note also that, if

29 The set of poorly endowed economies can be similarly analyzed.

30 See Section 5.1 for details.

31 *I.e.,* $F_{k_1} = \frac{f_{11} + f_{22}}{\phi_1 \alpha} < 0$.

k_1 is fixed, an increase in total capital endowment implies an increase in $k_2 = k - k_1$. Hence, diminishing returns to k_2 implies that, ceteris paribus, the opportunity cost of fuel-intensive capital decreases as k increases. Hence, F is increasing in k.[32] In Figures 7 (a) and (b), this is depicted by an outward shift in the F schedule. Suppose the Pigou tax in an economy with capital endowment k' is τ'. As seen Figures 7 (a) and (b), this economy will choose its level of fuel intensive capital as k_1'. We know that an economy with higher capital endowment (say, k''), is associated with a higher Pigou tax (say, τ'').[33] The change in the level of fuel intensive capital depends on how the increase in F due to increase in capital compares with the increase in the level of the Pigou tax. As seen in Figure 7 (b) (respectively, 7 (a)), if the increase in F is greater (respectively, less) than the increase in Pigou tax at k_1', then the level of fuel intensive capital increases (respectively, decreases) from k_1' to k_1''. Thus, a counterintuitive outcome becomes possible: higher Pigou tax levels in higher capital economies can result in increased allocation of capital to its fuel intensive use, and hence contribute to higher local emission levels !

The net effect on emission level due to an increase in the Pigou tax as we move from low capital to high capital economies in the group of richly endowed economies depends on the relative magnitudes of changes in abatement strategies (1) and (2). If the allocation of capital to fuel intensive use either falls or increases modestly as capital level increases, the concomitant increase in the cleaning up level will result in a decrease in the level of net emission. On the other hand, if increases in capital are associated with greater and greater amount of capital being allocated to the fuel-intensive use, then it is possible that, after some threshold level of capital in the group of richly endowed economies, the increases in cleaning-up may not be able to offset the resulting increases in the gross emission level. If such a threshold does not exist, then, combined with rising emission level in the poorly endowed group of countries, we have an *inverted* U-shaped EKC emerging. On the other hand, if such a threshold exists, then we obtain a N-shaped EKC across all countries.

<u>Supply of rights to emit.</u> In the set of richly endowed economies, the higher capital economies are associated with higher shadow prices of emission. It is as if the aggregate consumer

[32] *I.e.*, $F_k = -f_{22} > 0$.

[33] Recall, we are focussing on the group of richly endowed economies.

of a higher capital economy receives a higher price from selling emission rights and hence, under a welfare preserving income compensation, the supply of these rights increases as we move to higher capital economies. This is the substitution effect of an increase in capital. Higher capital economies are also associated with higher shadow incomes. Since emission is an inferior good in the set of richly endowed economies, the income effect of an increase in capital on emission level is negative. Emission levels permitted by aggregate consumers differ across countries depending on how the substitution effects compares with the income effects.

8.3. The case of inter-fuel substitution.

A third special case that we can study with our working model is the case of inter-fuel substitution. To keep the analysis simple, we assume $I = 2$, the firm's technology does not permit cleaning-up, and that the fuel inputs are not directly productive in intended production. Rather, both fuels are employed only to run capital equipment and all capital is fuel intensive. Hence, $f_{x_i^2} = 0$ for $i = 1, 2$, $h(a, y) = y$, and $f_2 = 0$. It is clear that, at an optimum of (5.1), $k_1 = k$ and $x^2 = 0$. Define the restriction of f to $k_1 = k$ and $x^2 = 0$ as $\hat{f}(k, l) \equiv f(k, 0, l, 0)$. Then $\hat{f}_k = f_1$. Rewrite $\phi_1 \equiv \phi_k$. Suppose ψ and ϕ are linear functions. The effect of increase in capital on welfare is obtained as the derivative of the value function $\mathcal{V}$ of problem (5.1) with respect to k. Employing the envelope theorem, this is the partial derivative of the Lagrangian (5.2) with respect to k: $\frac{\partial \mathcal{V}}{\partial k} = \frac{\partial L}{\partial k} = \hat{f}_k - \phi_k \frac{\mu}{\lambda}$. In the theorem below, we only focus on the range of capital where welfare is increasing in capital at a solution to (5.1).

Theorem INTER-FUEL SUBSTITUTION (IFS): *Suppose $I = 2$, $f_{x^2} = 0$, $f_2 = 0$, $\hat{f}(k, l) \equiv f(k, 0, l, 0)$, $h(a, y) = y$, ψ and ϕ are linear functions, and the solution mapping φ of (5.1) is a differentiable function. Suppose Assumption 1 holds and, for any $g \in \mathcal{G}$, there exists a $\overset{*}{k}(g) > 0$ such that $\langle \hat{y}(k, g), \hat{z}(k, g)\rangle \in \mathcal{N}(g)$ for all $k < \overset{*}{k}(g)$ and $\langle \hat{y}(k, g), \hat{z}(k, g)\rangle \notin \mathcal{N}(g)$ for all $k \geq \overset{*}{k}(g)$. Suppose $\frac{c_1}{c_2} < \frac{\psi_1}{\psi_2} < \frac{\alpha_1}{\alpha_2}$. Then for any $g \in \mathcal{G}$ and for all $k > 0$ such that $\hat{f}_k - \phi_k \frac{\mu}{\lambda} > 0$ at a solution to (5.1), the following are true:*

(i) $\frac{\partial \hat{x}_1}{\partial k} = \frac{1}{[\alpha_2\psi_1 - \alpha_1\psi_2]Q} \left[u_y \mathcal{A} \psi_2 \left[\hat{f}_k - \phi_k \frac{\mu}{\lambda} \right] + \phi_k \alpha_2 Q \right]$.

(ii) $\frac{\partial \hat{x}_2}{\partial k} = -\frac{1}{[\alpha_2\psi_1 - \alpha_1\psi_2]Q} \left[u_y \mathcal{A} \psi_1 \left[\hat{f}_k - \phi_k \frac{\mu}{\lambda} \right] + \phi_k \alpha_1 Q \right]$.

(iii) $\frac{\partial \hat{z}}{\partial k} = -\frac{u_y \mathcal{A}}{Q}\left[\hat{f}_k - \phi_k \frac{\mu}{\lambda}\right]$.

(iv) $\frac{\partial \hat{y}}{\partial k} = \frac{u_y \mathcal{B}}{Q}\left[\hat{f}_k - \phi_k \frac{\mu}{\lambda}\right]$.

(v) $\frac{\partial \hat{z}}{\partial k} > 0$ *whenever* $\langle \hat{z}(k,g), \hat{y}(k,g)\rangle \in \mathcal{N}(g)$ *and*
$\frac{\partial \hat{z}}{\partial k} \leq 0$ *whenever* $\langle \hat{z}(k,g), \hat{y}(k,g)\rangle \notin \mathcal{N}(g)$.

(vi) $\frac{\partial \hat{y}}{\partial k} > 0 \ \forall\ k$.

(vii) $\frac{\partial \hat{x}_1}{\partial k} < 0$ *and* $\frac{\partial \hat{x}_2}{\partial k} > 0$ *whenever* $\langle \hat{z}(k,g), \hat{y}(k,g)\rangle \notin \mathcal{N}(g)$.

(viii) $\frac{\partial \hat{x}_2}{\partial k} > 0$ *and* $\frac{\partial \hat{x}_1}{\partial k} > 0$ *whenever* $\langle \hat{z}(k,g), \hat{y}(k,g)\rangle \in \mathcal{N}(g)$.

(ix) If $\langle \overset{*}{z}^1, \ldots, \overset{*}{z}^S, \overset{*}{g}\rangle$ *is a Nash equilibrium emission outcome, then* $\overset{*}{k}(\overset{*}{g})$ *is a maximum of the function* $\hat{z}(\cdot, \overset{*}{g})$ *and (ii) of Theorem* Nk, zR *holds.*

The assumption $\frac{c_1}{c_2} < \frac{\psi_1}{\psi_2} < \frac{\alpha_1}{\alpha_2}$ implies that, while the extraction cost per unit of energy generated is lower for the first fuel ($\frac{c_1}{\psi_1} < \frac{c_2}{\psi_2}$), the emission per unit of energy generated by the first fuel is also higher ($\frac{\alpha_1}{\psi_1} > \frac{\alpha_2}{\psi_2}$). In this special case, the abatement strategy available to countries is substituting the first (*i.e.,* the dirtier) fuel by the second.[34] Theorem IFS shows that this is precisely what the richly endowed economies do even though the relative extraction cost of the relatively cleaner input is high: in this group of countries, as capital increases, the usage of the second input rises, while that of the first falls. The poor economies increase usage of both fuels.

9. Conclusions.

The by-production-cum-preference based approach adopted above reveals the importance of the roles of income and substitution effects as well as the types of strategies available for abatement for studying the relation between growth and environmental quality. By modeling the causes of emissions in nature explicitly, we obtain a plethora of abatement strategies. By not restricting emissions to exhibit global inferiority, we can capture the high and increasing marginal valuation of consumption relative to environment in the developing and subsistence economies. Increases in endowment imply not only increases in the shadow income level but also changes in the MWTP (which reflects both the shadow price of emission and the Pigou tax level), hence they induce both income and substitution effects on consumption and emission levels. While, emission is an

[34] Since all capital is fuel intensive, it is not possible to reduce usage of both fuels as capital increases.

inferior good in richly endowed economies, it is a normal good in subsistence economies. If MWTP increases (respectively, decreases) with increase in endowment, substitution effect implies an increase (respectively, a decrease) in the emission level. Typically, in poorly (respectively, richly) endowed economies, MWTP decreases (respectively, increases) with increase in endowment. In the case of emission, income and substitution effects hence work in opposite directions.

Several implications follow for the EKC phenomenon from the different features of the model. Suppose countries differ with respect to endowments of some resources. If these resources can substitute emission-causing inputs in production, then we show that the income effects dominate the substitution effects and the phenomenon of an *inverted U*-shaped EKC arises. If, on the other hand, the use of fuels is complementary to the use of these resources, then an *inverted U* or an *N*-shaped EKC can arise. If inter-fuel substitution possibilities are open to countries, then the richer countries will choose to exercise them even if they involve greater fuel costs. On the other hand, in the poorly endowed economies, emission levels increase because, in their bid to produce more of the consumption good even at the cost of lower environmental quality, they increase the usage of all fuels.

The case of the *N*-shaped EKC is particularly revealing. Perhaps it sheds some light on cross-country differences in emission levels between the developing world, the European block of countries, and countries such as the U.S. Suppose we assume that U.S. and countries in the European block belong to the group of richly endowed economies, while the developing world is the set of poorly endowed economies. Our theory predicts that emission levels rise with wealth levels in the developing world. It also provides an explanation for the higher emission levels of countries such as the U.S. as compared to those in the European block, even though environmental policies (such as the Pigou tax levels) could become tighter as we move to the richer of these rich economies. On the production side, this is because countries such as the U.S. have lower opportunity costs of employing their resources in their fuel intensive forms: given their bigger resource-base as compared to the European block, diminishing returns implies much lower returns from any non-fuel intensive use of their resources. From the point of view of preferences, emission is an inferior good for both the European block and countries such as the U.S. However, while the increases in shadow income are enough to offset the substitution effects in the European block of countries, in the group of countries such as the U.S., substitution effects are significantly greater and the increases in shadow income levels are not enough to offset them. The possibility of an N-shaped EKC reveals that there are limits to increasing affluence resulting in a cleaner and cleaner environment under non-cooperative behaviour.

TECHNICAL APPENDIX

Proof of Lemma ΔMRS:

(i) Strict quasiconcavity of u implies that $Q \equiv Q(y, z, g) > 0$.

(ii) The Lagrangian of utility maximization is: $L = u(y, z, g) - \kappa[p_z z + y - m]$. The FOCs for an interior optimum are

$$u_y - \kappa = 0, \quad u_z - \kappa p_z = 0, \quad \text{and} \quad -[y + p_z z - m = 0]. \tag{9.1}$$

Standard comparative static exercise with respect to p_z, m, and g yields:

$$\begin{bmatrix} 0 & -1 & -p_z \\ -1 & u_{yy} & u_{yz} \\ -p_z & u_{yz} & u_{zz} \end{bmatrix} \begin{bmatrix} d\kappa \\ dy \\ dz \end{bmatrix} = - \begin{bmatrix} -z & 1 & 0 \\ 0 & 0 & u_{yg} \\ -\kappa & 0 & u_{zg} \end{bmatrix} \begin{bmatrix} dp_z \\ dm \\ dg \end{bmatrix}. \tag{9.2}$$

Noting from FOCs (9.1) that $\kappa = u_y$ and $p_z = \frac{u_z}{\kappa}$ and employing Crammer's rule on (9.2), we obtain

$$\begin{aligned} \frac{\partial z}{\partial m} &= \frac{u_y}{Q}[u_y u_{yz} - u_z u_{yy}] = -\frac{u_y^3}{Q}\frac{\partial r}{\partial y} = -\frac{u_y \mathcal{A}}{Q} \text{ and} \\ \frac{\partial y}{\partial m} &= -\frac{u_y}{Q}[u_y u_{zz} - u_z u_{yz}] = \frac{u_y^3}{Q}\frac{\partial r}{\partial z} = \frac{u_y \mathcal{B}}{Q}. \end{aligned} \tag{9.3}$$

(iii) Employing Crammer's rule, once again, on (9.2), we obtain

$$\begin{aligned} \frac{\partial z}{\partial p_z} &= -z\frac{u_y}{Q}[u_y u_{yz} - u_z u_{yy}] - \frac{u_y^3}{Q} = -z\frac{\partial z}{\partial m} - \frac{u_y^3}{Q} \text{ and} \\ \frac{\partial y}{\partial p_z} &= z\frac{u_y}{Q}[u_y u_{zz} - u_z u_{yz}] + \frac{u_y^2 u_z}{Q} = -z\frac{\partial y}{\partial m} + \frac{u_y^2 u_z}{Q}. \end{aligned} \tag{9.4}$$

Slutsky decomposition yields the substitution effects:

$$\begin{aligned} \frac{\partial z^H}{\partial p_z} &= \frac{\partial z}{\partial p_z} + z\frac{\partial z}{\partial m} = -\frac{u_y^3}{Q} \leq 0 \text{ and} \\ \frac{\partial y^H}{\partial p_z} &= \frac{\partial y}{\partial p_z} + z\frac{\partial y}{\partial m} = \frac{u_y^2 u_z}{Q} \leq 0. \end{aligned} \tag{9.5}$$

(iv) $-u_z\mathcal{A} + u_y\mathcal{B} = -u_z[-u_y u_{yz} + u_z u_{yy}] + u_y[-u_y u_{zz} + u_z u_{yz}]$
$= -u_z^2 u_{yy} - u_y^2 u_{zz} + 2u_y u_z u_{yz} = Q.$

(v) Since u exhibits global strategic substitutability with respect to global emissions, we have $\frac{\partial r}{\partial g} \equiv -\frac{u_y u_{zg} - u_z u_{yg}}{u_y^2} \geq 0$. Employing Crammer's rule on (9.2), we obtain

$$\frac{\partial z}{\partial g} = -\frac{u_y}{Q}[u_z u_{yg} - u_y u_{zg}] = -\frac{u_y^3}{Q}\frac{\partial r}{\partial g} \leq 0 \quad \text{and} \quad \frac{\partial y}{\partial g} = \frac{u_z}{Q}[u_z u_{yg} - u_y u_{zg}] = \frac{u_y^2 u_z}{Q}\frac{\partial r}{\partial g} \leq 0. \; \blacksquare \tag{9.6}$$

The first-order conditions of (5.1) at an interior optimum:

$$u_y - \lambda h_y = 0, \tag{9.7}$$

$$u_z + \delta = 0, \tag{9.8}$$

$$-\lambda h_a + \delta\theta = 0, \tag{9.9}$$

$$\lambda f_l - \gamma = 0, \tag{9.10}$$

$$\lambda[f_1 - f_2] - \mu\phi_1 = 0, \tag{9.11}$$

$$\lambda f_{x_i^1} + \mu\psi_{x_i^1} = 0, \text{ and} \tag{9.12}$$

$$\lambda f_{x_i} - \gamma c_i - \delta\alpha_i = 0, \ \forall\ i = 1, \ldots, I, \tag{9.13}$$

Proofs of both decentralization theorems in Section 5 are similar. Here we present only one of the proofs.

Proof of Theorem D2: At the allocation $\langle y, a, k_1, l, x, x^1, z\rangle \in \varphi(k, g)$, (5.7) holds. The Lagrangian of problem (5.11) is $L = y' - \tau z' - p_l l - p_x x' - \kappa_1[h(y', a') - f(k_1', k - k_1', l', x'^2)] - \kappa_2[\phi(k_1') - \psi(x'^1)] - \kappa_3[-z' + \sum_{i=1}^{I}\alpha_i[x_i'^1 + x_i'^2] - \theta a']$. It can be checked that (5.7) implies that all the FOCs for an interior optimum of (5.11) will hold at $\langle y, a, k_1, l, x^1, x^2, z\rangle$ if prices and Pigou tax levels are fixed as in (5.13). (5.7) also implies (5.14) under the maintained assumptions on h and f. For all $i = 1, \ldots, I$, under (5.13), we have $\frac{p_{x_i}}{p_l} = c_i$. This is the rate of product transformation in the mining sector at $\langle\sum_{i=1}^{I} c_i x_i, x\rangle$. Hence,

$\langle\tilde{\tilde{l}}, \tilde{\tilde{x}}\rangle \equiv \langle\sum_{i=1}^{I} c_i x_i, x\rangle$ is profit maximizing in this sector. At a solution to problem (5.12), we have

$$\begin{aligned}\tilde{\tilde{y}} &= p_l\bar{l} + p_k k + \tau z + \Pi(p_l, p_x, \tau, k)\\ &= f_l\bar{l} + \tau z + [y - f_l l - \sum_{i=1}^{I} c_i f_l x_i - \tau z]\\ &= f_l[l + \sum_{i=1}^{I} c_i f_l x_i] + \tau z + [y - f_l l - \sum_{i=1}^{I} c_i f_l x_i - \tau z] = y.\end{aligned} \tag{9.14}$$

Thus, the allocation $\langle y, a, k_1, l, x^1, x^2, z, \tilde{\tilde{y}}, \tilde{\tilde{l}}, \tilde{\tilde{x}}\rangle$ is a competitive equilibrium with a Pigouvian tax if prices and Pigou tax levels are fixed as in (5.13). ■

Proof of Theorem NE: Define the function $b : \mathcal{Z}^S \times \mathcal{G} \longrightarrow \mathcal{Z}^S \times \mathcal{G}$ with image $b(z^1, \ldots, z^S, g) = \prod_{s=1}^{S} \hat{z}(k^s, g) \times \sum_{s=1}^{S} z^s$. Since $\mathcal{Z}^S \times \mathcal{G}$ is a convex and compact set, Brouwer's fix point theorem implies that there is a fixed point $\overset{*}{\nu} = \langle\overset{*}{z}^1, \ldots, \overset{*}{z}^S, \overset{*}{g}\rangle$ of B. Clearly, $\overset{*}{\nu}$ is a non-cooperative Nash equilibrium emission outcome of the global economy. ■

Proof of Theorem N$k$$z$R: If $\hat{z}(\cdot,\ \overset{*}{g})$ is non-increasing in k, then $\overset{*}{z}^s \equiv \hat{z}(k^s, \overset{*}{g}) \geq \hat{z}((k^{s+1}, \overset{*}{g}) \equiv \overset{*}{z}^{s+1}$. Similarly, we can prove the non-decreasing case. This proves (i). (ii) follows from (i) when we note that $\hat{z}(\cdot,\ \overset{*}{g})$ is non-decreasing in the interval $(0, \overset{*}{k}]$ and non-increasing in the interval $[\overset{*}{k}, k^S]$. (iii) follows from (i) when we note that $\hat{z}(\cdot,\ \overset{*}{g})$ is non-decreasing in the interval $(0, \overset{*}{k}]$, and non-increasing in the interval $[\overset{*}{k}, \hat{k}]$ and non-decreasing in the interval $[\hat{k}, k^S]$. ■

Proof of Lemma NFCk: Production-wise feasible changes in local emission and the consumption good levels induced by a change in the level of the capital endowment are obtained by differentiating (5.5), (5.6), and (5.3) with respect to k under the maintained assumptions:[35]

$$\begin{aligned}\frac{\partial \hat{z}}{\partial k} &= \alpha\frac{\partial \hat{x}}{\partial k} - \theta\frac{\partial \hat{a}}{\partial k} \quad \text{and}\\ \frac{\partial \hat{y}}{\partial k} &= \hat{f}_k + [\hat{f}_x - c\hat{f}_l]\frac{\partial \hat{x}}{\partial k} - h_a\frac{\partial \hat{a}}{\partial k}.\end{aligned} \tag{9.15}$$

Differentiating (5.7) with respect to k under the maintained assumptions yields

$$\frac{h_{aa}}{\theta}\frac{\partial \hat{a}}{\partial k} = \frac{\hat{f}_{xx}}{\alpha}\frac{\partial \hat{x}}{\partial k} \quad \Longrightarrow \quad \frac{\partial \hat{x}}{\partial k} = \frac{\alpha}{\theta}\frac{h_{aa}}{\hat{f}_{xx}}\frac{\partial \hat{a}}{\partial k}. \tag{9.16}$$

35 Differentiating (5.6) with respect to k we obtain $\frac{\partial \hat{l}}{\partial k} = -c\frac{\partial \hat{x}}{\partial k}$. This is employed in deriving $\frac{\partial \hat{y}}{\partial k}$.

Differentiating (5.7) with respect to k and employing (9.15) yields:

$$\begin{aligned}
\frac{\partial r}{\partial y}\frac{\partial \hat{y}}{\partial k}+\frac{\partial r}{\partial z}\frac{\partial \hat{z}}{\partial k}=\frac{h_{aa}}{\theta}\frac{\partial \hat{a}}{\partial k} \quad &\Longrightarrow \quad \frac{\mathcal{A}}{u_y^2}\frac{\partial \hat{y}}{\partial k}+\frac{\mathcal{B}}{u_y^2}\frac{\partial \hat{z}}{\partial k}=\frac{h_{aa}}{\theta}\frac{\partial \hat{a}}{\partial k} \\
\Longrightarrow \quad \frac{\mathcal{A}}{u_y^2}\left[\hat{f}_k+[\hat{f}_x-c\hat{f}_l]\frac{\partial \hat{x}}{\partial k}-h_a\frac{\partial \hat{a}}{\partial k}\right]&+\frac{\mathcal{B}}{u_y^2}\left[\alpha\frac{\partial \hat{x}}{\partial k}-\theta\frac{\partial \hat{a}}{\partial k}\right]=\frac{h_{aa}}{\theta}\frac{\partial \hat{a}}{\partial k} \qquad (9.17)\\
\Longrightarrow \quad \frac{\mathcal{A}}{u_y^2}\hat{f}_k-\frac{\theta}{u_y^2}\frac{\partial \hat{a}}{\partial k}\left[\mathcal{A}\frac{h_a}{\theta}+\mathcal{B}\right]&+\frac{\alpha}{u_y^2}\frac{\partial \hat{x}}{\partial k}\left[\mathcal{A}\left[\frac{\hat{f}_x-c\hat{f}_l}{\alpha}\right]+\mathcal{B}\right]=\frac{h_{aa}}{\theta}\frac{\partial \hat{a}}{\partial k}.
\end{aligned}$$

Employing (5.7), conclusion (iv) of Lemma ΔMRS, and (9.16) in (9.17), we obtain:

$$\begin{aligned}
\frac{\mathcal{A}}{u_y^2}\hat{f}_k-\frac{\theta Q}{u_y^3}\frac{\partial \hat{a}}{\partial k}+\frac{\alpha Q}{u_y^3}\frac{\partial \hat{x}}{\partial k}&=\frac{h_{aa}}{\theta}\frac{\partial \hat{a}}{\partial k} \\
\Longrightarrow \quad \frac{\mathcal{A}}{u_y^2}\hat{f}_k-\frac{\theta Q}{u_y^3}\frac{\partial \hat{a}}{\partial k}+\frac{\alpha^2 Q}{\theta u_y^3}\frac{h_{aa}}{\hat{f}_{xx}}\frac{\partial \hat{a}}{\partial k}&=\frac{h_{aa}}{\theta}\frac{\partial \hat{a}}{\partial k} \qquad (9.18)\\
\Longrightarrow \quad \frac{\partial \hat{a}}{\partial k}&=\frac{-\mathcal{A}\hat{f}_k u_y \hat{f}_{xx}\theta}{Q\left[h_{aa}\alpha^2-\hat{f}_{xx}\theta^2\right]-h_{aa}\hat{f}_{xx}u_y^3}.
\end{aligned}$$

Thus, we obtain $\frac{\partial \hat{a}}{\partial k}$ as in conclusion (i) of (1) of the lemma. Substitute (9.18) into (9.15) and (9.16) to obtain $\frac{\partial \hat{x}}{\partial k}$, $\frac{\partial \hat{z}}{\partial k}$, and $\frac{\partial \hat{y}}{\partial k}$ as in conclusions (ii) to (iv) of (1) of the lemma.

Proceeding in a similar manner, after recalling that r is a function of y, z, and g and differentiating (5.7) with respect to g, we obtain

$$\begin{aligned}
\frac{\partial r}{\partial y}\frac{\partial \hat{y}}{\partial g}+\frac{\partial r}{\partial z}\frac{\partial \hat{z}}{\partial g}+\frac{\partial r}{\partial g}&=\frac{h_{aa}}{\theta}\frac{\partial \hat{a}}{\partial g} \\
\Longrightarrow \quad \frac{\mathcal{A}}{u_y^2}\left[(\hat{f}_x-c\hat{f}_l)\frac{\partial \hat{x}}{\partial g}-h_a\frac{\partial \hat{a}}{\partial g}\right]+\frac{\mathcal{B}}{u_y^2}\left[\alpha\frac{\partial \hat{x}}{\partial g}-\theta\frac{\partial \hat{a}}{\partial g}\right]+\frac{\partial r}{\partial g}&=\frac{h_{aa}}{\theta}\frac{\partial \hat{a}}{\partial g} \\
\Longrightarrow \quad -\frac{\theta Q}{u_y^3}\frac{\partial \hat{a}}{\partial g}+\frac{\alpha^2 Q}{\theta u_y^3}\frac{h_{aa}}{\hat{f}_{xx}}\frac{\partial \hat{a}}{\partial g}+\frac{\partial r}{\partial g}&=\frac{h_{aa}}{\theta}\frac{\partial \hat{a}}{\partial g} \qquad (9.19)\\
\Longrightarrow \quad \frac{\partial \hat{a}}{\partial g}=-\frac{\partial r}{\partial g}&\left[\frac{u_y^3\hat{f}_{xx}\theta}{Q\left[h_{aa}\alpha^2-\hat{f}_{xx}\theta^2\right]-h_{aa}\hat{f}_{xx}u_y^3}\right].
\end{aligned}$$

Thus, we obtain $\frac{\partial \hat{a}}{\partial g}$ as in conclusion (i) of (1) of the lemma. From this, $\frac{\partial \hat{x}}{\partial g}$, $\frac{\partial \hat{z}}{\partial g}$, and $\frac{\partial \hat{y}}{\partial g}$ in conclusions (ii) to (iv) of (1) of the lemma follow in a straightforward way.

Conclusions (2) and (3) of the lemma follow in a straightforward way from conclusion (1). $\frac{\partial \tau}{\partial k}$ and $\frac{\partial \tau}{\partial g}$ in conclusion (4) of the lemma follow from differentiating (5.14) with respect to k and g, respectively, in Theorem D2. Since $\tau=-\frac{u_z}{u_y}=-p_z$, the substitution effects of a change in capital in conclusion (5) of the lemma follow from (7.4) and part (iii) of Lemma ΔMRS. The income effects of a change in capital in conclusion (5) of the lemma follow from part (ii) of Lemma ΔMRS, (7.5), and the fact that

$$p_z \frac{\partial \hat{z}}{\partial k} + \frac{\partial \hat{y}}{\partial k} = p_z \left[\alpha \frac{\partial \hat{x}}{\partial k} + \theta \frac{\partial \hat{a}}{\partial k} \right] + \hat{f}_k + [\hat{f}_x - c\hat{f}_l] \frac{\partial \hat{x}}{\partial k} - h_a \frac{\partial \hat{a}}{\partial k}. \tag{9.20}$$

Noting that $p_z = \frac{u_z}{u_y} = -\frac{h_a}{\theta} = -\frac{[\hat{f}_x - c\hat{f}_l]}{\alpha}$, on simplification, (9.20) yields $p_z \frac{\partial \hat{z}}{\partial k} + \frac{\partial \hat{y}}{\partial k} = \hat{f}_k$. ■

Proof of Lemma FIk:

(1) Under the maintained assumptions, (5.7) implies

$$-\frac{u_z}{u_y} = \frac{h_a}{\theta} = \frac{\hat{f}_1 - \hat{f}_2 - \hat{f}_l \phi_1 c}{\phi_1 \alpha}. \tag{9.21}$$

From this it follows that

$$\begin{aligned} \frac{h_{aa}}{\theta} \frac{\partial \hat{a}}{\partial k} &= \frac{1}{\alpha \phi_1} \left[\frac{\partial \hat{k}_1}{\partial k} [\hat{f}_{11} + \hat{f}_{22}] - \hat{f}_{22} \right] \\ \implies \frac{\partial \hat{k}_1}{\partial k} &= \left[\alpha \phi_1 \frac{h_{aa}}{\theta} \frac{\partial \hat{a}}{\partial k} + \hat{f}_{22} \right] \frac{1}{\hat{f}_{11} + \hat{f}_{22}}. \end{aligned} \tag{9.22}$$

Under the maintained assumptions, differentiating the FOCs (5.3) to (5.6), we obtain

$$\frac{\partial \hat{z}}{\partial k} = \alpha \phi_1 \frac{\partial \hat{k}_1}{\partial k} - \theta \frac{\partial \hat{a}}{\partial k} \quad \text{and} \quad \frac{\partial \hat{y}}{\partial k} = \left[\hat{f}_1 - \hat{f}_2 - \hat{f}_l \phi_1 c \right] \frac{\partial \hat{k}_1}{\partial k} + \hat{f}_2 - h_a \frac{\partial \hat{a}}{\partial k}. \tag{9.23}$$

(9.21), the definitions of $\mathcal{A}$ and $\mathcal{B}$, and (9.23) imply

$$\begin{aligned} &\frac{\partial r}{\partial y} \frac{\partial \hat{y}}{\partial k} + \frac{\partial r}{\partial z} \frac{\partial \hat{z}}{\partial k} = \frac{h_{aa}}{\theta} \frac{\partial \hat{a}}{\partial k} \implies \frac{\mathcal{A}}{u_y^2} \frac{\partial \hat{y}}{\partial k} + \frac{\mathcal{B}}{u_y^2} \frac{\partial \hat{z}}{\partial k} = \frac{h_{aa}}{\theta} \frac{\partial \hat{a}}{\partial k} \\ &\implies \frac{\mathcal{A}}{u_y^2} \left[[\hat{f}_1 - \hat{f}_2 - \hat{f}_l \phi_1 c] \frac{\partial \hat{k}_1}{\partial k} + \hat{f}_2 - h_a \frac{\partial \hat{a}}{\partial k} \right] + \frac{\mathcal{B}}{u_y^2} \left[\alpha \phi_1 \frac{\partial \hat{k}_1}{\partial k} - \theta \frac{\partial \hat{a}}{\partial k} \right] = \frac{h_{aa}}{\theta} \frac{\partial \hat{a}}{\partial k} \\ &\implies \frac{\alpha \phi_1}{u_y^2} \left[\mathcal{A} \frac{[\hat{f}_1 - \hat{f}_2 - \hat{f}_l \phi_1 c]}{\alpha \phi_1} + \mathcal{B} \right] \frac{\partial \hat{k}_1}{\partial k} - \frac{\theta}{u_y^2} \left[\mathcal{A} \frac{h_a}{\theta} + \mathcal{B} \right] \frac{\partial \hat{a}}{\partial k} + \frac{\mathcal{A}}{u_y^2} \hat{f}_2 = \frac{h_{aa}}{\theta} \frac{\partial \hat{a}}{\partial k}. \end{aligned} \tag{9.24}$$

Employing (9.21) and Part (iv) of Lemma ΔMRS, (9.24) implies

$$\frac{\alpha \phi_1 Q}{u_y^3} \frac{\partial \hat{k}_1}{\partial k} - \frac{\theta Q}{u_y^3} \frac{\partial \hat{a}}{\partial k} + \frac{\mathcal{A}}{u_y^2} f_2 = \frac{h_{aa}}{\theta} \frac{\partial \hat{a}}{\partial k}. \tag{9.25}$$

Substituting for $\frac{\partial \hat{k}_1}{\partial k}$ in (9.25) from (9.22) and solving for $\frac{\partial \hat{a}}{\partial k}$, we obtain (i) of (1) of the lemma. (ii) of (1) follows from substituting for $\frac{\partial \hat{a}}{\partial k}$ in (9.22). (iii) of (1) follows from (ii) of

(1). (iv) and (v) of (1) follow from substituting for $\frac{\partial \hat{a}}{\partial k}$ and $\frac{\partial \hat{k}_1}{\partial k}$ in the two expressions in (9.23) and noting from Part (iv) of Lemma ΔMRS that $u_y\mathcal{B} = Q + u_z\mathcal{A}$.

(2) $\frac{\partial \tau}{\partial k}$ follows from differentiating (5.14) in Theorem D2. The substitution effects and income effects of changing capital on emission and the consumption good follow from Lemma ΔMRS, (7.4), and (7.5), when we note that $\frac{\partial p_z}{\partial k} = -\frac{\partial \tau}{\partial k}$ and $p_z\frac{\partial \hat{z}}{\partial k} + \frac{\partial \hat{y}}{\partial k} = \hat{f}_2$. ■

Proof of Theorem IFS: The FOCs (9.10), (9.12), and (9.13) we obtain

$$\mu\psi_1 - \lambda\hat{f}_l c_i - \delta\alpha_i = 0, \ \forall\ i = 1, 2. \tag{9.26}$$

For $i = 1$, we obtain

$$\mu = \frac{\lambda\hat{f}_l c_1 + \delta\alpha_1}{\psi_1} \implies -\hat{f}_l c_1 = -\psi_1\frac{\mu}{\lambda} + \frac{\delta}{\lambda}\alpha_1 = -\psi_1\frac{\mu}{\lambda} + \frac{-u_z}{u_y}\alpha_1. \tag{9.27}$$

Plug into $i = 2$ of (9.26), we obtain

$$\left[\frac{\lambda\hat{f}_l c_1 + \delta\alpha_1}{\psi_1}\right]\psi_2 - \lambda\hat{f}_l c_2 = \delta\alpha_2. \tag{9.28}$$

From (9.28), (9.7), and (9.8) we obtain

$$\hat{f}_l\left[\frac{c_1\psi_2 - c_2\psi_1}{\alpha_2\psi_1 - \alpha_1\psi_2}\right] = \frac{\delta}{\lambda} = \frac{-u_z}{u_y}. \tag{9.29}$$

From (5.5) we obtain

$$\frac{\partial z}{\partial k} = \alpha_1\frac{\partial \hat{x}_1}{\partial k} + \alpha_2\frac{\partial \hat{x}_2}{\partial k}. \tag{9.30}$$

From (5.6) we obtain $\frac{\partial l}{\partial k} = -c_1\frac{\partial \hat{x}_1}{\partial k} - c_2\frac{\partial \hat{x}_2}{\partial k}$. Combining this with (5.3) we obtain

$$\frac{\partial \hat{y}}{\partial k} = \hat{f}_k - \hat{f}_l\left[c_1\frac{\partial \hat{x}_1}{\partial k} + c_2\frac{\partial \hat{x}_2}{\partial k}\right]. \tag{9.31}$$

Differentiating (5.4) with respect to k we obtain

$$\frac{\partial \hat{x}_1}{\partial k} = \left[\phi_k - \psi_2\frac{\partial \hat{x}_2}{\partial k}\right]\frac{1}{\psi_1}. \tag{9.32}$$

Combining this with (9.30) and (9.29) we obtain:

$$\begin{aligned}\frac{\partial \hat{z}}{\partial k} &= \frac{\alpha_1\phi_k}{\psi_1} + \frac{\partial \hat{x}_2}{\partial k}\left[\frac{\alpha_2\psi_1 - \alpha_1\psi_2}{\psi_1}\right] \\ \frac{\partial \hat{y}}{\partial k} &= \hat{f}_k - \frac{\hat{f}_l c_1\phi_k}{\psi_1} + [c_1\psi_2 - c_2\psi_1]\,\hat{f}_l\frac{\partial \hat{x}_2}{\partial k}.\end{aligned} \tag{9.33}$$

From (9.29) and (9.33) we have

$$
\begin{aligned}
&\frac{\partial \frac{-u_z}{u_y}}{\partial y}\frac{\partial \hat{y}}{\partial k} + \frac{\partial \frac{-u_z}{u_y}}{\partial z}\frac{\partial \hat{z}}{\partial k} = 0 \\
\Longrightarrow\ &\frac{\mathcal{A}}{u_y^2}\frac{\partial \hat{y}}{\partial k} + \frac{\mathcal{B}}{u_y^2}\frac{\partial \hat{z}}{\partial k} = 0 \\
\Longrightarrow\ &\frac{\mathcal{A}}{u_y^2}\left[\hat{f}_k - \frac{\hat{f}_l c_1 \phi_k}{\psi_1}\right] + \frac{\mathcal{B}}{u_y^2}\frac{\phi_k \alpha_1}{\psi_1} + \frac{\partial \hat{x}_2}{\partial k}\frac{[\alpha_2\psi_1 - \alpha_1\psi_2]}{\psi_1 u_y^2}\left[\mathcal{A}\hat{f}_l\left[\frac{c_1\psi_2 - c_2\psi_1}{\alpha_2\psi_1 - \alpha_1\psi_2}\right] + \mathcal{B}\right] = 0 \\
\Longrightarrow\ &\mathcal{A}\left[\hat{f}_k - \frac{\hat{f}_l c_1 \phi_k}{\psi_1}\right] + \mathcal{B}\frac{\phi_k \alpha_1}{\psi_1} = -\frac{\partial \hat{x}_2}{\partial k}\left[\frac{\alpha_2\psi_1 - \alpha_1\psi_2}{\psi_1 u_y}\right] Q.
\end{aligned}
\tag{9.34}
$$

Now employing (9.27) and substituting out for $-\hat{f}_l c_1$ we obtain, after some simplification,

$$
\frac{\partial \hat{x}_2}{\partial k} = -\frac{1}{[\alpha_2\psi_1 - \alpha_1\psi_2]\,Q}\left[u_y \mathcal{A}\psi_1\left[\hat{f}_k - \phi_k\frac{\mu}{\lambda}\right] + \phi_k\alpha_1 Q\right]. \tag{9.35}
$$

Using (9.32) we obtain

$$
\frac{\partial \hat{x}_1}{\partial k} = \frac{1}{[\alpha_2\psi_1 - \alpha_1\psi_2]\,Q}\left[u_y \mathcal{A}\psi_2\left[\hat{f}_k - \phi_k\frac{\mu}{\lambda}\right] + \phi_k\alpha_2 Q\right]. \tag{9.36}
$$

Thus, we obtain (i) and (ii) of this theorem. Using (9.33), (9.35), and (9.36) and simplifying we obtain conclusions (iii) and (iv) of this theorem. (v) and (vi) follow in an obvious manner from (iii). (ix) follows in an obvious manner from (v).

Under the maintained assumption $\frac{c_1}{c_2} < \frac{\psi_1}{\psi_2} < \frac{\alpha_1}{\alpha_2}$, we have $\alpha_2\psi_1 - \alpha_1\psi_2 < 0$. Hence, (vii) follows from (i) and (ii) in an obvious manner. To prove (viii) of the theorem, let $\langle y, z\rangle \in \mathcal{N}(g)$. Suppose $\frac{\partial \hat{x}_2}{\partial k} < 0$. Then (ii) implies that $\phi_k\frac{\alpha_1}{\psi_1}Q < |u_y\mathcal{A}[\hat{f}_k - \phi_k\frac{\mu}{\lambda}]|$. But since $\frac{\alpha_1}{\psi_1} > \frac{\alpha_2}{\psi_2}$, we have $\phi_k\frac{\alpha_2}{\psi_2}Q < |u_y\mathcal{A}[\hat{f}_k - \phi_k\frac{\mu}{\lambda}]|$. Hence, (i) implies $\frac{\partial \hat{x}_1}{\partial k} > 0$. Suppose $\frac{\partial \hat{x}_1}{\partial k} > 0$. Then $\phi_k\frac{\alpha_1}{\psi_1}Q > |u_y\mathcal{A}[\hat{f}_k - \phi_k\frac{\mu}{\lambda}]|$. This implies $\phi_k\frac{\alpha_2}{\psi_2}Q > |u_y\mathcal{A}[\hat{f}_k - \phi_k\frac{\mu}{\lambda}]|$. Hence, from (ii), we have $\frac{\partial \hat{x}_2}{\partial k} > 0$. Hence, $\frac{\partial \hat{x}_2}{\partial k} > 0 \iff \frac{\partial \hat{x}_1}{\partial k} > 0$. But since, $\frac{\partial \hat{z}}{\partial k} > 0$ whenever $\langle y, z\rangle \in \mathcal{N}(g)$, it cannot be the case that $\frac{\partial \hat{x}_2}{\partial k} < 0$ and $\frac{\partial \hat{x}_1}{\partial k} < 0$. Hence, $\frac{\partial \hat{x}_2}{\partial k} > 0$ and $\frac{\partial \hat{x}_1}{\partial k} > 0$. ■

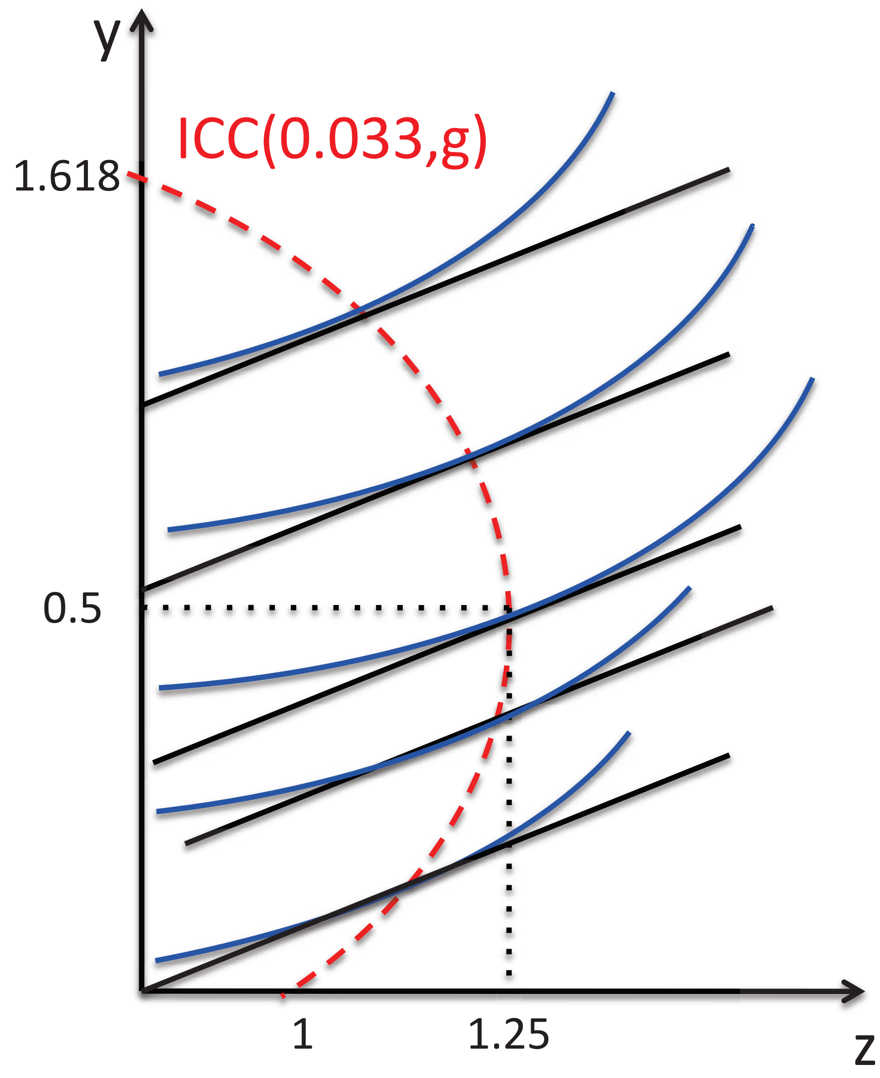
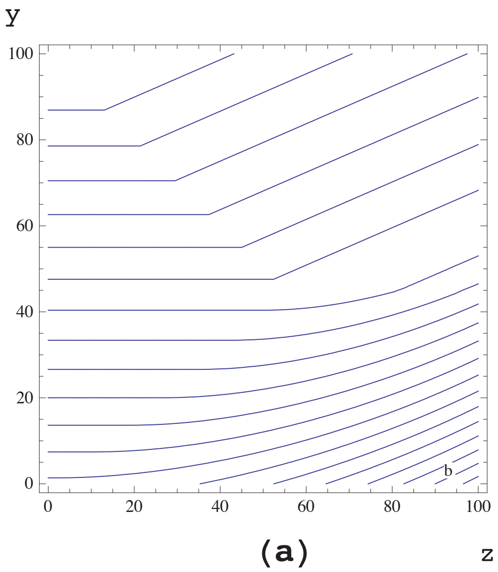
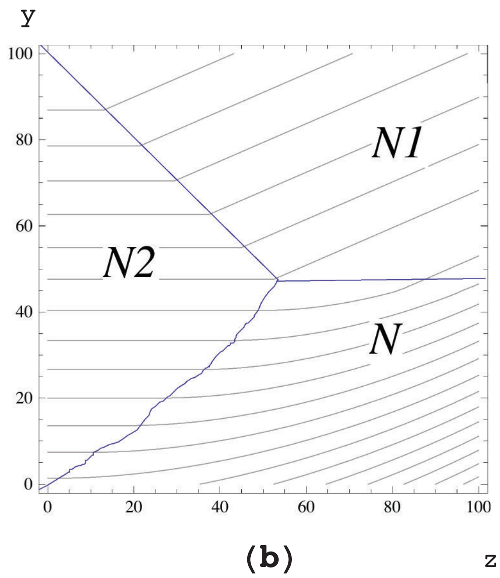


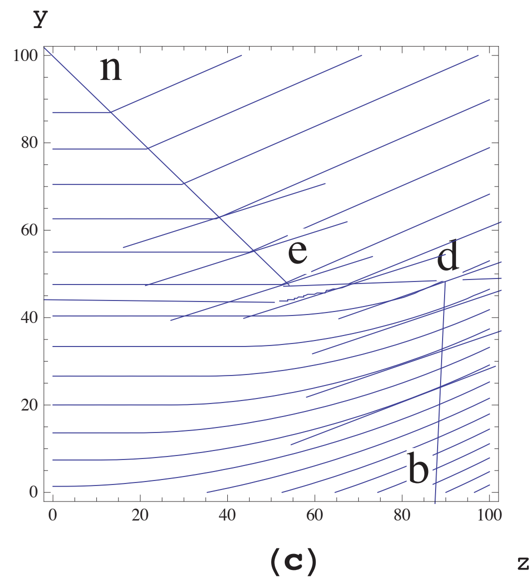
Figure 1


y
100
80
60
40
20
0
b
0
20
40
60
80
100
z

(a)


y
100
80
60
40
20
0
N1
N2
N
0
20
40
60
80
100
z

(b)


y
100
80
60
40
20
0
n
e
d
b
0
20
40
60
80
100
z

(c)

Figure 2

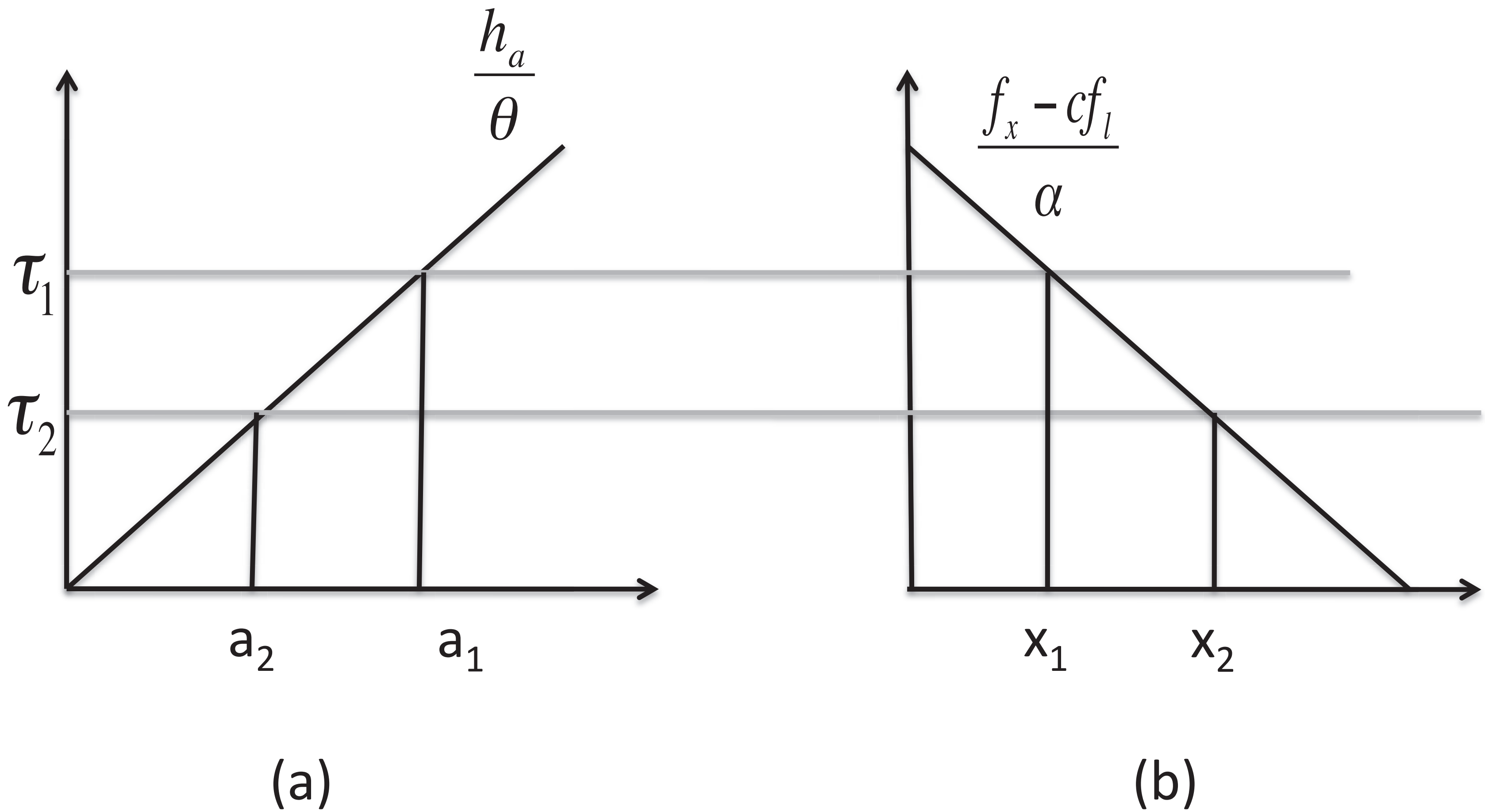


Figure 3

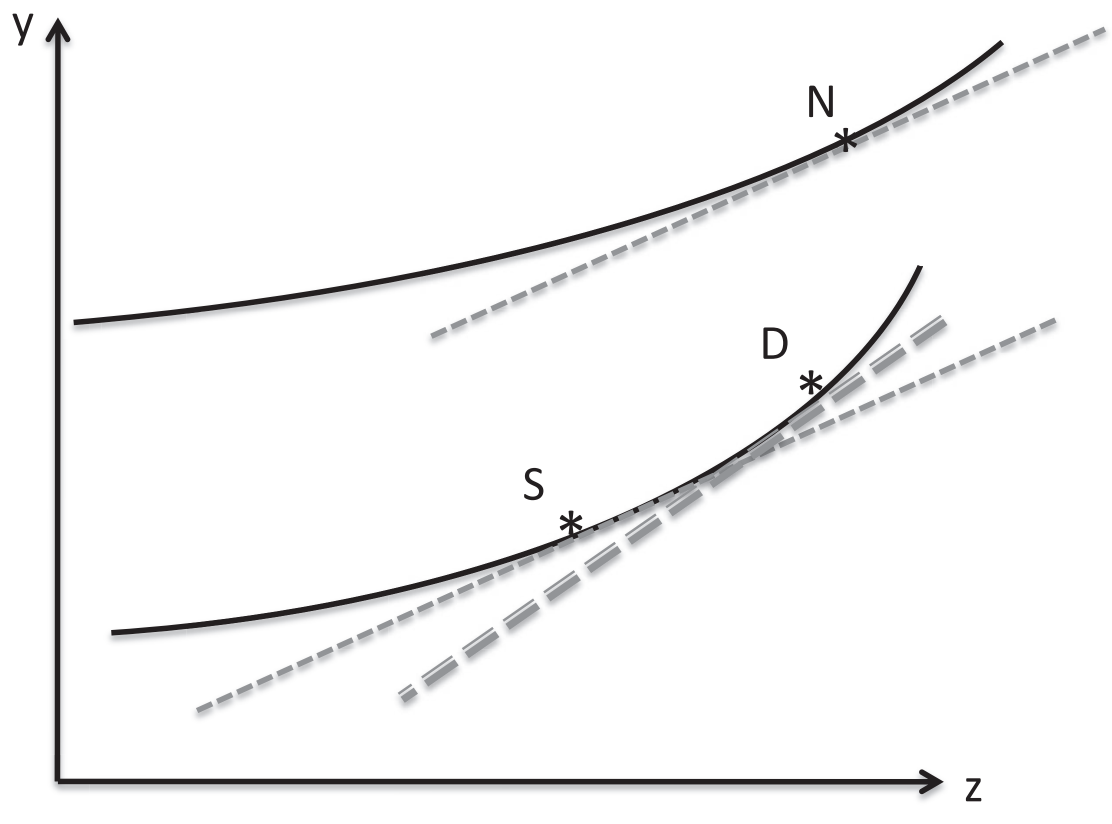
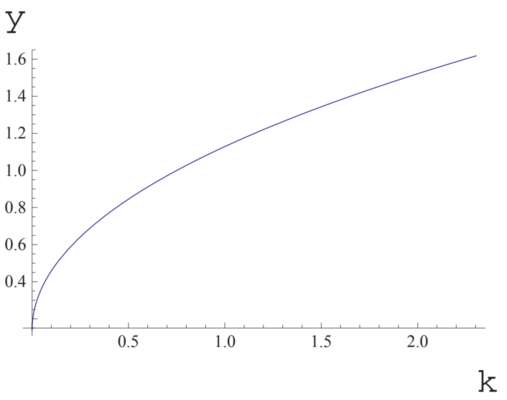
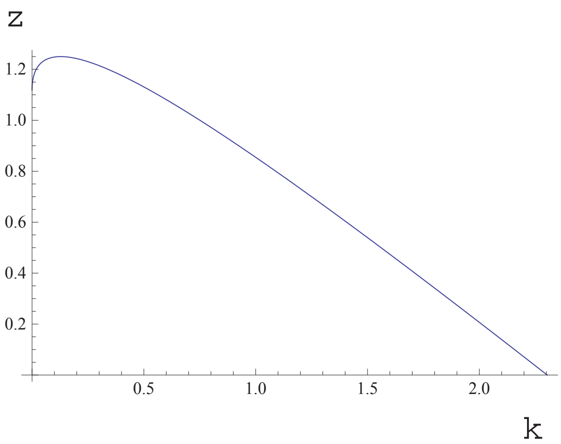


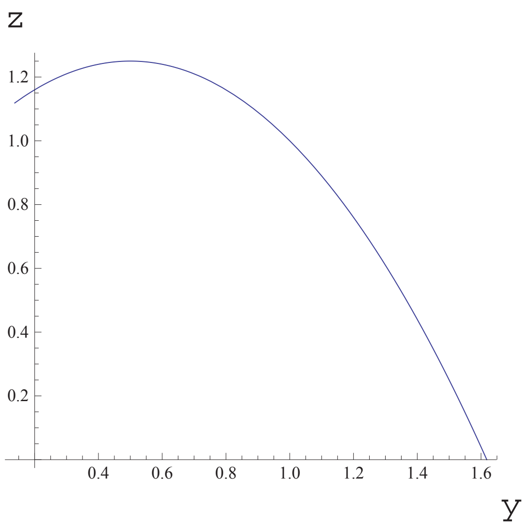
Figure 4


y
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.5
1.0
1.5
2.0
k

(a)

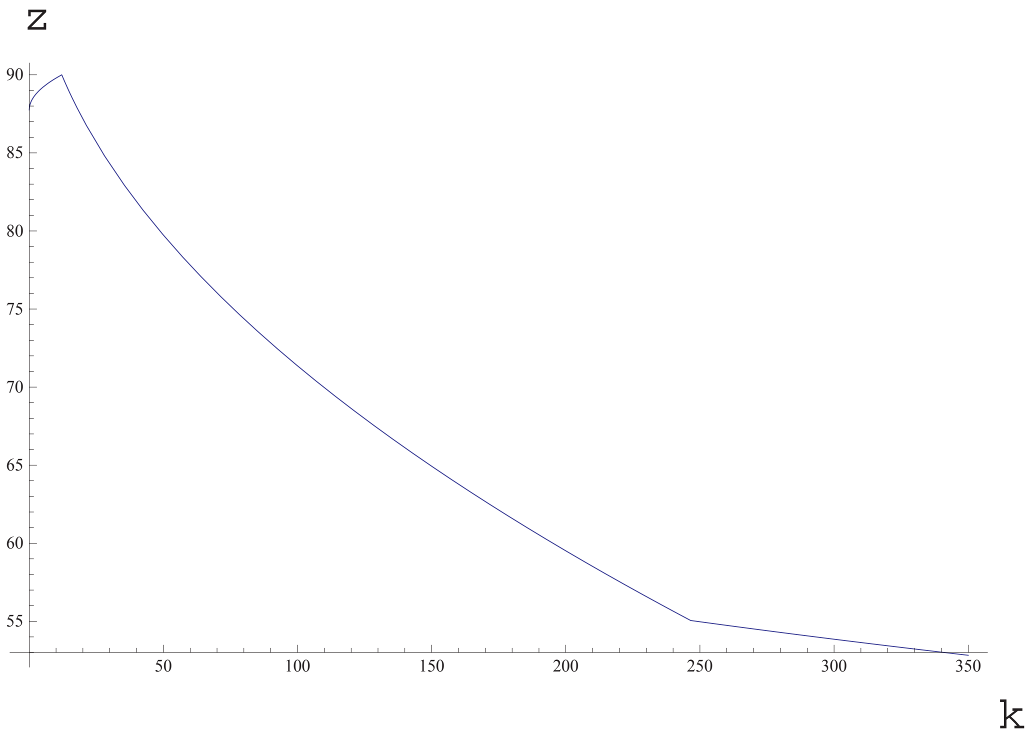

z
1.2
1.0
0.8
0.6
0.4
0.2
0.5
1.0
1.5
2.0
k

(b)

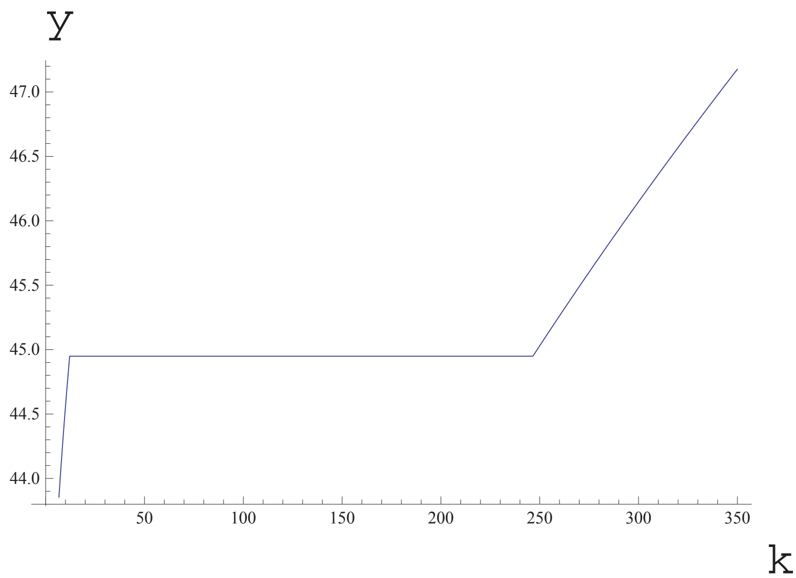

z
1.2
1.0
0.8
0.6
0.4
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
y

(c)

Figure 5


z
90
85
80
75
70
65
60
55
50
100
150
200
250
300
350
k

(a)


y
47.0
46.5
46.0
45.5
45.0
44.5
44.0
50
100
150
200
250
300
350
k

(b)

Figure 6

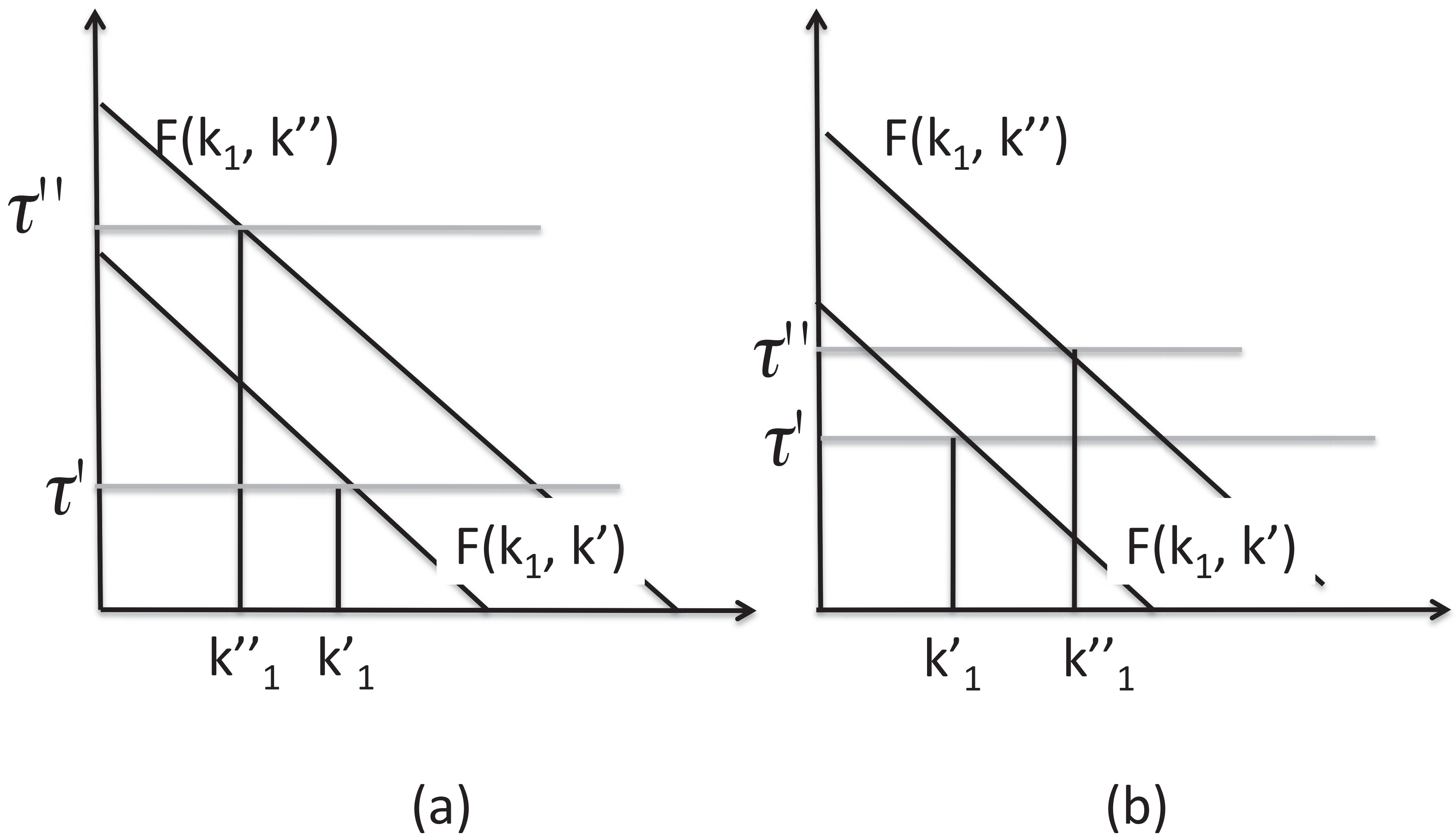


Figure 7

REFERENCES

Andreoni, J. and A. Levinson, “The simple analytics of the environmental Kuznets curve,” *Journal of public economics*, 80: 269–286, 2001.

Chambers, R. G. and T. Melkonyan, “Production technologies, material balance, and the income-environmental quality trade-off,” University of Exeter Business School – Economics discussion paper series: 2012.

Copeland, B. R. and M. S. Taylor, *Trade and the environment: theory and evidence*, Princeton university press, Princeton, New Jersey, 2003.

Doi, J., K. Iwasa, and K. Shimomura, “Giffen behaviour independent of the wealth level,” *Economic theory*, 41: 247–267, 2009.

Grossman, G. M. and A. B. Krueger, “Environmental impacts of a North-American free trade agreement,” *The U. S. - Mexico free trade agreement*, P. Garber, ed., The MIT press, Cambridge Massachusetts, 1993.

Grossman, G. M. and A. B. Krueger, “Economic growth and the environment,” *Quarterly journal of economics*, 110: 353–377, 1995.

Guenerie R., “The design of post-Kyoto climate schemes: an introductory analytical assessment,” *The Design of Climate Policy*, R. Guesnerie and H. Tulkens (eds.), CESifo seminar series, The MIT press, Cambridge Massachusetts, 2008.

Holz-Eakin, D. and T. M. Selden, “Stoking the fires? CO_2 emissions and economic growth,” *Journal of public economics*, 57: 85–101, 1995.

Israel, D. and A. Levinson, “ Willingness to pay for environmental quality: testable implications of the growth and environment literature,” *Contributions to economic analysis and policy*, 3: article 2, 2004.

John, A. and R. Pecchenino, “An overlapping generations model of growth and the environment,” *Economic journal*, 104: 1393–1410, 1994.

Jones, L. E. and R. E. Manuelli, "Endogenous policy choice: the case of pollution and growth," *Review of economic dynamics*, 4: 369–405, 1994.

Mason R. and T. Swanson, "A Kuznets curve analysis of ozone-depleting substances and the Montreal protocol," *Oxford economic papers*, 55: 1–24, 2003.

Murty, S., "Externalities and fundamental nonconvexities: a reconciliation of approaches to general equilibrium externality modeling and implications for decentralization," *Journal of economic theory*, 145: 331-53, 2010.

Murty, S., "The theory of by-production of emissions and capital-constrained non-cooperative Nash outcomes of a global economy," University of Exeter Business School – Economics discussion paper series: 11/10, 2011.

Murty, S., R. R. Russell, and S. B. Levkoff, "On modeling pollution-generating technologies," *Journal of environmental economics and management*, 64: 117-135, 2012.

Murty, S., "On the theory of by-production of emissions," A draft, 2012.

Selden, T. M. and D. Song, "Environmental quality and development: is there a Kuznets curve for air pollution emissions?" *Journal of environmental economics and management*, 27: 147–162, 1994.

Starrett, D., "Fundamental nonconvexities in the theory of externalities," *Journal of economic theory*, 4: 180–199, 1972.

Stokey, N. L., "Are there limits to growth?" *International economic review*, 39: 1–31, 1998.

Weitzman M. L., "Prices vs. quantities," *Review of economic studies*, 41: 477-491, 1974.