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Abstract

Distinguishing between intended ("good") production and unintended or residual ("bad") generation, we introduce the concept of by-production. In by-production technologies, pollution is an output that satisfies a "costly disposability" assumption and violates standard free disposability with respect to pollution-causing inputs. Our approach therefore differs substantially from standard approaches to modeling pollution-generating technologies. We show how by-production can be modeled using data envelopment analysis (DEA) methods. With an electric power plant database, we illustrate shortcomings under by-production of two popular efficiency indexes: the hyperbolic index and the directional distance function. We propose and implement an alternative efficiency index with superior properties.

Journal of Economic Literature Classification Number: D20, D24, D62, Q50

Keywords: pollution-generating technologies, free disposability, weak disposability, data envelopment analysis, environmental and technical efficiency measurement.

On modeling pollution-generating technologies.

1. Introduction.

Our reading of the environmental economics literature reveals three broad features of pollution that economists aim to capture. First, the generation of pollution/residuals seems to proceed hand-in-hand with the processes of consumption and production.[1] Second, the residuals so generated require the use of the assimilative capacity of the environment for their disposal. Third, the generation of the residuals and the consequent use of environmental resources for their disposal generate external effects on both consumers and producers and hence the need for policies to regulate the generation of pollution.

In this paper, we confine ourselves to addressing the first feature alone.[2] In particular, we focus on pollution generated by firms. We distinguish between outputs that firms intend to produce and outputs that unintentionally (incidentally) get generated by firms when they engage in the production of intended outputs. Pollution is such an unintended output. We are mainly concerned with studying the specification of technology sets that best captures the link between production of outputs intended by firms and the generation of pollution.

It is reasonable to say that, in the case of pollution generated by firms, there are some specific aspects about the process of transformation of inputs into intended outputs (*e.g.,* the use of certain inputs such as coal or the production of certain outputs such as varieties of cheese that release a strong odor) that trigger additional reactions in nature and (abstracting from abatement activities) *inevitably* result in the generation of pollution as a by-product. In this paper, we refer to these natural reactions, which occur alongside intended production by firms, as *by-production*[3] of pollution.

In the case of technologies exhibiting by-production, we observe an inevitability of a certain *minimal* amount of the incidental output (the by-product), given the quantities of certain inputs and/or certain intended outputs. Inefficiencies in production could generate more than this minimal amount of the unintended output. At the same time, in such technologies, we also observe the usual menu of *maximal* possible vectors of intended outputs, given an input vector. Such a menu generally reflects the negative tradeoffs in the production of intended outputs when inputs are held fixed, as production of each of these commodities is costly in terms of the inputs used. Inefficiencies in intended production may imply that less

[1] See, especially, Ayres and Kneese [1969] and Førsund [2009].
[2] See Murty [2010a] for a general equilibrium study of the second feature in the light of the first feature.
[3] A word that is not in the dictionary, but perhaps should be.

than this maximal amount may get produced. An increase in the amounts of the inputs used increases the menu of intended output vectors that are technologically feasible. At the same time, it increases the minimal amount of the unintended output that can be generated.[4]

The above underscores two crucial points to note about pollution-generating technologies:

(i) technologies of pollution-generating firms do not satisfy free disposability of by-products such as pollution (pollution cannot be disposed of below the minimal level described above if inputs and intended outputs are held fixed) and

(ii) in such technologies there is a mutual interdependence between changes in inputs, intended outputs, and pollution—an interdependence that we will argue is more correlation than causation.

In most of the existing literature, the standard building block employed in constructing pollution-generating technologies is the positive correlation between intended and unintended outputs that is usually observed in such technologies. This literature attributes this observed positive correlation to abatement activities by firms rather than directly to the phenomenon of by-production. Abatement activities of firms involve a diversion of resources (inputs) to mitigate or clean up the pollution they produce. In this paper, we model abatement activities as outputs of the firm. Examples are end-of-pipe treatment plants (that treat and clean water to remove the pollutant) and production of outputs like scrubbers (which reduce sulphur emissions).[5] The production of these abatement activities is hence costly, given fixed amounts of resources: the more resources are diverted to abatement activities, the less they are available for producing intended outputs. Hence, an increase in the level of abatement activities leads concomitantly to both lower residual generation and lower production of intended output.

In this literature, however, abatement activities are not explicitly modeled as another set of outputs produced by firms.[6] Rather, what is proposed is a "reduced form" of the technology in the space of inputs, by-products, and intended outputs. Special assumptions are made to allow the technology to exhibit a positive correlation between by-products and intended outputs, which is implicitly explained by abatement options open to firms. At the same time, it is also assumed that the technology satisfies the standard disposability

[4] *E.g.*, a greater amount of usage of coal increases the quantity of both smoke and electricity generated.

[5] We abstract from long-run abatement options of development, purchase, and installation of new technologies that generate less pollution. See *e.g.,* Barbera and McConnell [1998], where abatement activities include both a purchase of abatement capital and a diversion of some amounts of the usual inputs of a firm towards running of the abatement capital.

[6] For an exception, see Barbera and McConnell [1998].

assumptions with respect to *all* inputs and intended outputs. The approaches taken in the literature to model the positive correlation include: (a) treating pollution as a standard input (technology satisfies input free disposability with respect to pollution),[7] or (b) treating pollution as an output but with the technology satisfying the assumptions of weak disposability and null-jointness with respect to intended and unintended outputs.[8] In empirical works, both parametric and non-parametric specifications of such technologies are often employed for measuring technical efficiency, marginal abatement cost, productivity, and growth when economic units also produce incidental outputs like pollution. Both Data Envelopment Analysis (DEA)[9] and econometric approaches are employed in this literature.[10]

We propose a model of pollution-generating technologies that captures the salient features (i) and (ii) of the phenomenon of by-production identified above. Our model of technology, also called a by-production technology, is obtained as a composition of two technologies: an intended-production technology and a residual-generation technology. The former is a standard technology that describes how inputs are transformed into intended outputs in production. The latter reflects nature's residual generation mechanism, which is a relationship between pollution (an output) and commodities that cause pollution. Thus, if we assume that it is some inputs (*e.g.,* coal) that cause pollution, then an increase in the use of these inputs results (under standard assumptions) in an increase in intended outputs (say electricity). At the same time, such an increase in the use of these inputs causes also an increase in pollution via nature's residual generating technology. Thus, even without any reference to explicit abatement efforts by firms, the model generates a positive correlation between pollution generation and intended outputs.

[7] See, *e.g.,* Baumol and Oates [1988], Cropper and Oates [1992], Reinhard, Lovell, and Thijhssen [1999], and Reinhard, Lovell, and Geert [2000].

[8] A technology satisfies weak disposability of intended and unintended outputs if the latter can be disposed off only in strict tandem with the disposition of the former, and it satisfies null jointness if zero pollution implies all intended output quantities are zero as well. See Section 4 for formal definitions of these concepts.

[9] DEA is a mathematical programming approach to the construction of data-based technologies and the concomitant calculation of technological efficiency of individual firms (or other organizations). See Färe, Grosskopf, and Lovell [1994] for a basic description of DEA and Fried, Lovell, and Schmidt [2008] for surveys of more recent developments.

[10] For measurement issues based on parametric specifications of a technology that treats by-products as outputs and employs weak disposability and null jointness see, *e.g.,* Pittman [1983], Färe, Grosskopf, Noh, and Yaisawarng [1993], Coggins and Swinton [1994], Hailu and Veeman [1999], Murty and Kumar [2002, 2003], and Murty, Kumar, and Paul [2006]. For non-parametric set-theoretic approaches under similar assumptions on the technology see, *e.g.,* Färe, Grosskopf, and Pasurka [1986], Färe, Grosskopf, Lovell, and Pasurka [1989], Färe, Grosskopf, Noh, and Weber [2005], and Boyd and McClelland [1999]. See Zhou and Poh [2008] for a comprehensive survey of over a hundred papers employing this approach to the modeling of pollution-generating technologies.

We show that abatement options available to firms can also be explicitly factored into our model. When they are available, they form a part of both the intended production technology (as their production is also costly in terms of resources/inputs of the firm) and the residual generation mechanism (as they mitigate residual generation). Moreover, we show that the presence of abatement options implies that data generated by pollution-generating technologies can violate the null-jointness assumption that is often made in the literature, *i.e.,* positive levels of intended output may be consistent with zero levels of pollution. The weak-disposability restriction on pollution-generating technologies does not preclude regions of *negative* correlation between intended and unintended outputs.[11] On the other hand, in the by-production technology we formulate, no such regions of negative correlations will be observed.

The intended production technology satisfies standard free-disposability properties with respect to inputs and intended outputs and is assumed to be independent of the level of pollution.[12] As in Murty [2010a,b], the nature's residual generating technology treats pollution as an output that satisfies a new assumption of "costly disposability" and violates standard disposability properties with respect to goods that result in (affect) pollution generation. As a result, the by-production technology, which is an intersection of the intended production technology and nature's residual generating technology, violates standard disposability with respect to goods that cause (affect) pollution generation and exhibits costly disposability with respect to pollution. In these ways, our proposed by-production approach, is different from the standard input and output approaches to modeling pollution-generating technologies.

We show how our by-production technology can be constructed using DEA methods as the intersection of two DEA technologies, one for intended production and one for residual generation, and discuss the calculation of efficiency of individual firms using these methods. With the help of a simple example we show that the sets of (weakly) efficient points obtained from the weak-disposability approach usually employed in the DEA literature and the new by-production approach are generally different (the former will be a larger set of points than the latter). In the context of by-production, the conventional (in)efficiency indexes decompose nicely into an intended-output efficiency index and an environmental efficiency index. We use our example to show that the common indexes employed in this literature, the hyperbolic index and the directional distance function-based index, are seriously flawed when

[11] A fact already noted in the literature cited in Footnote 10 above.

[12] See Murty [2010b] for a generalization where pollution can affect intended production, *e.g.,* by deteriorating the quality of the labor input.

the technology satisfies by-production. In particular, standard indexes tend to overstate efficiency. We then propose an alternative index, a modification of an index proposed by Färe, Grosskopf, and Lovell [1994], for measurement of efficiency for by-production technologies. This index corrects for the flaws in the hyperbolic and directional distance function indexes. A comparison of the values of this index with those of the hyperbolic and directional distance indexes, using a data base for electric power firms, confirms our arguments about the inadequacies of the latter.

In Section 2, we show that a single implicit relation between outputs and inputs is not rich enough to capture, simultaneously, all the trade-offs between commodities that are implied by the phenomenon of by-production. In Section 3, we propose a model of a pollution-generating technology based on multiple production relations in which these inconsistencies in trade-offs are resolved. This is true regardless of whether or not abatement options are open to firms. Multiple production relations are required to distinguish between intended production by firms and nature's residual generation mechanism. In Section 4, we use a numerical example to show how by-production technologies can be constructed by DEA methods. Section 5 discusses issues related to efficiency measurement under by-production. In Section 6, we carry out an empirical analysis of efficiency measurement using an empirical data base. In Section 7, we extend our DEA formulation of a by-production technology to incorporate abatement efforts of firms. We conclude with Section 8.

2. Single-equation representation of pollution-generating technologies.

We show that a single implicit relation between outputs and inputs is not rich enough to capture, simultaneously, all the trade-offs between commodities that are implied by the phenomenon of by-production.

2.1. The case without abatement output.

The vectors of input quantities (indexed by $i = 1, \ldots, n$), intended-output quantities (indexed by $j = 1, \ldots, m$), and incidental-output quantities (indexed by $k = 1, \ldots, m'$), are given, respectively, by $y \in \mathbf{R}^m_+$, $z \in \mathbf{R}^{m'}_+$, and $x \in \mathbf{R}^n_+$.

Suppose pollution is caused by the use of certain inputs like coal or because of the production of certain intended outputs like cheese. Suppose also that the firm does not participate in any abatement activity to reduce the pollution that it generates. A single-equation formulation of such a pollution-generating technology, an extension of the standard functional representation of a multiple-output technology, is as follows:

$$T = \left\{ \langle x, y, z \rangle \in \mathbf{R}_+^{n+m+m'} \middle| \; f(x, y, z) \leq 0 \right\},$$

where f is differentiable, with derivatives with respect to inputs and intended outputs given by[13]

$$\begin{aligned} &\text{(a)} \quad f_i(x, y, z) \leq 0, \; i = 1, \ldots, n, \\ &\text{(b)} \quad f_j(x, y, z) \geq 0, \; j = 1, \ldots, m. \end{aligned} \tag{2.1}$$

The constraints (a) and (b) are standard differential restrictions to impose "free disposability" of, respectively, inputs and intended outputs:

$$\langle x, y, z \rangle \in T \;\wedge\; \bar{x} \geq x \Longrightarrow \langle \bar{x}, y, z \rangle \in T \tag{2.2}$$

and

$$\langle x, y, z \rangle \in T \;\wedge\; \bar{y} \leq y \Longrightarrow \langle x, \bar{y}, z \rangle \in T. \tag{2.3}$$

To capture the fact that pollution is an output of the production process for which disposal is not free, Murty [2010a,b] introduces and formalizes an assumption that is the polar opposite of free output disposability with respect to the unintended outputs:

$$\langle x, y, z \rangle \in T \;\wedge\; \bar{z} \geq z \Longrightarrow \langle x, y, \bar{z} \rangle \in T. \tag{2.4}$$

Following Murty, we refer to this property as "costly disposability" of residuals.[14] Costly disposability implies the possibility of inefficiencies in the generation of pollution (*e.g.,* if a given level of coal generates some level of smoke, then inefficiency in the use of coal may imply that this level of coal can also generate a greater amount of pollution. The differential restrictions required to impose costly disposability on T are

$$f_k(x, y, z) \leq 0, \; k = 1, \ldots, m'. \tag{2.5}$$

[13] Subscripts on f indicate partial differentiation with respect to the indicated variable.

[14] At this stage, though the assumption that the technology satisfies costly disposability of pollution seems similar to the assumption that it also satisfies input free disposability with respect to pollution, two differences between these assumptions and their implications will become clear later: (1) in our by-production approach this assumption is satisfied by nature's residual generation mechanism and not by the intended production technology and (2) the nature's residual generation mechanism treats pollution as an output of production and not as an input.

Quantity vectors satisfying $f(x, y, z) = 0$ are points on the frontier of the technology.[15] Those satisfying $f(x, y, z) < 0$ are inefficient: more intended output could be produced with given quantities of inputs and pollution; less pollution could be generated with given intended output and input quantities; and smaller input quantities could be used to produce the given output quantities, given the pollution level.

Assume, in this section and without loss of generality, that $m' = 1$. Suppose $f_k(\hat{x}, \hat{y}, \hat{z}) < 0$ for some $\langle \hat{x}, \hat{y}, \hat{z} \rangle$ satisfying $f(\hat{x}, \hat{y}, \hat{z}) = 0$. Then, from the implicit function theorem, there exist neighborhoods $U \subseteq \mathbf{R}_+^{m+n}$ and $V \subseteq \mathbf{R}_+$ around $\langle \hat{x}, \hat{y} \rangle \in \mathbf{R}_+^{n+m}$ and $\hat{z} \in \mathbf{R}_+$ and a function[16] $\zeta : U \to V$ such that

$$\hat{z} = \zeta(\hat{x}, \hat{y}) \tag{2.6}$$

and

$$f\big(x, y, \zeta(x, y)\big) = 0. \tag{2.7}$$

The trade-off between each intended output j and unintended output k (with inputs and all other outputs held fixed) implied by the implicit function theorem is

$$\frac{\partial \zeta(x, y)}{\partial y_j} = -\frac{f_j(x, y, z)}{f_k(x, y, z)} \geq 0, \quad j = 1, \ldots, m. \tag{2.8}$$

The trade-off between each input i and unintended output k (with intended outputs and all other inputs held fixed) is

$$\frac{\partial \zeta(x, y)}{\partial x_i} = -\frac{f_i(x, y, z)}{f_k(x, y, z)} \leq 0, \quad i = 1, \ldots, n. \tag{2.9}$$

Noting that all these trade-offs are evaluated at points in the technology set that are weakly technically efficient (that is, $f(x, y, z) = 0$), the foregoing formulation of a pollution-generating technology seems to be inconsistent with the phenomenon of by-production for the following reasons:

(a) The existence of the function ζ satisfying (2.8) as a strict inequality implies that there exists a rich menu (a manifold) of (weakly) technically efficient $\langle y, z \rangle$ combinations, with varying levels of z, that are possible with *given levels of all inputs*. If pollution is generated by input usage, this menu is contrary to phenomenon of by-production, since this phenomenon implies that at fixed levels of

[15] We adopt the following convention in this paper: A point $\langle x, y, z \rangle \in T$ lies on the frontier of T (or is a weakly efficient point of T) if there exists no other point $\langle \bar{x}, \bar{y}, \bar{z} \rangle \in T$ with $\bar{x}_i < x_i$ for all i, $\bar{y}_j > y_j$ for all j, and $\bar{z}_k < z_k$ for all k. A point $\langle x, y, z \rangle \in T$ lies on the efficient frontier of T (or is an efficient point of T) if there exists no other point $\langle \bar{x}, \bar{y}, \bar{z} \rangle \in T$ with $\bar{x} \leq x$, $\bar{y} \geq y$, and $\bar{z} \leq z$.

[16] See the appendix for a statement of the implicit function theorem.

inputs (*e.g.,* coal), there is only *one* (weakly) technically efficient (minimal) level of pollution.[17]

(b) Furthermore, if pollution is generated by inputs such as coal, as is very often the case, the non-positive trade-offs between pollution generation and inputs (derived by holding the levels of intended outputs fixed), apparent in (2.9), are inconsistent with by-production, which implies that this trade-off should be non-negative.

How should one interpret the trade-offs observed under single equation modeling of pollution-generating technologies when one abstracts from abatement options? As discussed above, these trade-offs are not reflective of the phenomenon of by-production. Rather, the non-negative trade-offs observed in (2.8) between each intended output and pollution and the non-positive trade-offs observed in (2.9) between each input and pollution suggest that this approach treats pollution like any other input in production: first, increases in its level, holding all other inputs fixed, increases intended outputs and, second, pollution is a substitute for all other inputs in intended production—the same level of intended outputs can be produced by decreasing other inputs and increasing pollution. This also does not seem to be intuitively correct: it is not a correct description of the role pollution plays in intended production.[18]

2.2. The case with abatement output.

Consider the case where the technology of a pollution-generating firm is defined by a single restriction on all inputs and outputs, including the abatement output:

$$T = \big\{ \langle x, y, z, y^a \rangle \in \mathbf{R}_+^{n+m+m'+1} \mid f(x, y, z, y^a) \leq 0 \big\}. \tag{2.10}$$

[17] If pollution is caused by some intended outputs (*e.g.,* strong odor from some varieties of cheese produced by a dairy) and (2.9) holds as a strict inequality, then it implies that there exists a rich menu of (weakly) technically efficient $\langle x, z \rangle$ combinations, with varying levels of z, that are possible with *given levels of all intended outputs*. Such a menu is inconsistent with by-production.

[18] In the literature, the treatment of pollution as any other productive input is often justified by equating pollution to the assimilative capacities of environmental resources such as air and water to absorb pollution. Murty [2010a,b] argue as follows: (1) qualitatively, these are two different goods—pollution is an output of the technology, while the assimilative capacity of environmental resource is an input. A given environmental resource like air can absorb different types of unintended outputs like CO_2, SO_2, *etc.*, and its assimilative capacity can be different for different pollutants. (2) The technological trade-off between an environmental resource and an input that causes pollution also does not follow the standard (negative) trade-offs between inputs. In particular, input free disposability is violated in the direction of inputs that cause pollution: *e.g.,* for every level of coal, there is a certain minimal level of smoke that is generated, and hence a certain minimal level of the environmental resource that is required. It is not technologically feasible to indefinitely increase usage of coal (as is required by the definition of input free disposability) keeping the levels of smoke and, hence, the environmental resource fixed.

We assume that

$$f_a(x, y, z, y^a) \geq 0. \tag{2.11}$$

This restriction captures the fact that the abatement output is also freely disposable:

$$\langle x, y, z, y^a \rangle \in T \ \wedge \ \bar{y}^a \leq y^a \Longrightarrow \langle x, y, z, \bar{y}^a \rangle \in T, \tag{2.12}$$

so that producing it is costly in terms of input usage, implying a non-positive trade-off between it and the other intended outputs. In that case, the implicit function theorem can again be invoked to show that the trade-off between the abatement output and pollution, evaluated in a neighborhood of a (weakly) technically efficient point $\langle \hat{x}, \hat{y}, \hat{z}, \hat{y}_a \rangle \in \mathbf{R}_+^{n+m+m'+1}$ such that $f(\hat{x}, \hat{y}, \hat{z}, \hat{y}_a) = 0$ and $f_k(\hat{x}, \hat{y}, \hat{z}, \hat{y}_a) < 0$, is

$$\frac{\partial \zeta(x, y, y_a)}{\partial y^a} = -\frac{f_a(x, y, z, y_a)}{f_k(x, y, z, y_a)} \geq 0 \tag{2.13}$$

whenever $f(x, y, z, y_a) = 0$, contradicting the fact that abatement output is produced by firms to mitigate, and not to enhance, pollution.

3. A by-production approach to modeling pollution.

Given the above analysis, a sound foundation must be identified for introducing multiple production relations to adequately capture the features of by-production. We feel that the resolution to the problem lies in early work of Frisch [1965] on production theory, in which he envisaged situations where the correct functional representation of a production technology may require more than one implicit functional relation between inputs and outputs. More recently, Førsund [2009] explores these ideas of Frisch.[19] We build on the works of Frisch and Førsund and show that the phenomenon of by-production requires distinguishing explicitly the by-product-generating mechanism from the production relation that describes the production of intended commodities. We show that when this is done the inconsistencies among trade-offs elucidated in Section 2 get resolved.

[19] He employs a welfare maximization problem to show that the optimal government policies are counter-intuitive and meaningless when a single production relation is used to represent a pollution-generating technology.

3.1. A by-production approach: the case without abatement.

In this sub-section, we abstract from explicit abatement efforts. The production of the intended output sets a residual-generation mechanism in motion, leading to the generation of the by-product. To fix our ideas on the salient aspects of by-production and to simplify notation, we continue to assume, without loss of generality, that $m' = 1$ and that the pollution is generated by usage of a single input (such an input could be coal), say input $\imath$.[20] Denote the input quantity vector purged of the quantity of input $\imath$ by x^1. Specify the technology as

$$T = T_1 \cap T_2, \tag{3.1}$$

where

$$T_1 = \left\{ \langle x^1, x_\imath, y, z \rangle \in \mathbf{R}_+^{n+m+1} \mid f(x^1, x_\imath, y) \le 0 \right\}, \tag{3.2}$$

$$T_2 = \left\{ \langle x^1, x_\imath, y, z \rangle \in \mathbf{R}_+^{n+m+1} \mid z \ge g(x_\imath) \right\}, \tag{3.3}$$

and f and g are continuously differentiable functions. The set T_1 is a standard technology set, reflecting the ways in which the inputs can be transformed into intended outputs. The standard free disposability properties (2.3) and (2.4) can be imposed on this set by assuming that f satisfies

$$\begin{aligned} &f_i(x, y) \le 0, \ i = 1, \ldots, n, \ \text{and} \\ &f_j(x, y) \ge 0, \ j = 1, \ldots, m. \end{aligned} \tag{3.4}$$

Note that (3.2) imposes no constraint on z, that is, it is implicitly assumed that the by-product does not affect the production of intended outputs.[21]

The set T_2 reflects nature's residual-generation mechanism. T_2 treats pollution as an output and satisfies costly disposability with respect to pollution as defined in (2.4), with the function g defining the minimal level of pollution that gets generated for given level of $x_\imath$.[22] The derivative of g satisfies

$$g'(x_\imath) \ge 0. \tag{3.5}$$

[20] The analysis can easily be extended to the case where pollution (such as a strong odour) is also caused by the production of an intended output (such as cheese). See Murty [2010b].

[21] This could be generalized, of course, allowing pollution to have an effect on intended production as well; *e.g.,* smoke could adversely affect the productivity of labor engaged in producing intended outputs. See Murty [2010b] for a generalization.

[22] Costly disposability, as defined in (2.4), could be considered to be too extreme. It implies that an infinite amount of pollution can be generated by given amount of input $\imath$. In general, there may also be an upper bound for the generation of the unintended output. See Murty [2010b] for a generalization.

The condition in (3.5) capture the fact that the efficient (minimal) level of pollution rises with the increase in the usage of input $\imath$. This means, however, that T_2 violates standard free disposability of input $\imath$. In fact it satisfies the polar opposite condition in this good:

$$\langle x^1, x_\imath, y, z\rangle \in T_2 \ \wedge \ \bar{z} \geq z \ \wedge \ \bar{x}_\imath \leq x_\imath \Longrightarrow \langle x^1, \bar{x}_\imath, y, \bar{z}\rangle \in T_2. \tag{3.6}$$

This implies that if a given level of coal generates some amount of pollution, then inefficiencies in residual generation may imply that lower amounts of the coal input can also generate the same level of pollution if the firm operates more efficiently.

It is easy to infer the disposability properties of T from the disposability properties of the intended production technology T_1 and the residual generation mechanism T_2

Theorem 1: *T satisfies free disposability with respect to all intended outputs and non-pollution-causing inputs. It, however, violates free disposability with respect to the pollution-causing input $\imath$. It satisfies costly disposability with respect to the quantity of pollution z.*

The technology violates standard disposability conditions with respect to the quantity of the pollution-causing input $x_\imath$ because, while T_1 satisfies standard free-disposability conditions in $x_\imath$, T_2 satisfies the polar opposite conditions with respect to this input.

Quantity vectors $\langle x, y, z\rangle \in T$ that satisfy $f(x, y) = 0$ and $z = g(x_\imath)$ are the weakly efficient points of T. If a quantity vector in $\langle x, y, z\rangle \in T$ is such that $f(x, y) < 0$, then it is technologically possible to decrease the levels of the non-pollution-causing inputs without changing the production levels of the remaining goods. If a quantity vector in $\langle x, y, z\rangle \in T$ is such that $z > g(x_\imath)$, then it is technologically possible to decrease the level of pollution without changing the production levels of all other goods.

To sign the trade-offs between pollution and a (non-pollution-causing) intended output j at a weakly efficient point of T, we invoke the implicit function theorem. Let $\langle \hat{x}, \hat{y}, \hat{z}\rangle$ be a weakly efficient point of T. Then

$$\begin{aligned} f(\hat{x}, \hat{y}) &= 0 \\ \hat{z} - g(\hat{x}_\imath) &= 0. \end{aligned} \tag{3.7}$$

Denote y^{-j} to be the vector obtained by purging the j^{th} element from the vector y. Suppose that $f_j(\hat{x}, \hat{y}) \neq 0$ and $g_\imath(\hat{x}_\imath) \neq 0$. Then the matrix

$$\begin{bmatrix} f_j(\hat{x}, \hat{y}) & f_\imath(\hat{x}, \hat{y}) \\ 0 & -g_\imath(\hat{x}_\imath) \end{bmatrix} \tag{3.8}$$

has full row rank. By the implicit function theorem, there exists a neighborhood U around $\langle \hat{x}^1, \hat{y}^{-j}, \hat{z}\rangle$ in $\mathbf{R}_+^{n+m-1}$, a neighborhood V around $\langle \hat{x}_\imath, \hat{y}_j\rangle$ in $\mathbf{R}_+^2$, and continuously differentiable mappings $\psi^j : U \to \psi^j(U)$ and $h : U \to h(U)$ with images

$$\begin{aligned} y_j &= \psi^j(x^1, y^{-j}, z) \\ x_\imath &= h(x^1, y^{-j}, z) \end{aligned} \tag{3.9}$$

such that $\big\langle h(x^1, y^{-j}, z),\ \psi^j(x^1, y^{-j}, z)\big\rangle \in V$ and

$$\begin{aligned} f(x^1, h(x^1, y^{-j}, z), \psi^j(x^1, y^{-j}, z), y^{-j}) &= 0 \\ z - g(h(x^1, y^{-j}, z)) &= 0. \end{aligned} \tag{3.10}$$

In that case, assuming that $g'(x_\imath) > 0$, the trade-off between y_j and z is[23]

$$\frac{\partial \psi^j(x^1, y^{-j}, z)}{\partial z} = -\frac{f_\imath\big(x, y\big)\ h_k(x^1, y^{-j}, z)}{f_j(x, y)} \geq 0. \tag{3.11}$$

How should one interpret this non-negative "trade-off" between y_j? Starting at a weakly efficient point in a local neighborhood of $\langle \hat{x}, \hat{y}, \hat{z}\rangle \in T$, an increase in z is attributable, because of the by-production phenomenon inherent in T_2, to an increase in $x_\imath$ (as $h_k(x^1, y^{-j}, z) > 0$). Under the conventional assumptions on intended production in (3.4), the trade-off between the pollution-generating input $\imath$ and intended output j is

$$-\frac{f_\imath(x, y)}{f_j(x, y)} \geq 0, \tag{3.12}$$

hence, the increase in $x_\imath$ implies an increase in y^j. The "trade-off" in (3.11), thus, reflects a non-negative *correlation* between the residual and an intended output via $x_\imath$, because a change in $x_\imath$ affects both y^j (non-negatively in intended production) and z (positively with respect to residual generation).

To summarize, the non-negative "trade-off" between an intended and an unintended output in the reduced form model is explained by (a) the phenomenon of by-production, which relates the use of inputs such as $\imath$ to the by-product, and (b) the non-negative marginal product of input $\imath$ in producing intended outputs like j.

3.2. A by-production approach: incorporating abatement activities.

We again keep the analysis simple by sticking to a single abatement output (as well as a single unintended output). On the other hand, we make the model more general to allow the possibility of input substitutability in the generation of the by-product.[24] We

[23] Note, as we have assumed a single unintended output, $h_k(x^1, y^{-j}, z)$ is the derivative of the function h with respect to z. Note also that h is the inverse of g, *i.e.*, $h(x^1, y^{-j}, z) = g^{-1}(z)$, so that, if $z = g(x_\imath)$ and $g'(x_\imath) > 0$, then $h_k(x^1, y^{-j}, z) = 1/g'(x_\imath) > 0$.

[24] For example, substituting a cleaner variety of coal for a less pure variety or vice-versa.

do so by partitioning the vector of all n inputs into n_1 non-residual-generating inputs and n_2 residual-generating inputs. Denote the respective input quantity vectors by x^1 and x^2. Let y^a denote the level of the firm's abatement activities, which are also costly in terms of the input resources of the firm. Without loss of generality, we assume that the intended outputs do not cause pollution. Similarly to the previous section, we specify the technology as $T = T_1 \cap T_2$, where

$$\begin{aligned} T_1 &= \{\langle x^1, x^2, y, z, y^a\rangle \in \mathbf{R}^{n+m+2} \mid f(x^1, x^2, y, y^a) \le 0\} \\ T_2 &= \{\langle x^1, x^2, y, z, y^a\rangle \in \mathbf{R}^{n+m+2} \mid z \ge g(x^2, y^a)\}. \end{aligned} \tag{3.13}$$

T reflects both the transformation of inputs into intended outputs and abatement output (as indicated by the definition of T_1) and the use of the abatement output by the firm to control the by-production of the residual that results from use of pollution-generating inputs in producing intended outputs (as indicated by the definition of T_2 in (3.13)). We confine ourselves again to a local analysis and posit the following signs of the partial derivatives at a weakly efficient point $\langle \hat{y}, \hat{y}_a, \hat{x}^1, \hat{x}^2, \hat{z}\rangle$ of T:

$$\begin{aligned} f_j(\hat{x}^1, \hat{x}^2, \hat{y}, \hat{y}_a) &\ge 0, \ \ j = 1, \dots m, \\ f_a(\hat{x}^1, \hat{x}^2, \hat{y}, \hat{y}_a) &> 0, \\ f_i(\hat{x}^1, \hat{x}^2, \hat{y}, \hat{y}_a) &\le 0, \ \ i = 1, \dots n, \\ g_a(\hat{x}^2, \hat{y}_a) &< 0, \\ g_\imath(\hat{x}^2, \hat{y}_a) &\ge 0 \quad \text{for all } \imath = n_1 + 1, \dots, n, \\ g_\imath(\hat{x}^2, \hat{y}_a) &> 0 \quad \text{for some } \imath = n_1 + 1, \dots, n. \end{aligned} \tag{3.14}$$

It is easy to see that (3.13) and (3.14) imply that T_1 satisfies standard free disposability conditions for inputs, abatement output, and intended outputs. In addition, there is a negative (or at least non-positive) trade-off between standard outputs and the abatement output and a positive (or a non-negative) trade-off between each intended output and the inputs in intended production.

With respect to residual generation, (3.13) and (3.14) imply that T_2 satisfies costly disposability for the unintended output and a condition that is the polar opposite of standard input and output free disposability for the abatement output and non-pollution-generating inputs:

$$\langle x^1, x^2, y, z, y^a\rangle \in T_2 \ \wedge \ \bar{z} \ge z \ \wedge \ \bar{x}^2 \le x^2 \ \wedge \ \bar{y}^a \ge y^a \Longrightarrow \langle x^1, \bar{x}^2, y, \bar{z}, \bar{y}^a\rangle \in T_2. \tag{3.15}$$

We call (3.15) "costly disposability of pollution, abatement output, and inputs that generate pollution."[25] The trade-offs between z and each of the pollution-generating input quantities $x_\imath^2$ implied by (3.14) are non-negative and that between z and abatement output y^a is negative. Thus, the sign of g_a captures the mitigating effect abatement has on residual generation and the sign of $g_\imath$ captures the increase in pollution attributable to the increase in inputs causing pollution.

It is easy to infer the disposability properties of T from the above characteristics of T_1 and T_2:

Theorem 2: *T satisfies free disposability with respect to all intended outputs and non-pollution-causing inputs. It, however, violates free disposability with respect to each of the pollution-causing inputs and the abatement output. It satisfies costly disposability with respect to pollution.*

Let the inequalities in (3.14) hold. We now sign the trade-off between z and an intended output y_j at a weakly efficient point of T. As in the previous section, we do so by employing the implicit function theorem. Let $\langle \hat{x}^1, \hat{x}^2, \hat{y}, \hat{z}, \hat{y}_a \rangle$ be a weakly efficient point of T. Then

$$\begin{aligned} f(\hat{x}^1, \hat{x}^2, \hat{y}, \hat{y}_a) &= 0 \\ \hat{z} - g(\hat{x}^2, \hat{y}^a) &= 0. \end{aligned} \tag{3.16}$$

Let $f_j(\hat{x}^1, \hat{x}^2, \hat{y}, \hat{y}_a) \neq 0$ and $g_a(\hat{x}^2, y^a) \neq 0$. Then the matrix

$$\begin{bmatrix} f_j(\hat{x}^1, \hat{x}^2, \hat{y}, \hat{y}_a) & f_a(\hat{x}^1, \hat{x}^2, \hat{y}, \hat{y}_a) \\ 0 & -g_a(\hat{x}^2, \hat{y}_a) \end{bmatrix} \tag{3.17}$$

is full-row ranked. The implicit function theorem implies that there exists a neighborhood U around $\langle \hat{x}, \hat{y}^{-j}, \hat{z} \rangle$ in $\mathbf{R}_+^{n+m}$, a neighborhood V around $\langle \hat{y}_j, \hat{y}^a \rangle$ in $\mathbf{R}_+^2$, and continuously differentiable mappings $\psi^j : U \to \psi^j(U)$ and $h : U \to h(U)$ with images

$$\begin{aligned} y_j &= \psi^j(x, y^{-j}, z) \\ y^a &= h(x, y^{-j}, z) = g^{-1}(z, x^2) \end{aligned} \tag{3.18}$$

such that $\left\langle \psi^j(x, y^{-j}, z),\ h(x, y^{-j}, z) \right\rangle \in V$ and

$$\begin{aligned} f(x, \psi^j(y^{-j}, z), y^{-j}, h(x, y^{-j}, z)) &= 0 \\ z - g(x^2, h(x, y^{-j}, z)) &= 0. \end{aligned} \tag{3.19}$$

[25] This assumption reflects the inefficiencies in the production of pollution: if given levels of coal and abatement activities generate some amount of pollution, then inefficiencies in the use of coal or abatement activities imply that a lower amount of the coal input or a higher level of abatement activities could generate the same level of pollution if the firm were to operate more efficiently.

In that case, the trade-off between y_j and z is

$$\frac{\partial \psi^j(x, y^{-j}, z)}{\partial z} = -\frac{f_a\big(x, y, y^a\big)\ h_k(x, y^{-j}, z)}{f_j(x, y, y^a)} \geq 0. \tag{3.20}$$

As in the previous section, this non-negative trade-off between an intended output and pollution at a weakly efficient point of T reflects a correlation between these commodities; in this case, this correlation is effected by abatement effort of the firm to mitigate by-production of pollution.[26] Precisely, holding the levels of all inputs (including pollution-causing inputs) fixed, an increase in z must have come about because of reductions in abatement efforts y^a by firms, and hence there is an increase in resources diverted towards production of other intended outputs y (assuming, of course, that firms are operating in a weakly efficient way).

From our analysis above, we can derive the reduced-form functional representation of the technology T. By substituting out abatement efforts from the function f in (3.13), we can rewrite T equivalently as

$$T = \big\{\langle x^1, x^2, y, z, y^a\rangle \in \mathbf{R}^{n+m+2} \mid \tilde{f}\big(x, y, z\big) \leq 0 \ \wedge \quad y^a \geq h(x, y^{-j}, z)\big\}, \tag{3.21}$$

where

$$\tilde{f}\big(x, y, z\big) := f\big(x, y, h(x, y^{-j}, z)\big). \tag{3.22}$$

Using (3.21), we can define a reduced-form technology in the space of intended and unintended outputs and inputs as

$$\tilde{T} := \{\langle x^1, x^2, y, z\rangle \in \mathbf{R}_+^{n+m+1} \mid \tilde{f}(x, y, z) \leq 0\}. \tag{3.23}$$

The input and output approaches in the conventional literature model a reduced-form technology—quite in the spirit of $\tilde{T}$—in the space of intended and unintended outputs and inputs that exhibits a positive correlation between intended and unintended outputs but satisfies *all* of the standard free disposability assumptions with respect to intended outputs and inputs. The technology is modeled only in reduced form because, although this literature attributes the positive correlation to abatement options available to firms, abatement activities are not explicitly modeled.

In the case of the by-production, it is easy to check that, in the neighborhood of a point $\langle x, y, z\rangle$ that satisfies $\tilde{f}(x, y, z) = 0$, the trade-off between an intended and an unintended output, $-\tilde{f}_j\big(x, y, z\big)/\tilde{f}_k\big(x, y, z\big)$, is given by (3.20) and hence is non-negative. This

[26] Note that, as in the previous section, a (generally different) non-negative correlation between the intended and unintended outputs effected by an input that causes pollution could also be derived.

is consistent with conventional modeling of the reduced form of a pollution-generating technology. However, the derivative of the function $\tilde{f}$ with respect to a pollution-causing input $i = n_1 + 1, \ldots, n$ is

$$\tilde{f}_i(x^1, x^2, y, z) = f_a(x^1, x^2, y, y^a) h_i(x, y^{-j}, z) + f_i(x^1, x^2, y, y^a). \tag{3.24}$$

Given (3.18) and the sign conventions in (3.14), the sign of $\tilde{f}_i$ is ambiguous, contrary to the conventional literature, where it is signed as per a normal input. As seen in Theorems 1 and 2 in Section 3, this follows from the fact that the residual generating technology T_2 (and hence the by-production technology $T = T_1 \cap T_2$) violates standard free disposability in such inputs.

4. Data-based pollution-generating technologies.

The foregoing analysis reveals that modeling the phenomenon of by-production requires more than one implicit production relation among inputs and outputs. One of these relations captures intended production activities of firms (that is, describes the set T_1), while the other captures the inevitability of residual generation when firms engage in intended production (that is, describes the set T_2). The former identifies an upper bound for the intended outputs of firms for every given level of inputs, while the latter identifies a lower bound for pollution generation given every level of intended outputs and inputs that are responsible for causing pollution.

The econometric approach must involve simultaneous estimation of two (or more) structural production relations that have the above features. In particular the production relation associated with intended production will be the upper frontier of T_1 and the production relation associated with residual generation will be the lower frontier of T_2. These production relations should satisfy the trade-offs implied by (3.14).

An alternative approach to constructing pollution-generating technologies, one employing data envelopment analysis (DEA) methods, has become increasingly common in recent years.[27] This approach essentially envelops the data in the "smallest" or "tightest fitting" convex set or convex cone. In the case of conventional inputs and outputs, the technology is the convex, free disposal hull of the data,[28] but the problem is more complicated in the case of pollution-generating technologies.

[27] The survey of these methods by Zhou, Ang, and Poh [2008] contains 150 references.

[28] Assuming non-increasing returns to scale. Under the assumption of constant returns to scale, it is the *conical*, convex, free disposal hull of the data. See, *e.g.*, Färe, Grosskopf, and Lovell [1994] for details.

To lay out these concepts formally, we consider a more general model than the one presented above, incorporating multiple pollution-generating inputs and multiple pollutants. We restrict ourselves to the case where pollution is caused by the use of certain inputs by firms.[29]

First augment the notation in Section 2 as follows:

(i) p decision making units (DMUs),[30] indexed by d.

(ii) m intended outputs, indexed by j, with quantity vector $y \in \mathbf{R}_+^m$. The $p \times m$ matrix of observations on intended output quantities is denoted by Y.

(iii) n inputs, indexed by i. The first n_1 are non-pollution-generating, while the remaining $n_2 = n - n_1$ are pollution generating. The quantity vector is $x = \langle x^1, x^2 \rangle \in \mathbf{R}_+^n$. The $p \times n$ matrix of observations on the input quantities is denoted by $X = \langle X^1, X^2 \rangle$.

(iv) m' pollutants, indexed by k, with quantity vector $z \in \mathbf{R}_+^{m'}$. The $p \times m'$ matrix of observations on pollutants is denoted by Z.

For illustrative purposes, we posit an example for a very simple special case with five decision making units, one intended output, one unintended output, and one input:

Example 1: $p = 5$, $m = 1$, $n = n_1 = 1$, and $m' = 1$. The (artificial) data are as follows:

DMU	x	y	z
1	1	2	4
2	1	3/2	1
3	1	2/3	2
4	2	3	5
5	2	2	3

(4.1)

In the conventional output approach to modeling pollution-generating technologies, all intended outputs and inputs are assumed to satisfy standard disposability conditions, but two key assumptions are made regarding the unintended outputs. The first,

$$\langle x, y, z \rangle \in \tilde{T} \;\wedge\; \lambda \in [0,1] \implies \langle x, \lambda y, \lambda z \rangle \in \tilde{T}, \tag{4.2}$$

is called "weak disposability", a concept originally attributable to Shephard [1953, 1974]. The second,

$$\langle x, y, z \rangle \in \tilde{T} \;\wedge\; z = 0 \implies y = 0, \tag{4.3}$$

[29] The data set used below for our empirical application does not contain information on abatement. Extension to the case where some intended outputs also cause pollution is straightforward. In Section 7 we consider a numerical example to illustrate the extension to the case with abatement efforts of firms.

[30] Here we follow the standard nomenclature in the literature on technical efficiency measurement. The generic DMU could be a firm, a plant belonging to a specific firm, or any of a number of types of units of study.

is called "null jointness". Weak disposability and null-jointness imply that (a) while pollution is not freely disposable, it is possible to jointly and proportionately decrease pollution and the intended outputs and (b) production of *any* positive level of intended output always results in positive amounts of the residual being generated. This literature is predicated on the belief that these two assumptions can capture the fact that, starting at any efficient point of the technology, it is not possible to decrease pollution without decreasing the production of the intended outputs, and hence that, together, they model the positive reduced-form correlation between pollution and other intended outputs. The standard DEA construction of a pollution-generating technology (based on the assumptions of weak disposability and null-jointness) first formulated by Färe, Grosskopf, and Pasurka [1989], is given by[31]

$$\tilde{T}_{WD} = \Big\{ \langle x, y, z \rangle \in \mathbf{R}_+^{n+m+m'} \mid \lambda X \leq x \ \wedge \ \lambda Y \geq y \ \wedge \ \lambda Z = z \text{ for some } \lambda \in \mathbf{R}_+^p \Big\}. \quad (4.4)$$

The production possibility set satisfying weak disposability for Example 1, with $x = 1$ is shown in Panel 4 of Figure 1 (where points A and B are the $\langle z, y \rangle$ combinations for DMUs 2 and 1, respectively, and the other DMU vectors fall below the frontier.

Denote the overall technology $T_1 \cap T_2$ that satisfies by-production by T_{BP}. We assume that T_1 satisfies free disposability of inputs and intended outputs (as defined in (2.2) and (2.3)) and that it is closed, convex, and satisfies constant returns to scale. In addition, T_1 satisfies the following assumption, which we call "independence of T_1 from z" and which states that pollution does not directly affect production of intended outputs:

$$\langle x, y, z \rangle \in T_1 \Longrightarrow \langle x, y, \bar{z} \rangle \in T_1 \ \forall \ \bar{z} \in \mathbf{R}_+^{m'}. \quad (4.5)$$

(This assumption would have to be relaxed if, *e.g.*, the presence of pollution could adversely affect labor productivity in producing intended outputs. See Murty [2010b].) The intended-output technology T_1 that satisfies these assumptions is obtained in a standard way using DEA techniques as follows:

$$T_1 = \Big\{ \langle x, y, z \rangle \in \mathbf{R}_+^{n+m+m'} \mid \lambda X \leq x \ \wedge \ \lambda Y \geq y \text{ for some } \lambda \in \ \mathbf{R}_+^p \Big\}. \quad (4.6)$$

We assume T_2 satisfies costly disposability of pollution and inputs that cause pollution (as defined in (3.6)) and constant returns to scale. Also note that, since we have assumed that only x^2 affects residual generation, T_2 also satisfies "independence of T_2 from x^1 and y":

$$\langle x, y, z \rangle \in T_2 \Longrightarrow \langle \bar{x}^1, x^2, \bar{y}, z \rangle \in T_2 \ \forall \ \langle \bar{x}^1, \bar{y} \rangle \in \mathbf{R}_+^{n_1+m}. \quad (4.7)$$

[31] This formulation assumes constant returns to scale for inputs and intended outputs; for non-increasing returns to scale, impose the additional constraint $\sum_d \lambda_d \leq 1$.

The DEA version of T_2, which satisfies these assumptions, is obtained as

$$T_2 = \left\{ \langle x^1, x^2, y, z \rangle \in \mathbf{R}_+^{n_1+n_2+m+m'} \mid \mu X^2 \geq x^2 \ \wedge \ \mu Z \leq z \ \text{ for some } \mu \in \mathbf{R}_+^p \right\}. \tag{4.8}$$

The first inequality in (4.8) reflects costly disposability of inputs that cause pollution and the second reflects costly disposability of pollution. Since T_2 is independent of x^1 and y, no inequalities need to be specified for x^1 and y.

A data set coming from pollution-generating units must simultaneously belong to both T_1 and T_2. The overall technology that exhibits by-production is the intersection of T_1 and T_2:

$$\begin{aligned} T_{BP} = \Big\{ \langle x^1, x^2, y, z \rangle \in \mathbf{R}^{n_1+n_2+m+m'} \mid\ & \lambda [X^1 \ X^2] \leq \langle x^1,\ x^2 \rangle,\ \lambda Y \geq y, \\ & \mu X^2 \geq x^2,\ \mu Z \leq z, \\ & \text{for some } \langle \lambda, \mu \rangle \in \mathbf{R}_+^{2p} \Big\}. \end{aligned} \tag{4.9}$$

The above construction of T_{BP} using activity analysis involves two sets of production relations. These are reflected in the two different intensity vectors λ and μ, each of which is applied to the same data set.

These sets under the assumptions of Example 1 are depicted in the first three panels of Figure 1. Noting that T_1 is independent of z and T_2 is independent of y, Panels 1 and 3 of Figure 1 show the DEA constructions of projections of T_1 (in the space of the input and the intended output) and T_2 (in the space of the input and the unintended output), respectively.[32]

Panels 2 and 4 of the same figure show the combinations of intended and unintended outputs that are feasible with $x = 1$, under the by-production (BP) and the weak disposability (WD) approaches, respectively. It is clear from Panel 2 that, in the case of BP, the output possibility set has only one efficient point, $e = \langle 1, 2 \rangle$ (the efficient frontier of the output possibility set is a singleton). This gives the minimum level of the unintended output and the maximum level of the intended output that can be produced when $x = 1$ and corresponds to efficient points of T_1 and T_2 as seen in Panels 1 and 3.[33] On the other hand, Panel 4 shows that the efficient frontier of the output possibility set satisfying weak disposability OAB has a far greater number of points. This illustrates that the efficient frontier of the output possibility set under the BP approach is smaller than under the WD approach.

[32] With an abuse of notation, but with no confusion, we also call these projections T_1 and T_2 in Figure 1. Panels 1 and 3 of this figure are drawn under the maintained assumption of constant returns to scale.

[33] Note that, while e is not a point in our artificial data set, the data are used to find e. The rest of the frontier of the output possibility set in Panel 2 reflects the fact that T_{BP} satisfies standard output free disposability in the direction of the intended output and costly disposability in the direction of pollution.

5. Measuring technical efficiency.

Two conventional efficiency indexes have been extensively employed in the DEA pollution literature: the output-oriented hyperbolic (HYP) index employed in the original DEA pollution study of Färe, Grosskopf, and Pasurka [1986] and the output-oriented directional-distance-function (DDF) index employed in more recent studies (*e.g.*, Färe, Grosskopf, Noh, and Weber [2005]).[34] These indexes are "output-oriented" because they measure efficiency in (intended and unintended) output space (*i.e.*, in the output direction).

For each technology $T = \tilde{T}_{WD}, T_{BP}$ and for each decision making unit $(d = 1, \ldots, p)$, the output-oriented HYP efficiency index is defined by[35]

$$E_H\left(x^d, y^d, z^d, T\right) = \min_{\beta > 0}\left\{\beta \mid \left\langle x^d, y^d/\beta, \beta z^d\right\rangle \ \in \ T\right\}, \tag{5.1}$$

and the output-oriented DDF index of *inefficiency* is defined by

$$I_{DD}\left(x^d, y^d, z^d, T\right) = \max\left\{\beta \mid \left\langle x^d,\ y^d + \beta g_y,\ z^d - \beta g_z\right\rangle \ \in \ T\right\}, \tag{5.2}$$

where $g = \langle g_y, g_z \rangle \in \mathbf{R}_+^{m+m'}$ is the arbitrary (output) "direction vector." E_H maps into the (0,1] interval, while E_{DD} maps into $\mathbf{R}_+$. For points on the frontier of T, $E_H(x, y, z, T) = 1$ and $I_{DD}(x, y, z, T) = 0$.[36] The vectors $\left\langle x^d, y^d/\overset{*}{\beta}, \overset{*}{\beta} z^d\right\rangle$ and $\left\langle x^d,\ y^d + \overset{*}{\beta} g_y,\ z^d - \overset{*}{\beta} g_z\right\rangle$, where $\overset{*}{\beta}$ is the solution value in each case, are referred to as "reference points"; they are comparison vectors for assessing the efficiency of a particular production vector.

5.1. Inadequacies of conventional efficiency indexes for the by-production approach: the hyperbolic and directional-distance-function indexes.

When using our proposed BP approach, the HYP and DDF efficiency indexes in (5.1) and (5.2) implicitly decompose total (in)efficiency into (in)efficiency in intended production

[34] The HYP efficiency index was formulated for standard technologies by Färe, Grosskopf, and Lovell [1985, pp. 110–111]. The DDF index was adapted from the shortage function of Luenberger [1992] to the measurement of efficiency by Chambers, Chung, and Färe [1996] and Chung, Färe, and Grosskopf [1997]. For a comparison of the properties of these two efficiency indexes, among others, see Russell and Schworm [2010].

[35] Provided all outputs are positive.

[36] Note that an HYP output-oriented index of *inefficiency* can be defined by $1/E_H(x, y, z, T)$, which lies in the interval $[1, \infty)$.

(β_1) and environmental (in)efficiency (β_2). The decompositions follow from the important facts that T_1 is independent of z and T_2 is independent of y. Thus we have

$$\begin{aligned}
E_H(x,y,z,T_{BP}) &= \min_{\beta>0} \{\beta|\ \langle x, y/\beta, \beta z\rangle \in T_{BP}\} \\
&= \min_{\beta>0}\{\beta|\langle x, y/\beta, \beta z\rangle \in T_1 \text{ and } \langle x, y/\beta, \beta z\rangle \in T_2\} \\
&= \max\{\beta_1, \beta_2\}, \text{ where} \qquad (5.3)\\
\beta_1 &= \min_{\beta>0}\{\beta|\langle x, y/\beta, z\rangle \in T_1\} =: E_H^1(x,y,z,T_{BP}) \text{ and} \\
\beta_2 &= \min_{\beta>0}\{\beta|\langle x, y, \beta z\rangle \in T_2\} =: E_H^2(x,y,z,T_{BP})
\end{aligned}$$

and

$$\begin{aligned}
I_{DD}(x,y,z,T_{BP}) &= \max_{\beta}\{\beta|\ \langle x, y+g_y\beta, z-g_z z\rangle \in T_{BP}\} \\
&= \max_{\beta}\{\beta|\langle x, y+g_y\beta, z-g_z\beta\rangle \in T_1 \text{ and } \langle x, y+g_y\beta, z-g_z\beta\rangle \in T_2\} \\
&= \min\{\beta_1, \beta_2\}, \text{ where} \\
\beta_1 &= \max_{\beta}\{\beta|\langle x, y+g_y\beta, z\rangle \in T_1\} =: I_{DD}^1(x,y,z,T_{BP}) \text{ and} \\
\beta_2 &= \max_{\beta}\{\beta|\langle x, y, z-g_z\beta\rangle \in T_2\} := I_{DD}^2(x,y,z,T_{BP}).
\end{aligned} \qquad (5.4)$$

If max $\{\beta_1, \beta_2\} = \beta_1 \neq \beta_2$ for the HYP output-oriented measure of efficiency, the data point is compared to a reference point that is weakly efficient in intended production but is not weakly environmentally efficient. If max $\{\beta_1, \beta_2\} = \beta_2 \neq \beta_1$, the reference point is weakly environmentally efficient but not weakly efficient in intended production. A similar logic applies in an obvious way for the DDF measure of inefficiency. Thus, the reference points with which different data points are compared to measure (in)efficiency may not be fully efficient when the BP approach is used, and we argue below that they typically are *not* fully efficient.

Consider the quantity vector of DMU 3 in Example 1, represented by point $a = \langle a_z, a_y\rangle = \langle 2, 2/3\rangle$ in the output possibility set corresponding to $x = 1$ in Panel 2. If the BP approach is used to measure HYP efficiency, (5.3) and Panels 1 to 3 show that $\beta_1 = 1/3$ and $\beta_2 = 1/2$ so that $\beta = \beta_2$.[37] This implies that the reference point that is being used to measure efficiency of $\langle 2, 2/3\rangle$ is $e' = \langle 1, 4/3\rangle$. In contrast to the fully efficient point e, e' is

[37] The intuition as to why the efficiency measure chooses $\beta = \beta_2 = \max\{\beta_1, \beta_2\}$ as a full measure of output efficiency is that, while $\langle 2\beta_2, \frac{2}{3\beta_2}\rangle$ is feasible both with respect to T_1 and T_2 with $x = 1$, $\langle 2\beta_1, \frac{2}{3\beta_1}\rangle$ is feasible only with respect to T_1 and not T_2, as it implies a reduction in the level of the unintended output z below the minimum that $x = 1$ can produce.

environmentally efficient but not efficient in intended production. On the other hand, the HYP efficiency of a using the WD approach in Panel 4 is .47, and the reference point is e'', which is technologically efficient with respect to the WD technology.[38]

Suppose that, as is common in the literature, we adopt a direction vector $g = \langle g_z, g_y \rangle = \langle 1, 1 \rangle =: \mathbf{1}$ to compute the DDF index of inefficiency for DMU 3. If the BP approach is employed, then β_1 is implicitly defined by $\frac{2}{3} + \beta_1 = 2$, so that $\beta_1 = 4/3$. Similary, β_2 is implicitly defined by $2 - \beta_2 = 1$ so that $\beta_2 = 1$. Thus, the DDF inefficiency score of DMU 3 is $\beta = \beta_2 = 1$, and this leads to a reference point $\langle 1, 5/3 \rangle$ that is environmentally efficient but not efficient in intended production.

Now consider the quantity vector of DMU 2 represented by point $b = \langle 1, 3/2 \rangle$ in the output possibility set corresponding to $x = 1$ in Panel 2. For the HYP measure, program (5.3) and Panels 1 to 3 of Figure 1 imply that $\beta_2 = 1$ while $\beta_1 = 3/4 < 1$. Thus, the conventional HYP measure computed using the BP approach gives DMU 2 an efficiency score $\beta = 1$ even though DMU 2 is not efficient in both the environmental and the intended output dimensions: it is only environmentally efficient.[39]

These examples illustrate a fundamental problem with the conventional measures of efficiency when using the BP approach for constructing the technology: the efficiency score for a firm may take the value 1 for HYP measures or 0 for the DDF measure even though the firm is not weakly efficient in both environmental and intended output directions. In addition, the reference point with which the firm is compared may not be weakly efficient in both these dimensions, resulting in an understatement (overstatement) of overall inefficiency (efficiency).

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38 The frontier of the output possibility set in Panel 4 can be represented functionally by

$$\begin{aligned} y &\leq \frac{3}{2}z \qquad 0 \leq z \leq 1 \\ y &\leq \frac{z}{6} + \frac{4}{3} \quad 1 \leq z \leq 4. \end{aligned} \tag{5.5}$$

The HYP efficiency index in this case will choose a reference point that either lies on line-segment OA (for $z \in [0,1]$) or on line-segment AB (for $z \in [1,4]$). The reference point will be of the form $\langle 2\beta, \frac{2}{3\beta} \rangle$. Suppose, the reference point is on OA, then (5.5) implies that it should solve $\frac{2}{3\beta} = \frac{3}{2}2\beta$ and this yields $\beta = \frac{\sqrt{2}}{3} = 0.471$. If it is on AB then (5.5) implies that it should solve $\frac{2}{3\beta} = \frac{2\beta}{6} + \frac{4}{3}$ and this yields $\beta = \sqrt{6} - 2 = 0.449$. However, for this case, (5.5) implies that the underlying reference point, $\langle 0.899, 1.483 \rangle$, is not feasible. Hence, HYP efficiency associated with a is 0.471, which takes us to the reference point $e'' = \langle 0.943, 1.414 \rangle$ lying on OA.

39 Similarly, it is easy to verify that the conventional DDF measure of inefficiency also gives DMU 2 an inefficiency score of 0.

It is well known that the HYP and DDF indexes do not satisfy the indication condition: score equal to 1 or 0, respectively, if and only if the point is (fully) efficient.[40] But, because of another problem, the DDF is particularly unsuitable for use as an inefficiency index for a BP technology. The inefficiency scores obtained from the DDF measure are very sensitive to the choice of the direction vector $g = \langle g_z, g_y \rangle$. While computing the DDF index of inefficiency, the direction and size of vector g are held fixed across all data points.[41] For this choice of g, the DDF inefficiency indexes for DMUs, 1, 2, and 3 in Example 1, are obtained as below:

$$\begin{array}{cccc}
\text{DMU} & \beta_1 & \beta_2 & \beta = \min\{\beta_1, \beta_2\} \\
1 & 0 & \frac{3}{g_z} & \beta_1 = 0 \\
2 & \frac{1/2}{g_y} & 0 & \beta_2 = 0 \\
3 & \frac{4/3}{g_y} & \frac{1}{g_z} & \begin{array}{ll} \beta_2 & \text{if } g_y < 4g_z/3 \\ \beta_1 & \text{if } g_y > 4g_z/3 \end{array}
\end{array} \tag{5.6}$$

Thus, except when a DMU is environmentally efficient or efficient in intended production, the DDF measure chooses β_1 or β_2 as the overall measure of inefficiency depending on the choice of the direction vector g. It is a common practice in the literature to choose $g = \mathbf{1}$. In this example with $g = \mathbf{1}$, the DDF measure selects the environmental inefficiency component for DMU 3 as the overall measure of inefficiency. It is, of course, obvious that the DDF inefficiency score is sensitive, in general, to the choice of the direction vector. This sensitivity seems to be more salient in the BP approach, however, since the choice of g is typically tantamount to predetermining a choice between the selection of the environmental or the intended production inefficiency components as the measure of overall inefficiency.[42]

Many (in)efficiency indexes have been proposed in the literature.[43] In empirical work on pollution-generating technologies, however, HYP and DDF are among the more widely used of these conventional indexes. Given the above problems with these two indexes under the BP approach, we propose, in the next subsection, a modification of another conventional efficiency index that is better behaved for use in measuring efficiency on BP production technologies.

[40] See Russell and Schworm [2010].

[41] The components g_y and g_z of g are interpreted to be measured in the units in which intended output and pollution are measured, respectively, so that the inefficiency scores can be interpreted to be independent of units of measurement.

[42] It is well known that the HYP inefficiency index can be interpreted as an alternative kind of DDF inefficiency index in which the direction vector varies across DMUs and, in particular, is equated to the quantity vector $\langle z, y \rangle$. This alteration alleviates the above problem with the conventional DDF index (where g is held fixed across all DMUs). (See Chambers, Chung, and Färe [1996].)

[43] See Russell and Schworm [2010] for an analysis of these indexes and their properties.

5.2. A proposed efficiency index for by-production technologies: modification of the Färe-Grosskopf-Lovell index.

The previous subsection shows that the principal problem with the widely used hyperbolic and directional-distance-function efficiency indexes applied to BP technologies is the endemic understatement of the degree of inefficiency.

The index we propose for measuring efficiency on by-production technologies is motivated by the input-oriented index proposed by Färe and Lovell [1978] and extended to the full $\langle$input, output$\rangle$ space for standard technologies (with no unintended outputs) by Färe, Grosskopf, and Lovell [1985, pp. 153–154]. The key feature of this index is that the reference points it uses to assign efficiency scores to the DMUs are all efficient, in contrast to the the HPY and DDF indexes, for which the reference points are all weakly efficient.[44] Define

$$y \oslash \beta = \langle y_1/\beta_1, \ldots, y_m/\beta_m \rangle \tag{5.7}$$

and

$$\gamma \otimes z = \langle \gamma_1 z_1, \ldots, \gamma_{m'} z_{m'} \rangle. \tag{5.8}$$

The output-oriented Färe-Grosskopf-Lovell index is defined by[45]

$$E_{FGL}(x, y, z, T) := \min_{\beta,\gamma} \left\{ \frac{\sum_j \beta_j + \sum_k \gamma_k}{m + m'} \middle| \ \langle x, y \oslash \beta, \gamma \otimes z \rangle \in T \right\}. \tag{5.9}$$

This index maps into the (0,1] interval and is equal to 1 if and only if the output vectors are technically efficient. The modification we propose facilitates decomposition of overall efficiency into intended production efficiency and environmental efficiency on by-production technologies. As our modification is minor, we continue to refer to it as the (output oriented) Färe-Grosskopf-Lovell index and define it as follows:

$$E_{FGL}(x, y, z, T) := \frac{1}{2} \min_{\beta,\gamma} \left\{ \frac{\sum_j \beta_j}{m} + \frac{\sum_k \gamma_k}{m'} \middle| \ \langle x, y \oslash \beta, \gamma \otimes z \rangle \in T \right\}. \tag{5.10}$$

[44] This feature is attributable to the fact that The Färe-Grosskopf-Lovell index involves a maximal contraction/expansion of inputs/outputs in coordinate-wise directions (rather than in a maximal radial or hyperbolic direction). Hence, all the slack in inputs and outputs is removed. (Of course, the input-oriented or output-oriented version of this index takes up all slack only in the input or output space, leaving the possibility of residual slack in outputs or inputs.)

[45] Although the constraint set for β in the following minimization problem is not closed, the min exists so long as intended outputs are all strictly positive. See Levkoff, Russell, and Schworm [2010] for an analysis of boundary issues for this efficiency index.

In the case of BP technologies, and under the assumption that T_1 is independent of z and T_2 is independent of y, the index decomposes as follows:

$$\begin{aligned}
E_{FGL}(x,y,z,T_{BP}) &:= \frac{1}{2}\min_{\beta,\gamma}\left\{\frac{\sum_j \beta_j}{m} + \frac{\sum_k \gamma_k}{m'}\,\middle|\, \langle x, y\oslash\beta, \gamma\otimes z\rangle \in T_{BP}\right\} \\
&= \frac{1}{2}\min_{\beta,\gamma}\left\{\frac{\sum_j \beta_j}{m} + \frac{\sum_k \gamma_k}{m'}\,\middle|\, \langle x, y\oslash\beta, \gamma\otimes z\rangle \in T_1 \;\wedge\; \langle x, y\oslash\beta, \gamma\otimes z\rangle \in T_2\right\} \\
&= \frac{1}{2}\min_{\beta}\left\{\frac{\sum_j \beta_j}{m}\,\middle|\, \langle x, y\oslash\beta, z\rangle \in T_1\right\} + \frac{1}{2}\min_{\gamma}\left\{\frac{\sum_k \gamma_k}{m'}\,\middle|\, \langle x, y, \gamma\otimes z\rangle \in T_2\right\} \\
&=: \frac{1}{2}\left[E^1_{FGL}(x,y,z,T_1) + E^2_{FGL}(x,y,z,T_2)\right],
\end{aligned} \tag{5.11}$$

where the third identity follows from independence of T_1 from z and independence of T_2 from y. This index is one-half of the sum of the average maximal coordinate-wise expansions of intended-output quantities and the average maximal coordinate wise contractions of unintended-output quantities subject to the constraint that the expanded/contracted output-quantity vector remain in the production possibility set for a given input vector. Under our independence assumptions, the index decomposes into the sum of a standard intended-output-oriented index defined on T_1, E^1_{FGL}, and an environmental index defined on T_2, E^2_{FGL}.[46]

The properties of this proposed index can be illustrated using the artificial data in Example 1 above. Consider first the case of DMU 3, represented by point a in Panel 3 of Figure 1. It is clear that $E^1_{FGL}(1,2/3,2,T_1) = 1/3$ and $E^2_{FGL}(1,2/3,2,T_2) = 1/2$, so that $E_{FGL}(1,2/3,2,T_{BP}) = 5/12 < E_H(1,2/3,2,T_{BP}) = 1/2$. Moreover, the reference point for a is the fully efficient point e in Panel 3; thus, unlike the HYP and DDF indexes, this proposed index takes up all the slack in the measurement of efficiency. Consider now the quantity vector of DMU 2 represented by point $b = \langle 1, 3/2\rangle$ in Panel 3. Although this point is not fully efficient, the values of both HYP and DDF are equal to 1. On the other hand, for this DMU, $E^2_{FGL}(1,3/2,1,T_2) = 1$ but $E^1_{FGL}(1,3/2,1,T_1) = (3/2)/2 = 3/4$, so that $E_{FGL}(1,3/2,1,T_{BP}) = 7/8$. These examples illustrate the fact that the proposed index corrects the principal problem with the HYP and DDF indexes in the measurement of efficiency on BP technologies. In particular, the FGL efficiency scores will typically be lower than the HYP efficiency scores.[47]

[46] Note that, instead of weighting each index equally, one could adopt different weights (summing to 1) if there were a reason to give more importance to one type of efficiency than the other.
[47] This is true for both WD and BP technologies.

It can also be verified that, for DMU 3, $E_{FGL}(1, 2/3, 2, T_{WD}) = .47$, and the associated reference point is e'' in Panel 4 of Figure 1. Hence, the FGL efficiency score for DMU 3 under the WD approach is higher than under the BP approach. Further, e'' is technologically infeasible under the BP approach, while the analogous reference point e for DMU 3 under the BP approach is technologically infeasible under the WD approach. The output quantity vector associated with DMU 2 is efficient under the WD approach ($E_{FGL}(1, 2/3, 2, T_{WD}) = 1$ and it involves no slack viz-a-viz the WD technology). But, this vector is only weakly efficient under the BP approach and hence FGL gives DMU 2 a lower efficiency score. Thus, the efficiency scores for DMUs and the associated reference points for FGL efficiency index are typically quite different across the BP and WD approaches. In particular, if a DMU is judged efficient by the FGL index under the BP approach, this index will also judge it efficient under the WD approach. But the converse is not true. This implies that the FGL efficiency scores under the WD approach will typically be at least as high as those under the BP approach.

6. Empirical application.

We now illustrate the implementation of the modified FGL index on a BP technology constructed with an actual data base. We use annual data for 92 coal-fired electric power plants from 1985 to 1995.[48] This data base includes observations for one intended output: net electricity generation (in kWh); two unintended outputs: sulfur dioxide (SO_2) and nitrogen oxide (NO_x) (in short-tons); two non-polluting inputs: the capital stock and the number of employees; and three pollution-generating inputs: the heat content (in Btu) of coal, oil, and natural gas consumed at each power plant. Thus $p = 92$, $m = 1$, $m' = 2$, $n_1 = 2$, and $n_2 = 3$.

The various efficiency indexes are calculating by executing mathematical programming problems. In particular, the appropriate objective function in (5.1), (5.2), or (5.10) is optimized subject to the constraints in (4.4), (4.6), or (4.8), respectively.[49] For example,

[48] The data set is that used by Pasurka [2006], and a detailed description can be found in that paper.
[49] Recall that the BP approach involves decompositions of (in)efficiency indexes (see (5.3), (5.4), and (5.11)).

$E^1_{FGL}(x, y, z, T_1)$ is calculated by solving

$$\begin{aligned}
\min_{\beta,\lambda} \beta \quad \text{s.t.} \quad & \sum_{d=1}^{92} \lambda^d x_i^d \le x_i^{d'},\ i = 1, 2, 3, 4, 5, \\
& \sum_{d=1}^{92} \lambda^d y^d \ge y^{d'}/\beta \\
& \lambda^d \ge 0,\ d = 1, \dots, 92,
\end{aligned} \tag{6.1}$$

$E^2_{FGL}(x, y, z, T_2)$ is calculated by solving

$$\begin{aligned}
\min_{\gamma,\mu} \frac{\gamma_1 + \gamma_2}{2} \quad \text{s.t.} \quad & \sum_{d=1}^{92} \mu^d x_i^d \ge x_i^{d'},\ i = 3, 4, 5, \\
& \sum_{d=1}^{92} \mu^d z_k^d \le \gamma_k z_k^{d'},\ k = 1, 2, \\
& \mu^d \ge 0,\ d = 1, \dots, 92,
\end{aligned} \tag{6.2}$$

and $E_{FGL}(x, y, z, T_{BP})$ is obtained as the simple average of the two value functions.[50]

The results depicted in Table 1 underscore the sensitivity of the the DDF measure to the choice of the direction vector (illustrated above using Example 1). In our data set, the consequence of choosing $g = \mathbf{1}$ is that the DDF measure of inefficiency picks up the environmental inefficiency component as the overall measure for most DMUs. Table 1 reports the (in)efficiency scores of a sample of ten DMUs for the year 1985 under the BP approach. The magnitudes of the HYP efficiency figures for β_1 and β_2 for these firms are reasonably comparable (ranging from 0.7416 to 1.000 for β_1 and from 0.3052 to 1.000 for β_2), so that the operation $\beta = \max\{\beta_1, \beta_2\}$ is, in some sense, non-discriminatory in choosing between β_1 and β_2. The magnitudes of β_1 and β_2 for the DDF measure, however, are in orders ranging from 10^8 to 10^{10} and from 10^3 to 10^5, respectively, so that, except when $\beta_1 = 0$, the operation $\beta = \min\{\beta_1, \beta_2\}$ predominantly favors β_2 over β_1. Primarily for this reason we do not present further results for the DDF measure of inefficiency.

Table 2 contains the mean values of the HYP and FGL efficiency indexes for each year in our sample. Columns (1) and (2) pertain to the WD technology and Columns (3)–(8)

[50] We should note, however, that calculation of the HYP and FGL indexes on WD technologies employ the linear approximation used by Färe, Grosskopf, and Pasurka [1986] and much of the subsequent literature. Owing to the relatively large dimensionality of our data set, calculation of solutions to the nonlinear programs needed to calculate these indexes explicitly on WD technologies is impractical. In the case of the BP approach, however, the programs required to computations all three (in)efficiency indexes—HYP, DDF, and FGL—are linear and hence pose no such calculation problems.

pertain to the BP technology underlying our data set. The BP approach is our proposed method of constructing pollution-generating technologies and the FGL index is our proposed method of calculating efficiency on BP technologies.

Columns (1) and (2) and Columns (5) and (8) of Table 2 show that, under both the WD and BP approaches, the HYP index runs higher than the FGL index. As in Example 1, this comparison reflects the fact that the expansion/contraction to the frontier of the latter takes up all the slack in outputs, thus comparing the output quantity vector to a reference vector on the efficient frontier, whereas the expansion/contraction of the former leaves some slack, comparing the output quantity vector to a point on the frontier but not necessarily in its efficient subset.

Table 2 also indicates that, for our data set, both the HYP and FGL efficiency estimates are consistently higher for the WD technology than for the BP technology, a phenomenon that we explained above using Example 1. These differences in the efficiency scores across the BP and WD technologies suggest that, for both HYP and FGL measures, the reference points with respect to which efficiency is measured are different under the two approaches. In particular, in the FGL case, all the reference points are efficient, whereas for the HYP case, all are only weakly efficient. Thus, our results show that the sets of efficient and the sets of weakly efficient points differ across WD and BP technologies.

In the case of our particular data set, regardless of the index used, Table 2 also shows that the degree of inefficiency in the pollution technology T_2 is much larger than that in the intended-production technology T_1: apparently, the DMUs in our data set are less concerned about the environmental dimension of their production activities or environmental efficiency is more difficult to achieve.

The FGL index records greater pollution-generation inefficiency than does the HYP index. An obvious expanation could again be the differences in the way in which the two indexes treat slacks in outputs.[51]

Table 3 provides counts of weakly efficient and efficient firms using the HYP and FGL indexes, respectively, for the two technologies. Columns (1) and (6) and Columns (2) and (10) provide a comparison across WD and BP technological specifications of numbers of firms that receive an efficiency score of 1 under the HYP and FGL measures, respectively. The table shows that, for both the HYP and FGL indexes, the WD technological specification results in a larger number of firms receiving an efficiency score 1 than does the BP technological

[51] Note that the (output oriented) HYP and FGL indexes take the same values for the intended production technology T_1 because, with only a single intended output, they collapse to the same index.

specification. This seems consistent with the findings from Example 1: the frontier of the output possibility set is larger under the WD specification than under the BP specification. Hence, the probability of a DMU being assigned an efficiency value of 1 is greater under the WD approach than under the BP approach.

Columns (3)–(10) of Table 3 also help to compare the performance of FGL and HYP indexes under the BP approach. First, it is not surprising that the HYP index, which allows slack to remain in reference output vectors, judges at least as many DMUs to be efficient (environmentally, in intended production, and overall) as does the FGL measure. This comparison is indicated by comparing Column (3) with Column (7), Column (4) with Column (8), and Column (6) with Column (10).[52] Second, it follows that all the DMUs that are judged environmentally efficient by FGL are a subset of the DMUs judged environmentally efficient by HYP. Finally, as demonstrated by Example 1, the HYP index gives efficiency score 1 to DMUs that are efficient in intended outputs *or* are environmentally efficient *or* are both. Hence, Column (6) is obtained by adding Columns (3) and (4) and subtracting Column (5) from this sum. On the other hand, as also demonstrated by Example 1, FGL is more demanding in judging a DMU efficient: it gives efficiency score 1 to a DMU if and only if it is efficient both environmentally and in intended production. Thus, Column (10) is equal to Column (9).

Table 4 shows how the rankings of firms on the basis of their efficiency scores compare across the two efficiency indexes HYP and FGL, across the two technological specifications, and across the environmental and intended-production efficiency scores. Columns (1) and (2) of Table 4 show that, for both HYP and FGL, the Spearman correlation coefficients between the efficiency scores under the WD and BP approaches are moderately high and positive: the rank correlation coefficients lie in the range .5 to .71 and .66 to .89 for the HYP and FGL measures, respectively. In the light of the significant conceptual differences between the two approaches (in particular, the differences in the frontiers of the BP and WD technologies), which are reinforced strongly by our empirical findings above, the BP approach seems to make a larger difference in the levels than in the ranking of the efficiency scores of the DMUs.

Table 4 also allows comparison of rankings under the the HYP and FGL indexes applied to BP technologies. Given that in our data set there is only a single intended output, there are no differences in the efficiency scores for intended production obtained from the HYP and

[52] In particular, with respect to the intended production technology T_1, since there is only one intended output, there is no slack remaining in the reference vector when the HYP index gives a DMU an efficiency score of 1. Hence, Columns (3) and (7), are identical.

FGL measures. Hence, the Spearman correlation coefficients in Column (4) are all equal to 1. Our data set also exhibits high rank correlations between environmental efficiency scores obtained from the FGL and HYP measures: as seen in Column (5), the rank correlation coefficients lie in the range .87 to .99. Nevertheless, the rank correlation coefficients between overall efficiency scores obtained under FGL and HYP are on the lower side: as seen in Column 3, these lie in the range .42 to .72. This could be explained by the differences in the way HYP an FGL indexes aggregate over environmental and intended output efficiency scores. In Example 1, we saw that the HYP gives an efficiency score of 1 to a DMU that is environmentally efficient but not efficient in intended production or vice-versa. The FLG index, however, penalizes such DMUs for the slack in production of the intended or the unintended output and gives them a lower score. Thus, the strength of the association between the rankings of DMUs on the basis of their overall efficiency under the HYP and FGL measures is not clear: in our particular data set, the association is low.

Columns (7) and (8) of Table 4 show the rank correlation coefficients between efficiency scores in intended and unintended productions for the HYP and FGL indexes under the BP approach. These values are all negative and low; *e.g.,* the Spearman correlation coefficients range between -.08 to -.28 and -.01 to -.27 for the HYP and FGL indexes, respectively. Negative correlation values indicate that DMUs that are more efficient in intended production are also likely to be more environmentally inefficient, and vice-versa. This may suggest that the DMUs face some trade-offs between efficiency in intended production and in pollution generation. In our data set, however, these trade-offs are weak, as the correlation values are very low. Thus, one may conclude that most DMUs in our data set do not face significant trade-offs between intended production and residual generation and can improve simultaneously on both environmental and intended output efficiencies.

7. By-production versus weak disposability: Comparisons of DEA formulations in the presence of abatement efforts.

The WD approach explains the positive correlation between intended outputs and pollution through abatement efforts of firms that are not modeled. Hence, it considers only a reduced form of the overall technology in the space of inputs and all unintended and intended outputs other than the abatement output. In this section, we extend the DEA formulation of a BP technology to include abatement efforts made by firms and derive the DEA analogue of its reduced form defined in (3.23). With the help of an example, we then compare the reduced forms of the two technologies.

A DEA version of the BP technology in the presence of an abatement output is derived as follows: With respect to the intended technology T_1, abatement is a standard output that satisfies standard output free disposability. The residual-generating mechanism T_2, on the other hand, satisfies costly disposability of abatement output. Thus,

$$\begin{aligned} T_{BP} &= T_1 \cap T_2, \text{ where} \\ T_1 &= \Big\{ \langle x^1, x^2, y, y^a, z \rangle \in \mathbf{R}_+^{n_1+n_2+m+1+m'} \mid \lambda[X^1 \ X^2] \leq \langle x^1, \ x^2 \rangle, \ \lambda Y \geq y, \ \lambda A \geq y^a, \\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{for some } \lambda \in \mathbf{R}_+^p \Big\}, \text{ and} \\ T_2 &= \Big\{ \langle x^1, x^2, y, y^a, z \rangle \in \mathbf{R}_+^{n_1+n_2+m+1+m'} \mid \mu X^2 \geq x^2, \ \mu A \leq y^a, \ \mu Z \leq z, \\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{for some } \mu \in \mathbf{R}_+^p \Big\}, \end{aligned} \tag{7.1}$$

where A is the vector of abatement outputs for the p firms.

Holding all input quantities fixed at x, we next derive a DEA version of the reduced form of T_{BP}. Precisely, this is the projection of the output possibility set of T_{BP} (corresponding to input-quantity level x) defined in the $\langle z, y, y^a \rangle$ space into the $\langle z, y \rangle$ space.

Noting that technology T_1 is independent of z, the DEA construction of the projection of the output-possibility set for technology T_1 (corresponding to input level x) into the $\langle y^a, y \rangle$ space is denoted by $\hat{P}_1(x)$.[53] In a similar manner, noting that technology T_2 is independent of y, we define the DEA construction of the projection $\hat{P}_2(x)$ of T_2 into the $\langle y^a, z \rangle$ space.[54]

The DEA versions of the WD technology (see (4.4)) and the reduced form of T_{BP} in the $\langle z, y \rangle$ space, for a fixed level x of input quantities, are defined as follows:

$$\begin{aligned} \hat{P}_{BP}(x) &= \Big\{ \langle z, y \rangle \in \mathbf{R}_+^{m+m'} \mid \exists \ y^a \in \mathbf{R}_+ \text{ such that } \langle y^a, y \rangle \in \hat{P}_1(x) \ \wedge \ \langle y^a, z \rangle \in \hat{P}_2(x) \Big\} \\ \tilde{P}_{WD}(x) &= \Big\{ \langle z, y \rangle \in \mathbf{R}_+^{m+m'} \mid \langle x, y, z \rangle \in \tilde{T}_{WD} \Big\}. \end{aligned} \tag{7.2}$$

In Example 2 below, we compare $\hat{P}_{BP}(x)$ and $\tilde{P}_{WD}(x)$. It is assumed that $n_2 = 1, \ n_1 = 0, \ m = m' = 1$, and $x = 1$.

[53] This is the set of all combinations $\langle y^a, y \rangle$ that are possible with input level x for technology T_1.

[54] This is the set of all combinations $\langle y^a, z \rangle$ that are possible with input level x for technology T_2.

Example 2: $p = 8$. The (artificial) data are as follows:

$$
\begin{array}{ccccc}
\text{DMU} & x & y^a & y & z \\
1 & 1 & 0 & 8 & 9 \\
2 & 1 & 1 & 7 & 6 \\
3 & 1 & 2 & 6 & 8 \\
4 & 1 & 3 & 6 & 3 \\
5 & 1 & 4 & 1 & 2 \\
6 & 1 & 5 & 4 & 0 \\
7 & 1 & 6 & 2 & 0 \\
8 & 1 & 7 & 1 & 11
\end{array} \tag{7.3}
$$

After plotting the data, we find that $\hat{P}_1(1)$ and $\hat{P}_2(1)$ can be represented functionally by piece-wise linear functions:

$$
\begin{aligned}
\rho^1(y^a) &= 8 - \frac{2}{3}y^a, && y^a \in [0,3] \\
&= 9 - y^a, && y^a \in [3,5] \\
&= \frac{23}{2} - \frac{3}{2}y^a, && y^a \in [5,7]
\end{aligned} \tag{7.4}
$$

and

$$
\begin{aligned}
\rho^2(y^a) &= 9 - 3y^a, && y^a \in [0,1] \\
&= \frac{15}{2} - \frac{3}{2}y^a, && y^a \in [1,5] \\
&= 0, && y^a \geq 5.
\end{aligned} \tag{7.5}
$$

The sets $\hat{P}_1(1)$ and $\hat{P}_2(1)$ are shown in Panels 1 and 2 of Figure 2. (7.2) implies that $\hat{P}_{BP}(1)$ (shown in Panel 3 of Figure 2) is constructed as follows:

$$\hat{P}_{BP}(1) = \big\{ \langle z, y \rangle \in \mathbf{R}^2_+ |\ z \geq \rho^2(y^a) \ \wedge \ y \leq \rho^1(y^a) \ \wedge \ y^a \in [0,7] \big\}. \tag{7.6}$$

Note that the construction of $\hat{P}_{BP}(1)$ involves explicit reference to the abatement output[55] No reference was made, however, to data on y^a in the DEA construction of $\tilde{P}_{WD}(1)$ in Panel 4 of Figure 2.

Moreover, while weak disposability holds for $\tilde{P}_{WD}(1)$, the data are such that null jointness is violated. This can be rationalized by the fact that the abatement output of a firm can completely mitigate pollution even when it is producing positive amounts of the intended outputs.[56] Further, the boundary of $\tilde{P}_{WD}(1)$ has a negatively sloped region, indicating a negative correlation between intended and unintended outputs in that region. The frontier of $\hat{P}_{BP}(1)$, on the other hand, is everywhere non-negatively sloped.

[55] In particular, we have been able to express the frontier of $\hat{P}_{BP}(1)$ as a vector-valued function of y^a.

[56] This could be true, *e.g.,* in the presence of abatement activities such as recycling of wastes or if all wastes are biodegradable and can hence be completely eliminated.

8. Conclusions.

Pollution is an unintended output that cannot be freely disposed of. Underlying its production are a set of chemical and physical reactions that take place in nature when firms engage in the production of intended outputs. These natural reactions define nature's residual generation mechanism, which is a relation between the residuals generated and some inputs that are used or some intended outputs that are produced by the firm: hence, the inevitability of a certain minimal amount of pollution being generated when firms engage in intended production. We call this phenomenon by-production of pollution. The larger is the scale of intended production, the greater are the pollution-causing inputs being used or the greater are the pollution-causing intended outputs being produced, and hence, the more is the pollution generated. This provides the fundamental explanation for the positive correlation that is observed between intended production and residual generation.[57]

Standard approaches in the existing literature, on the other hand, usually attribute the observed positive correlation between pollution generation and intended production to resource-costly abatement options of firms. Such options, however, are not explicitly modeled, and only a reduced form of the technology is considered. Pollution is either treated as an input satisfying standard input free disposability or is considered as an output that is weakly disposable.

To capture the phenomenon of by-production, we model pollution-generating technologies as a composition of two technologies: an intended-production technology and a residual-generation technology. The former describes how inputs are transformed into intended outputs, is assumed to be independent of the level of pollution, and satisfies standard free-disposability properties.[58] The latter reflects nature's residual generation, violates standard disposability properties with respect to goods that result in (affect) pollution generation, and exhibits costly disposability with respect to pollution. As a result, the overall technology violates standard disposability with respect to inputs that cause (affect) pollution generation and exhibits costly disposability with respect to pollution. In these ways, the technology we propose is substantially different from the standard input and output approaches to modeling pollution-generating technologies.

[57] Some of the literature has adopted physical science terminology to describe these relationships in terms of the "material balance" condition (see Ayres and Kneese [1969] and, more recently, Coelli, Lauwers, and van Huylenbroeck [2007].

[58] See Murty [2010b] for a generalization where pollution can affect intended production, *e.g.,* by deteriorating the quality of the labor input.

We formulate DEA specifications of technologies that satisfy by-production, with or without pollution-abatement activities, and employ them to measure technical efficiency of firms. In the context of by-production, standard measures of efficiency decompose very naturally into environmental and intended output efficiencies. However, we find that, in the context of by-production, the commonly used indexes of (in)efficiency, the hyperbolic and the directional distance function-based index, overstate efficiency. In the existing set of (in)efficiency indexes proposed in the literature, we find that a modification of an index proposed by Färe, Grosskopf, and Lovell [1994] corrects the flaws in the hyperbolic and directional distance function indexes for measurement of efficiency for by-production technologies. A comparison of the values of this index with those of the hyperbolic and directional distance indexes, using a database for electric power firms, supports our arguments about the inadequacies of the latter.

APPENDIX

Implicit function theorem: Let $f : \mathbf{R}^n_+ \times \mathbf{R}^m_+ \to \mathbf{R}^m$ be a continuously differentiable vector valued function with image $f(x, y) = z$, where $x \in \mathbf{R}^n_+$ and $y \in \mathbf{R}^m_+$. Let $\langle \bar{x}, \bar{y} \rangle \in \mathbf{R}^{n+m}_+$ be such that $f(\bar{x}, \bar{y}) = 0$ and the $m \times m$ matrix $\nabla_y f(\bar{x}, \bar{y})$ is full-row ranked (has a non-zero determinant). Then there exist neighborhoods U and V around $\bar{x}$ and $\bar{y}$ in $\mathbf{R}^n_+$ and $\mathbf{R}^m_+$, respectively, and a continuously differentiable function $\Phi : U \to V$ with image $\Phi(x) = y$ such that, for all $x \in U$, we have $f(x, \Phi(x)) = 0$ and

$$\nabla_x \Phi(x) = - \left[\nabla_y f(x, \Phi(x))\right]^{-1} \nabla_x f(x, \Phi(x)).$$

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TABLES

Table 1: HYP and DDF (in)efficiency indexes for BP technology.

HYP β_1	HYP β_2	HYP $\max\{\beta_1, \beta_2\}$	DDF β_1	DDF β_2	DDF $\min\{\beta_1, \beta_2\}$
1	0.3425	β_1	0	43996.691	β_1
1	0.3052	β_1	0	46160.821	β_1
0.9024	0.4192	β_1	377628130	12283.274	β_2
0.8440	1	β_2	256197050	0	β_2
0.8412	0.6914	β_1	1226089000	7287.4051	β_2
0.7734	0.4540	β_1	304909890	3174.1978	β_2
0.8191	0.8304	β_2	89913470	252.83995	β_2
0.8909	0.5577	β_1	127072340	1888.807	β_2
0.8754	0.9262	β_2	511323320	814.31455	β_2
0.7416	0.5194	β_1	125872500	933.20682	β_2

Notes: Results in this table pertain to a sample of 10 DMUs for the year 1985. The direction vector employed for computing DDF is $g = \mathbf{1}$.

Table 2: Mean efficiency values.

	WD technology		BP technology					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Year	HYP	FGL	HYP1	HYP2	HYP	FGL1	FGL2	FGL
1985	.94	.78	.89	.64	.90	.89	.52	.70
1986	.94	.78	.87	.62	.88	.87	.49	.68
1987	.95	.79	.90	.65	.92	.90	.54	.72
1988	.95	.81	.88	.63	.90	.88	.60	.74
1989	.95	.82	.90	.63	.92	.90	.60	.75
1990	.94	.82	.88	.62	.91	.88	.59	.74
1991	.95	.80	.89	.59	91	.89	.54	.71
1992	.95	.79	.89	.58	.91	.89	.53	.71
1993	.95	.79	.89	.60	.91	.89	.54	.72
1994	.94	.77	.88	.60	.90	.88	.56	.72
1995	.91	.74	.80	.61	.84	.80	.55	.68

Table 3: Counts of efficient DMUs.

	WD technology		BP technology							
	HYP	FGL	HYP				FGL			
Year	(1) $\beta = 1$	(2) $\beta = 1$	(3) $\beta_1 = 1$	(4) $\beta_2 = 1$	(5) $\beta_1 = 1$ $\beta_2 = 1$	(6) $\beta = 1$	(7) $\beta_1 = 1$	(8) $\beta_2 = 1$	(9) $\beta_1 = 1$ $\beta_2 = 1$	(10) $\beta = 1$
1985	35	3	9	9	1	17	9	4	0	0
1986	36	3	5	6	1	10	5	4	0	0
1987	43	3	12	10	1	11	12	6	0	0
1988	41	7	8	8	0	16	8	5	0	0
1989	41	6	9	11	1	19	9	9	0	0
1990	36	6	7	11	0	18	7	8	0	0
1991	39	4	8	10	2	16	8	7	1	1
1992	38	5	10	8	1	17	10	7	1	1
1993	44	5	7	7	0	14	7	5	0	0
1994	43	3	6	6	0	12	6	5	0	0
1995	34	3	9	9	0	18	9	5	0	0

Table 4: Spearman rank correlation coefficients among efficiency indexes.

	Across BP and WD technologies		Within BP technology				
			ρ(HYP,FGL)			$\rho(\beta_1, \beta_2)$	
Year	(1) HYP	(2) FGL	(3) β	(4) β_1	(5) β_2	(6) HYP	(7) FGL
1985	.71	.82	.60	1.00	.89	-.08	-.01
1986	.70	.89	.53	1.00	.87	-.12	-.09
1987	.60	.78	.54	1.00	.91	-.13	-.12
1988	.60	.77	.42	1.00	.97	-.23	-.23
1989	.63	.66	.45	1.00	.99	-.28	-.27
1990	.58	.71	.50	1.00	.98	-.24	-.24
1991	.52	.79	.46	1.00	.96	-.20	-.17
1992	.57	.87	.43	1.00	.94	-.21	-.13
1993	.50	.82	.42	1.00	.94	-.18	-.18
1994	.54	.76	.47	1.00	.96	-.13	-.16
1995	.59	.78	.72	1.00	.96	-.18	-.14

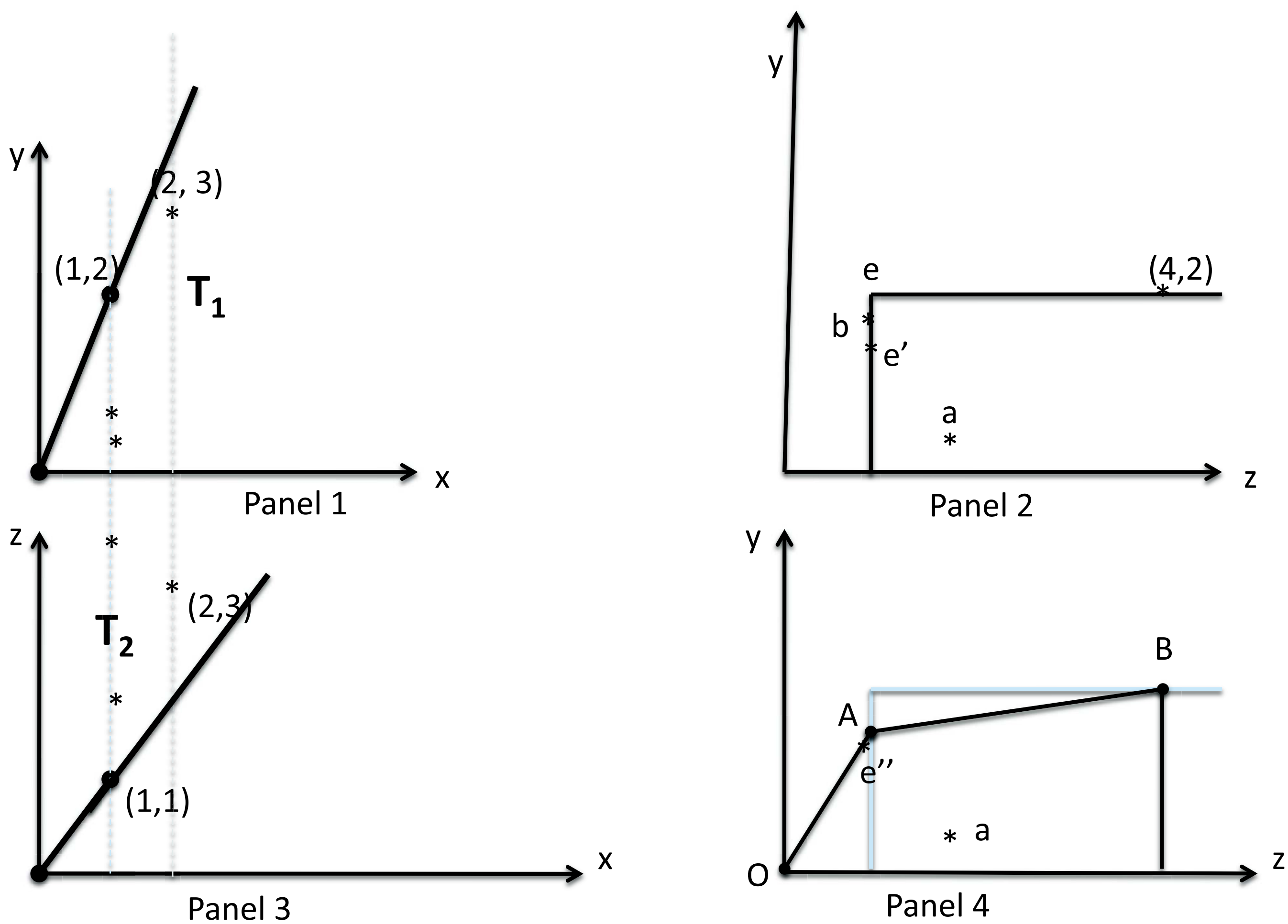
y
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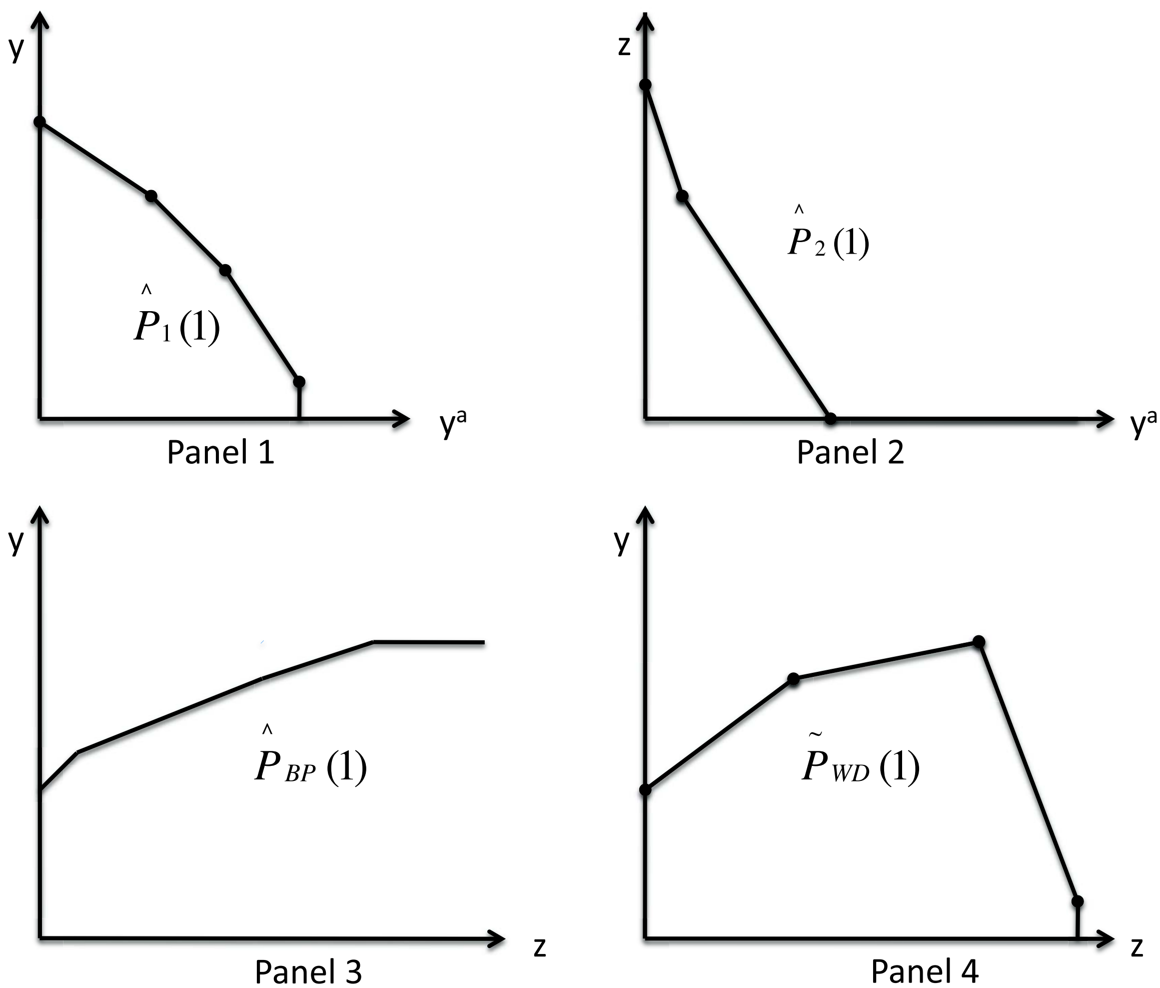
Figure 1


y
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Panel 3
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$\tilde{P}_{WD}(1)$
z
Panel 4

Figure 2