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Abstract

We consider a model where a judgment-proof firm needs finance to realize a project. This project might cause an environmental hazard with a probability that is the private knowledge of the firm. Thus there is asymmetric information with respect to the environmental riskiness of the project. We consider the implications of a simple joint and strict liability rule on the lender and the firm where, in case of a damage, the lender is responsible for that part of the liability which the judgment-proof firm cannot pay. We use a weighted version of the neutral bargaining solution (Myerson 1983 / 1984) to determine the financial contract between the lender and the firm. In the given model we show that either full or a punitive liability is optimal.

Keywords: judgement proofness, extended liability, neutral bargaining solution.

JEL *classification number*: K13, K32, Q38, G33.

1 Introduction

Liability rules are an important element in many systems of law. Liability laws are appealing to practitioners because they are easy and seemingly costless to implement. However, their impact on the incentives of economic agents remains

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much a matter of debate, in particular when one of the parties concerned can be judgement-proof. A firm becomes judgment proof if the damage costs of an environmental accident caused by the firm exceeds her own capital base. For such cases many countries considered the introduction of extended liability where lenders to such firms can be made liable for the residual damage costs which the firm cannot pay (see Boyer and Laffont (1997)). Starting with Pitchford (1995), Heyes (1996) and Boyer and Laffont (1997) a growing literature has studied the incentive effects of extended liability. Pitchford (1995) and Boyer and Laffont (1997) study the impact on the incentives of firms to prevent accidents. Boyer and Laffont (1997) also consider the role of private information with respect to the profitability of projects. Heyes (1996) studies a model with adverse selection concerning the environmental riskiness of projects and with moral hazard in regard to the efforts firms invest to prevent accidents. Boyer and Laffont as well as Hayes assume that the lender has all the bargaining power when selecting the lending contract. However, this is at odds with much of the finance literature where lenders are typically assumed to make zero profits. As shown in Balkenborg (2001) assumptions about the bargaining power a lender has vis-a-vis a firm when bargaining on a loan contract can be crucial when evaluating the impact of extended liability.

This paper determines the optimal extended liability rule for the adverse selection problem that arises when the firm has private information on the accident probability of the project she seeks to get financed. We will use the same base model as used in Balkenborg (2001) to study moral hazard, except that the probability of an environmental accident does not depend on the effort of the firm, but is exogenously given and known only to the firm.

The comparative statics result for the moral hazard model and for our adverse selection model could not be more different. A joint analysis of the moral hazard and the adverse selection problem in a single model promises hence to be rather subtle and is not attempted here.

The first difference concerns the question whether social welfare is higher when the lender or when the firm has all the bargaining power. In the moral hazard model the accident probability is decreasing in the bargaining power of the lender. A monopoly lender is the worst-case scenario because a monopoly lender extracting all the surplus from the project is bad for the incentives of the firm to take care (see Balkenborg (2001), Shavell (1997)).

In contrast, only a monopoly lender guarantees a first-best outcome in the adverse selection case. In our model, a monopoly lender is able to extract all the surplus, regardless of the private knowledge of the firm. Hence a firm who knows that the accident probability of her project is high cannot gain by pretending to have a low accident probability. Therefore high-risk projects do not jeopardize low-risk projects, the potential distortions due to adverse selection do not arise and first best can be achieved. As soon as the firm can capture some of the surplus, adverse selection causes distortions away form first best.

The second difference concerns the optimal joint liability. If the firm has all the bargaining power it is in the moral hazard model never socially optimal to use the deep pockets of the lender. The lender should contribute zero to the damage costs (see Pitchford (1995)). In the adverse selection model, however, not only should the lender contribute to the damage costs, but the overall joint liability imposed on firm and lender should typically exceed the actual damage costs (the joint liability should be "punitive"). Conversely, when the lender has all the bargaining power, the optimal liability is punitive in the moral hazard model but full joint liability (i.e., a joint liability equal to the actual damage costs) is optimal in the adverse selection case.

The distortion in our adverse selection model is, in essence, due to signalling. Bargaining should settle on a contract where it is not possible for low-risk types of the firm to propose an alternative contract that harms high-risk types and makes low-risk types of the firm and the lender better off. Such alternative contracts, if offered, would not lead the lender to conclude that he is facing a high-risk type and would hence be accepted. This would destabilize the initial agreement. Considerations of this type lead in our model to the selection of the contract that maximizes the expected payoff of the type of firm with the lowest accident probability subject to the constraint that the lender gets a fixed share of the maximal joint surplus.[1] This contract is shown to be typically an option contract with three different options.[2]

1. The first option is to run the project and give a high share of the surplus to the firm when no accident occurs and nothing when an accident occurs. This option favors low-risk types of the firm. In equilibrium it is taken up by those firms for whom the project promises (after taking account of the joint liability) a surplus in expectation.

2. The second option is a compensation payment to prevent firms with a high accident probability from running the project. This is necessary here because firms are assumed to have no own wealth and can hence only gain from running the project.[3] In equilibrium, this option is taken by the high-risk types of the firm for whom the project would yield a loss in expectation.

3. There is a third option where the project is only run with a small probability. The third option is for the medium-risk types of the firm for whom the project yields a loss in expectation but where it is cheaper to have them run

[1] I skip here the case of a joint liability that the firm could pay on its own.

[2] In the formal model we work with "direct mechanisms" where the firm first announces the accident probability of her project and a neutral mediator then selects the option.

[3] To have such "bribes" in a loan contract may seem odd at first, but all it means is that the lender finances a nice new office and a rather decent salary to the owner / manager that does not have to be paid back if the firm later (after some further costly "research") decides to withdraw from the project.

the project than to pay them a compensation for staying out of business. They would have to be given a higher compensation than the high-risk types. However, this higher compensation, if offered, would be taken by all high-risk types as well. The total expected payment for bribes could become so high that the surplus available for the low-risks or the lender would have to be reduced.

It is the third option and the medium-risk types which create the distortion in our model.[4] Suppose the joint liability to the firm and the lender is set equal to the actual damage costs. Then the low-risk types are those producing a social surplus in expectation and they run the project. For the high-risk types the project would yield a social loss, but they take the compensation. All this is first-best. However, for the medium types the project also yields a social loss and for them the project is run with positive probability. This would not happen if the accident probability were public information.

In Balkenborg (2001) we used a weighted version of the Nash bargaining solution (Nash (1950), Myerson (1991)) to solve the bargaining stage of the model. In this paper we have to analyze a bargaining problem with incomplete information to which the Nash bargaining solution does not apply. In particular, when the firm has all the bargaining power we have a bargaining problem with an informed principal (see Myerson (1983), Maskin and Tirole (1990) and Maskin and Tirole (1992)). In this paper we use a weighted version of the neutral bargaining solution (Myerson (1983), Myerson (1984)) to solve the bargaining stage, primarily because it is the most generally applicable solution concepts for such problems. Technically, the determination of this solution is the main contribution of the paper. The relation to other approaches is considered in Subsection 2.

Section 2 introduces the model and the notations. Section 3 determines the weighted neutral bargaining solutions and compares it with other approaches. Comparative statics results are given in Section 4. In the conclusions in Section 5 we discuss some limitations of our approach. The appendix contains most of the proofs.

2 The model

A wealth-constrained risk-neutral firm with no own wealth would like to run an environmentally risky project. This project requires an initial investment of size K and would yield a gross profit $v + K$. The net value of the project is hence v. Since the firm has no own wealth she needs a loan to run the project. A risk

[4]The precise classification into low-risk, medium-risk and high risk types is part of the definition of the optimal contract and hence dependent on the joint liability.

neutral lender with large but finite wealth $X > 0$ would be able to finance the project.

As stated, the project is environmentally risky. With probability p the project might cause an environmental accident. An accident would be a pure externality causing damage costs $h > v$ on potential victims. We assume that the wealth of the lender plus the surplus generated by the project would be more than enough to pay for the total damage costs, so $v + X > h > v$.

The accident will not directly affect the firm or the lender unless some form of liability is imposed. We study here the effects of a *joint and strict liability* $c \geq 0$. By this we mean that, if an accident occurs, the firm and the lender are liable with all their joint wealth for the amount c. First the firm is liable with all her wealth up to the amount c. If the firm cannot cover the full amount c, the lender has to pay for the remainder. Effectively the joint liability cannot exceed $v + X$, the total amount of cash available. Hence we assume $c \leq v + X$ in the following.

In this paper we study the case of adverse selection with respect to the accident probability $0 \leq p \leq 1$. Thus the accident probability p is the private knowledge of the firm. Firms with different accident probabilities correspond to different *types* of the firm. We consider here the case of finitely many types

$$0 \leq p_0 < p_1 < \cdots < p_t < \cdots < p_T \leq 1$$

and denote by $0 < q_t \leq 1$ the ex-ante probability for the firm to be of type p_t. Sometimes we refer to the index t rather than the probability p_t as "the type" of the firm.

We assume either that there are only three types or that the following *monotone hazard rate* condition is satisfied for $t < T$:[5]

$$\frac{(p_{t+1} - p_t) \sum_{\tau=t+1}^{T} q_\tau}{q_t} \text{ is decreasing in } t. \tag{1}$$

We consider here the case of a bilateral monopoly with a single firm and a single lender. Admittedly, this analysis does not immediately carry over to a scenario where several firms compete for a loan and / or where several lenders compete to finance the project. The timing of the model is as follows:

1. The social planner announces the liability $0 \leq c \leq v + X$.

2. Nature selects with probability q_t the level of safety $p = p_t$ of the project which is then revealed to the firm.

[5]Notice that if the discrete distribution approximates a differentiable cdf $F(p)$ with density $f(p)$ then $\frac{1-F(p)}{f(p)} = \frac{(p_{t+1}-p_t)\sum_{\tau=t+1}^{T} q_\tau}{q_t}$. Thus our monotone hazard rate condition is the familiar one from the literature (see, e.g., Laffont and Martimort (2002)). It is made to ensure that it suffices to check the "local" incentive constraints (type t does not want to imitate type $t+1$ or $t-1$) in order to prove overall incentive compatibility. It turns out that we need this assumption only when there are four types or more.

3. The lender and the firm bargain over a mechanism. Following the revelation principle (Myerson (1979)), we assume that direct mechanisms are used.

4. If a mechanism is agreed upon, the firm announces a type $\hat{p}$ which, of course, does not have to be her true type.

5. The direct mechanism determines the outcome conditional on the type $\hat{p}$ announced by the firm. If the project is run, an accident occurs with probability p. If an accident occurs, the joint and strict liability c must be paid.

In our case a direct mechanism can be described by a 4-tuple of functions[6]

$$\mu = (\mu(\hat{p}))_{0\leq\hat{p}\leq1} = \left(Q(\hat{p}), w^{+}(\hat{p}), w^{-}(\hat{p}), w^{0}(\hat{p})\right)_{0\leq\hat{p}\leq1}$$

If the firm announces to be of type $\hat{p}$, i.e. if she claims that her project has accident probability $\hat{p}$, then $Q(\hat{p})$ is the probability with which the project is undertaken.

If, with probability $1 - Q(\hat{p})$, the project is *not* undertaken, the expected final wealth of the firm is $w^{0}(\hat{p})$. The lender loses this amount.

If, with probability $Q(\hat{p})$, the project is undertaken, then the firm's expected final wealth is $w^{+}(\hat{p})$ if no accident occurs and $w^{-}(\hat{p})$ if an accident occurs. The lender gains $v - w^{+}(\hat{p})$ or $v - c - w^{-}(\hat{p})$, respectively.

The firm can only end up with a non-negative wealth, i.e. $0 \leq w^{+}(\hat{p}), w^{-}(\hat{p}), w^{0}(\hat{p})$ must hold. Because the project generates at most the amount of capital v and since the lender's own wealth is X, we must overall impose the restrictions $0 \leq Q(\hat{p}) \leq 1$, $0 \leq w^{+}(\hat{p}) \leq v + X$, $0 \leq w^{-}(\hat{p}) \leq v + X - c$ and $0 \leq w^{0}(\hat{p}) \leq X$ on the mechanism.

Notice that the mechanism can be, but does not have to be, interpreted as menu of wage-contracts.

Let

$$U_{f}^{*}(\mu, \hat{p}|p) = (1 - Q(\hat{p}))\, w^{0}(\hat{q}) + Q(\hat{p})\left[(1-p)\, w^{+}(\hat{p}) + p w^{-}(\hat{p})\right]$$

denote the expected profit of the firm from the direct mechanism μ if she is of type p and announces to be of type $\hat{p}$. Let

$$U_{f}(\mu|p) = U_{f}^{*}(\mu, p|p)$$

denote her expected utility from truthfully announcing her type. If all types of the firm announce her type truthfully the expected profit to the lender is, conditional on the firm being of type p,

$$U_{l}(\mu|p) = Q(p)(v - pc) - U_{f}(\mu|p)$$

[6]Without loss of generality we can assume, whenever convenient, that the mechanism is defined for all accident probabilities $0 \leq p \leq 1$. See Footnote 8 below.

and overall

$$U_l(\mu) = \sum_{t=0}^{T} U_l(\mu|p_t)\, q_t$$

A mechanism is called *incentive compatible* if it satisfies for all p, $\hat{p} \in [0,1]$ the *incentive constraint*

$$U_f(\mu|p) \geq U_f(\mu, \hat{p}|p)$$

We assume that the reservation utility of the lender and of each type of the firm is zero. To be *individually rational* the mechanism must hence satisfy the *participation constraint* $U_l(\mu) \geq 0$ for the lender. Because the firm has no own wealth the participation constraint $U_f(\mu|p) \geq 0$ is automatically satisfied when the non-negativity constraints $0 \leq w^+(\hat{p}), w^-(\hat{p}), w^0(\hat{p})$ hold. We call a mechanism *feasible* if it is individually rational and incentive compatible.

It will be important to distinguish three types of interim efficiency. A mechanism μ is *interim efficient* (among all types of all players) if it is feasible and if there exists no feasible mechanism ν such that $U_l(\nu) \geq U_l(\mu)$ and such that $U_f(\nu|p_t) \geq U_f(\mu|p_t)$ holds for all $0 \leq t \leq T$ whereby at least one of these inequalities is strict. μ is *interim efficient for the firm* if it is feasible and if there exists no feasible mechanism ν such that $U_f(\nu|p_t) \geq U_f(\mu|p_t)$ holds for all $0 \leq t \leq T$ whereby at least one of these inequalities is strict. In our model $U_l(\mu) = 0$ must hold for all mechanisms μ that are interim efficient for the firm. Finally, a mechanism is *interim efficient for the lender* if it maximizes the profit of the lender among all feasible mechanisms.[7] In our model, mechanisms that are interim efficient for the firm or the lender are also overall interim efficient.

When setting the joint liability c the social planner wants to maximize the utilitarian social welfare

$$\sum_{t=0}^{T} q_t Q(p_t)(v - p_t h)$$

where $Q(p_t)$ is the probability that the project is run given the firm's type and given the direct mechanism chosen by the lender and the firm conditional on the joint liability being c. In our model, the first-best level of social welfare that can be achieved is

$$\sum_{t=0}^{T} q_t (v - p_t h)^+$$

where we make use of the notation

$$(a)^+ = \begin{cases} a & \text{for} \quad a > 0 \\ 0 & \text{for} \quad a \leq 0 \end{cases}.$$

[7] There is only one type of lender and hence no need to compromise between different types.

3 The Weighted Neutral Bargaining Solution

We start with a basic observation on incentive compatible (but not necessarily individually rational) mechanisms.[8]

Lemma 1 *The following holds for any incentive compatible mechanism* μ

a) The inequality

$$-Q(p)\left(w^{+}(p)-w^{-}(p)\right)\geq\frac{U_f(\mu|p)-U_f(\mu|\hat{p})}{p-\hat{p}}\geq-Q(\hat{p})\left(w^{+}(\hat{p})-w^{-}(\hat{p})\right)$$

is satisfied for any $0\leq\hat{p}<p\leq1$.

b) The firm's type contingent payoff $U_f(\mu|p)$ *is convex in* p, *i.e.*

$$\frac{U_f(\mu|p')-U_f(\mu|p)}{p'-p}\leq\frac{U_f(\mu|p'')-U_f(\mu|p')}{p''-p'}$$

is satisfied for all $0\leq p<p'<p''\leq1$.

c) The inequality

$$U_f(\mu|p)\geq\frac{1-p}{1-\hat{p}}U_f(\mu|\hat{p})$$

is satisfied for $0\leq\hat{p}<p\leq1$ *while the inequality*

$$U_f(\mu|p)\geq\frac{p}{\hat{p}}U_f(\mu|\hat{p})$$

is satisfied for $0\leq p<\hat{p}\leq1$.

Proof. ad a and b) From the definition of U_f^* and U_f we obtain for any $p\neq\hat{p}$

$$\begin{aligned}&U_f^*(\mu,\hat{p}|p)-U_f(\mu|\hat{p})\\=\;&(1-Q(\hat{p}))\,w^0(\hat{q})+Q(\hat{p})\left[(1-p)\,w^{+}(\hat{p})+pw^{-}(\hat{p})\right]\\&-\left((1-Q(\hat{p}))\,w^0(\hat{q})+Q(\hat{p})\left[(1-\hat{p})\,w^{+}(\hat{p})+\hat{p}w^{-}(\hat{p})\right]\right)\\=\;&-Q(\hat{p})(p-\hat{p})\left(w^{+}(\hat{p})-w^{-}(\hat{p})\right)\end{aligned}$$

and hence, since μ is incentive compatible,

$$U_f(\mu|p)-U_f(\mu|\hat{p})\geq-Q(\hat{p})(p-\hat{p})\left(w^{+}(\hat{p})-w^{-}(\hat{p})\right)\tag{2}$$

Interchanging the roles of p and $\hat{p}$ we get

$$U_f(\mu|\hat{p})-U_f(\mu|p)\geq-Q(p)(\hat{p}-p)\left(w^{+}(p)-w^{-}(p)\right)$$

[8] Strictly speaking, the mechanism has only to be defined and to be incentive compatible with respect to the values of p in the support of the distribution of types. However, one can always extend an incentive compatible mechanism μ defined only for the types $p_0,\cdots,p_T$ to an incentive compatible mechanism μ' defined for all types $0\leq p\leq1$ by choosing for any $0\leq p\leq1$ $\mu'(p)=\mu(\hat{p}_t)$ such that $U_f^*(\mu,\hat{p}_t|p)=\max_{0\leq t\leq T}U_f^*(\mu,p_t|p)$.

or

$$U_f(\mu|p) - U_f(\mu|\hat{p}) \leq -Q(p)(p-\hat{p})\left(w^+(p) - w^-(p)\right) \qquad (3)$$

Hence a) follows for $\hat{p} < p$ and then immediately b) follows for $p < p' < p''$.

ad c) The definition of U_f and the nonnegativity constraints on the mechanism yield $Q(\hat{p})(1-\hat{p})w^+(\hat{p}) \leq U_f(\mu|\hat{p})$. From Inequality (2) we obtain for $p > \hat{p}$

$$U_f(\mu|p) - U_f(\mu|\hat{p}) \geq -Q(\hat{p})(p-\hat{p})w^+(\hat{p}) \geq -\frac{p-\hat{p}}{1-\hat{p}}U_f(\mu|\hat{p})$$

and $U_f(\mu|p) \geq \frac{1-p}{1-\hat{p}}U_f(\mu|\hat{p})$. Symmetrically $Q(\hat{p})\hat{p}w^-(\hat{p}) \leq U_f(\mu|\hat{p})$ and Inequality (3) imply $U_1(\mu|p) \geq \frac{p}{\hat{p}}U_1(\mu|\hat{p})$ for $p < \hat{p}$. ■

Given any function $f(p) \geq 0$ that has all the properties described for the function $U_f(\mu|p)$ in the lemma, it is not difficult to construct an incentive compatible mechanism μ with $U_f(\mu|p) = f(p)$ for all $0 \leq p \leq 1$. Of course, the mechanism may violate the participation constraint of the lender. Still, one can construct a plethora of interim efficient mechanism where $U_f(\mu|p)$ is U-shaped or increasing or constant in the accident probability p. In particular, let $S(c) = \sum_{t=0}^{T} q_t (v - p_t c)^+$ denote the ex-ante maximally available surplus for the firm and the lender. Then the mechanism $\mu = (Q, w^+, w^-, w^0)$ defined by $Q(\hat{p}) = 1$ for $\hat{p} < v/c$, $Q(\hat{p}) = 0$ for $\hat{p} \geq v/c$, $w^+(\hat{p}) = w^-(\hat{p}) = w^0(\hat{p}) = S(c)$ for all $0 \leq \hat{p} \leq 1$ is interim efficient and distributes the gain $S(c)$ equally among all types. Mechanism like these are implausible because they do not reward productive types more than unproductive types.

To rule out such mechanisms and to select among the many interim efficient mechanisms we use the concept of the neutral bargaining solution, due to Myerson (1983) and Myerson (1984). Myerson's concept is an extension of the Nash bargaining solution to bargaining problems with incomplete information. It is based on an axiomatic approach and uses elements of both cooperative and noncooperative game theory. It is the solution concept with the smallest solution sets satisfying the axioms described below. We use here a weighted version depending on a parameter $0 \leq \alpha \leq 1$ which we interpret as the *bargaining power* of the lender. The firm has bargaining power $1-\alpha$. The case $\alpha = 0$ corresponds to the case where the firm has all the bargaining power and can essentially make a take-it-or-leave-it offer to the lender. The neutral bargaining solution for this case is developed in Myerson (1983). $\alpha = 0.5$ is the case of equal bargaining power discussed in Myerson (1984).

After we have described in this section the neutral bargaining solution and the mechanisms it selects in our model, we will compare it to other refinement approaches. I will take quite some freedom in describing the neutral bargaining solution and its axiom, partly to give an alternative exposition of the ideas and partly because some rephrasing is needed to fit with the model here. Sometimes

I have to be a bit vague because I do not want to develop Myerson's general framework. Readers familiar with Myerson's papers will not find it difficult to verify the equivalence with his original formulations.

3.1 Efficiency

Just as Nash required his bargaining solution to be Pareto-efficient, Myerson requires that *the neutral bargaining solution is interim efficient* as an axiom. If we expect the bargaining to end efficiently and if we assume that the firm knows her types when bargaining, this is clearly the right notion of efficiency. However, even if bargaining occurs ex-ante, before the firms learns her type, interim efficiency seems appropriate if there is the possibility to renegotiate.

3.2 Strong Solutions and Random Dictatorship

a) Strong solutions for the firm are of interest if the firm has all the bargaining power. A *strong solution* for the firm is a mechanism μ with the following three properties:

1. The payoff to the lender from the mechanism neither depends on the type announced by the firm nor on her true type: $U_l(\mu) = U_l^*(\mu, \hat{p}|p)$ for all $0 \leq p, \hat{p} \leq 1$. At a strong solution, asymmetric information is not a problem.

2. The lender's expected payoff is zero, as we would expect it if the firm has all the bargaining power: $U_l(\mu) = 0$.

3. The mechanism is interim efficient for the firm.

There is general agreement that the strong solution, if it exists, is the appropriate solution concept if the lender has all the bargaining power. It is an interim efficient Rothschild-Stiglitz-Wilson allocation relative to the initial endowment and yields hence the unique perfect Bayesian equilibrium outcome in the three-stage game where the informed principal can make a take-it-or-leave-it offer to select among direct mechanisms (see Maskin and Tirole (1992), in particular Section 9). It is consistent with many equilibrium refinements for this game (see Myerson (1983), Theorem 1 and Maskin and Tirole (1992), Proposition 7).

The next axiom imposed by Myerson is hence: *A strong solution for the firm is the neutral bargaining solution if the firm has all the bargaining power.*

If the joint liability c does not exceed the net value of the project, our model has a strong solution. Namely, the firm always pays for the liability herself and pays exactly the investment costs K back to the lender. The lender makes zero profit and his deep pockets are not employed for liability payments. Formally,

Proposition 2 *If $c \leq v$, the following mechanism $\mu_0 = \left(Q_0, w_0^+, w_0^-, w_0^0\right)$ is a strong solution for the firm and hence the neutral bargaining solution for the firm: $Q_0(\hat{p}) = 1$, $w_0^+(\hat{p}) = v$, $w_0^-(\hat{p}) = v - c$ for all $0 \leq \hat{p} \leq 1$.*

b) Because there is only one type of lender, the strong solution for the lender (and hence the neutral bargaining solution if the lender has all the bargaining power) is the mechanism which maximizes his expected payoff among all feasible mechanisms. In our model it is the following simple mechanism where the lender appropriates all surplus.

Proposition 3 *The strong solution for the lender (and hence the neutral bargaining solution if the lender has all the bargaining power) is the mechanism $\mu_1 = \left(Q_1, w_1^+, w_1^-, w_1^0\right)$ defined by $Q_1(\hat{p}) = 1$ if $v - \hat{p}c > 0$, $Q_1(\hat{p}) = 0$ if $v - \hat{p}c < 0$, $w_1^+(\hat{p}) = w_1^-(\hat{p}) = w_1^0(\hat{p}) = 0$ for $0 \leq \hat{p} \leq 1$.*

c) For intermediate bargaining power it is not clear what a "strong solution" should be. Myerson (1984) avoids the problem with his *"random dictatorship" axiom*, which extends to arbitrary bargaining power $0 \leq \alpha \leq 1$ as follows:

If μ_0 is a strong solution for the firm and μ_1 a strong solution for the lender, then the randomized mechanism $(1-\alpha)\mu_0 + \alpha\mu_1$, where μ_0 is played with probability $1-\alpha$ and μ_1 with probability α, is, if interim efficient, the weighted neutral bargaining solution for the bargaining power α of the lender.

It follows immediately

Proposition 4 *If $c \leq v$ the following mechanism $\mu_\alpha = (Q_\alpha, w_\alpha^+, w_\alpha^-, w_\alpha^0)$ is a neutral bargaining solution when the lender has bargaining power $0 \leq \alpha \leq 1$: $Q_\alpha(\hat{p}) = 1$, $w_\alpha^+(\hat{p}) = (1-\alpha)v$ and $w_\alpha^-(\hat{p}) = (1-\alpha)(v-c)$.*

Thus the project is always run and the proceeds, whatever they are, are divided according to the bargaining power.

3.3 Extended Models and Limits

When the joint liability c exceeds the net value v of the project, a strong solution for the firm does not exist for our model. When strong solutions do not exist, Myerson's idea is to look at extensions of the model for which a strong solution exists. He shows that there exists always an interim efficient mechanism μ of the original model for which one can find a sequence of extended models with strong solutions that approach in the limit the given mechanism "from below". A neutral bargaining solution for $\alpha = 0$ is any mechanism that can be approached in this way.

It suffices to consider here only extensions of our model where a "safe project" D is added that always gives the lender zero regardless of the type of firm, $u_l(D|p) = 0$ for all $0 \leq p \leq 1$, and where the payoff to the firm $u_f(D|p)$ can be dependent on the type, but not on any announcements of types. D is thus trivially a feasible mechanism. Provided it is interim efficient, it is a strong solution. A general mechanism of the extended model is in our case given by five functions $(R(\hat{p}), Q(\hat{p}), w^{+}(\hat{p}), w^{-}(\hat{p}), w^{0}(\hat{p}))$ where $1 - R(\hat{p})$ is the probability with which the safe project D is chosen if the firm announces to be of type $\hat{p}$ and where $(Q(\hat{p}), w^{+}(\hat{p}), w^{-}(\hat{p}), w^{0}(\hat{p}))$ determines, with the same interpretations as for the original model, what happens if D is not chosen. To connect the original model with the extended model, we need the following *Independence of Irrelevant Alternatives* axiom: *If the mechanism μ of the original model is not a neutral bargaining solution of the extended model, then it is also not a neutral bargaining solution of the original model.*

Next we have to consider a sequence of extensions for each integer $k = 1, 2, 3, \ldots$. The extensions in the sequence differ only with respect to the type-dependent payoffs $u_f^k(D|p)$ which the firm gets from the safe project D. We can assume that these payoffs converge to a limit $\lim_{k\to\infty} u_f^k(D|p) = u_f(D|p)$. To connect the solutions in the sequence of models with the solution for the limit model we impose the following *continuity axiom*: *Suppose that μ_α is an interim efficient mechanism of the limit model and that the k-th model in the sequence has the neutral bargaining solution μ_α^k for the bargaining power α. Then, if*

$$\lim_{k\to\infty} u_f\left(\mu_\alpha^k|p\right) \leq u_f(\mu_\alpha|p) \text{ holds for all } 0 \leq p \leq 1,$$

μ_α is a weighted neutral bargaining solution for the bargaining power α of the limit model.[9]

The natural way to find the neutral bargaining solution for $\alpha = 0$ is to approximate a solution candidate "from below" by a sequence of strong solutions in extended models. Only very few mechanisms can be approximated in this way, which is why Myerson's approach selects very sharply among the mechanisms.

It may be instructive to see in the simplest setting why the mechanism μ which distributes the surplus equally among all types cannot be approximated from below. Suppose there are only two types. For the low-risk type $t = 0$ the accident probability is p_0 is zero, for the high-risk type $t = 1$ it is strictly positive. Assume that the joint liability exceeds the net value of the project $(c > v)$ but that it is still worthwhile for the high-risk type to produce $(v - p_1 c > 0)$. Suppose the mechanism μ described above where both types get $S(c) = q_0 v + q_1(v - p_1 c)$ could be approached by strong solutions from below. Suppose first that $\lim_{k\to\infty} u_f^k(D|p_1) > (1 - p_1) S(c)$. Then the following mechanism would Pareto-dominate D in all extended models for large k. If the firm announces to be of the low-risk type, the original project is run. The firm receives $S(c) + \varepsilon$ if no accident occurs and zero otherwise. If she announces to be of the high risk type, the safe project D is selected. Hereby $\varepsilon > 0$ is chosen such that $u_f^k(D|p_1) > (1 - p_1)(S(c) + \varepsilon)$ for large k. This mechanism is feasible in the extended models

[9] Myerson (1984) imposes an additional probability invariance axiom which we do not need here and which I therefore skip.

and gives the low-risk type $S(c) + \varepsilon$ for arbitrarily large k. This contradicts the assumption that D is interim efficient and $\lim_{k\to\infty} u_f^k(D|p_0) \leq S(c)$.

So we must have $\lim_{k\to\infty} u_f^k(D|p_1) \leq (1-p_1)S(c)$. However, then we can consider a mechanism where the original project is always run and where both types get $S(c) + \varepsilon$, with $\varepsilon > 0$ sufficiently small, if no accident occurs and zero otherwise. We have $\lim_{k\to\infty} u_f^k(D|p_0) < S(c) + \varepsilon$ and $\lim_{k\to\infty} u_f^k(D|p_1) < (1-p_1)(S(c)+\varepsilon)$ for large k. Moreover, $(q_0 + q_1(1-p_1))(S(c)+\varepsilon) < S(c)$ for small $\varepsilon > 0$. Thus we obtain the contradiction that for k large D is not interim efficient for the firm.

3.4 Characterization of the Neutral Bargaining Solution for $c > v$.

To find, first for $\alpha = 0$, the neutral bargaining solution of the original model we want to choose the payoffs $u_f^k(D|p)$ such that D is a strong solution and such that

$$\omega(p) = \lim_{k\to\infty} u_f^k(D|p) \leq u_f(\mu|p)$$

holds for a mechanism μ of the original model. μ is then the neutral bargaining solution if the firm has all the bargaining power. The numbers $\omega(p)$ are called *warranted claims.*

Since types are ordered by the accident probability in our model it is not surprising that we can determine the warranted claims jointly with the neutral bargaining solution in an inductive procedure starting with the highest risk type.

To do this, recall that we have only finitely many types $p_0,\ldots,p_T$ in our model. For any type $0 \leq t \leq T$ we call the following model the t-bargaining problem. The model differs from the given one only in the prior over the types. The new prior (q_τ^t) is the posterior to which the lender would update if he learned that the type of the firm is t or higher. Thus

$$q_\tau^t = \begin{cases} 0 & \text{for} \quad 0 \leq \tau < t \\ \frac{q_\tau}{q_t+\ldots+q_T} & \text{for} \quad t \leq \tau \leq T \end{cases}$$

Theorem 5 *The neutral bargaining solutions $\mu^{0,t}$ in the t-bargaining problems when the firm has all the bargaining power and the warranted claims $\omega(p_t)$ are inductively defined for $t = T, T-1, \ldots, 0$ as follows:*

$\mu^{0,t}$ is the feasible mechanism in the t-bargaining problem that maximizes type p_t's expected payoff among all feasible mechanisms that give all types $\tau > t$ at least their warranted claims $\omega(p_\tau)$ as expected payoff. Type t's warranted claim is his expected payoff in the mechanism $\mu^{0,t}$.

In particular, $\omega(p_T) = (v - p_T c)^+$.

Notice that the theorem implies Proposition 2 for the case $v > c$. For $c > v$ we obtain the following result. Recall that a type t' with $t' > t$ can guarantee himself in any incentive compatible mechanism a fraction $\frac{1-p_{t'}}{1-p_t}$ of the expected payoff of type t.

Proposition 6 *Suppose $v < c$. Then the warranted claims satisfy $\omega(p_t) = 0$ for all t with $v - p_t c \leq 0$. For all $t < t'$ with $v - p_t c > 0$ and $p_{t'} < 1$ we have.*

$$\omega(p_t) > \frac{1-p_t}{1-p_{t'}} \omega(p_{t'})$$

Therefore the neutral bargaining solution when the firm has all the bargaining power is the mechanism that maximizes the expected payoff of type $t = 0$ among all feasible mechanisms.

We give next a more explicit description of this mechanism. This mechanism is basically an option contract with three options. The first option is chosen by low-risk types $t < t_1$, the second by medium-risk types $t_1 \leq t < t_2$ and the third by high-risk types $t_2 \leq t$. Hereby t_1 and t_2 are chosen as follows. If no t with $v - p_t c \leq 0$ exists (so the project yields a gain for all types), set $t_1 = t_2 = T + 1$. Otherwise, let t_1 be the smallest index with $v - p_t c \leq 0$ and define t_2 as the smallest integer for which

$$q_t (v - p_t c) + (p_{t+1} - p_t)(v + X) \sum_{\tau=t+1}^{T} q_\tau \leq 0 \tag{4}$$

Clearly, $t_2 \geq t_1$. Next, we determine a "wage" w^+ to be paid if the project is run without accident. Let

$$p_t^* = \begin{cases} p_t & \text{for} \quad t_2 \leq t_2 \\ p_{t_2} & \text{else} \end{cases}$$

and

$$r_t = \begin{cases} q_t \frac{p_t(c-v)+(1-p_t)X}{(1-p_t)(v+X)} & \text{for} \quad t_1 \leq t < t_2 \\ q_t & \text{else} \end{cases} .$$

Let

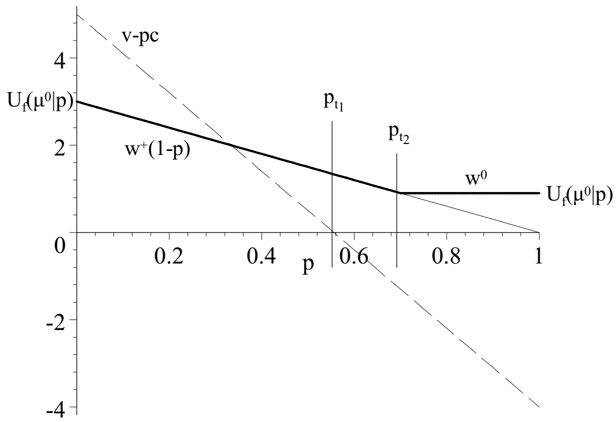
$$w^+ = \frac{\sum_{t=0}^{T} q_t (v - p_t c)^+}{\sum_{t=0}^{T} r_t (1 - p_t^*)}$$

and let $w^0 = (1 - p_{t_2}) w^+$

Theorem 7 *The neutral bargaining solution μ^0 when the firm has all the bargaining power is for $c > v$ the following mechanism $(Q(p), w^0(p), w^+(p), w^-(p))$:*

1. *$Q(p_t) = 1$, $w^+(p_t) = w^+$, $w^-(p_t) = 0$ for $0 \leq t < t_1$.*
2. *$Q(p_t) = \frac{w^+}{v+X}$, $w^+(p_t) = v + X$, $w^-(p_t) = w^0(p_t) = 0$ for $t_1 \leq t < t_2$.*
3. *$Q(p_t) = 0$, $w^0(p_t) = w^0$ for $t_2 \leq t \leq T$.*

The following figure shows the expected gain $U_f(\mu^0, p)$ of each type of the firm in this mechanism. The graph of this function is piecewise linear with a kink at $p = p_{t_2}$, it is downward sloping to the left and constant to the right of the kink.



The mechanism is an option contract with the following three options:

- One option is to run the project with certainty and to receive the net payment $w^+ > 0$ when no accident occurs and zero otherwise. This is the option chosen by all low-risk types $t < t_1$ for whom the project yields a gain in expectation $(v - pc \geq 0)$. Because this option is always available to the firm, each type p of the firm must gain at least $w^+(1-p)$.

- Another option is not to run a project and to receive a compensation. This is the option chosen by the high-risk types with accident probability $p \geq p_{t_2}$. For these types running the project would yield a loss $(v - pc < 0)$. If the compensation w^0 would not be offered in exchange for not running the project, these types would choose to run the project negligently since they have no own money at stake and can hence only win. The compensation is chosen such that type t_2 is indifferent between running the project and taking the compensation. Since the size of the compensation cannot be conditioned on the accident probability, the graph of $U_f(\mu^0, p)$ is flat for $p \geq p_{t_2}$.

- The types $t_1 < t < t_2$ are of "medium risk". For them the project is run with a small probability $Q = \frac{w^+}{v+X}$, in which case they receive all available wealth $v + X$. Running the project for this type brings a loss $(v - pc < 0)$ and therefore the project is run only with a small probability. If it is run and no accident occurs, the firm wins the maximal available prize $v+X$. This lottery is chosen such that the medium-risk types are indifferent between accepting the contract with the lottery and accepting the contract for the low-risk types. With both contracts they would gain $(1-p)\,w^{+}$ in expectation. In equilibrium they select the one with the lower accident probability.

The medium types are not "bribed out of business" because they require a higher compensation than the high-risk types. Since any compensation given to them can also be taken up by the high-risk types, it is, from the perspective of the low-risk types, cheaper to keep them working than to bribe them out of business. Even better is the described lottery.

When determining the solution, the interesting part is to determine t_2, i.e. to find out which types who run a loss ($v - pc < 0$) should play the "lottery" and which ones should be bribed out of business. Thanks to the monotone hazard rate condition (1), this critical number t_2 is determined by the inequality (4).

Example 1 Although our analysis is restricted to the case of finitely many types, we can, for instance, approximate the uniform distribution of types by setting $p_t = \frac{t}{T}$ and $q_t = \frac{1}{T+1}$ $(t = 0, \cdots, T)$ and then take the limit $T \to \infty$. In the limit p_{t_1} converges to v/c, p_{t_2} to $\frac{2v+X}{v+c+X}$ and w^{+} to

$$\frac{v^2\,(v+c+X)}{v^2+c^2+c\,(c-v)+cX}$$

Since $\frac{2v+X}{v+c+X} > \frac{v}{c}$ there is in the limit for $c > v$ *always* an interval of medium-risk types for whom the project is run with positive probability.

We have now described the neutral bargaining solutions μ_0 when the firm has all the bargaining power and μ_1 when the lender has all the bargaining power. It is not difficult to show that the mechanism $\mu_\alpha := (1-\alpha)\,\mu_0 + \alpha\mu_1$, where nature chooses with probabilities $1-\alpha$ and α between the two mechanisms μ_0 and μ_1, is interim efficient. The random dictatorship axiom hence implies:

Proposition 8 *The neutral bargaining solution when the lender has bargaining power* $0 \leq \alpha \leq 1$ *is the mechanism* μ_α *just described.*

3.5 Comparison with Other Approaches

3.5.1 Bargaining before the private information is received

Does the firm learn the accident probability before or after she bargains with the lender? Both scenarios are meaningful in our model, although the neutral bargaining solution seems to be tailored for bargaining problems where the firm already has her private information.

Let us again consider the case where the firm has all the bargaining power and let us suppose that she can make a take-it-or-leave-it offer before she learns her type. She is restricted, however to propose interim efficient contracts. Ex ante all the firm cares about is her expected payoff averaged over all her possible types. The maximal payoff she can gain is $S(c) = \sum_{t=0}^{T} q_t (v - p_t c)^+$ and this maximum is, for instance, achieved with the mechanism μ where $S(c)$ is distributed equally among all types. It is hence an equilibrium if the firm proposes this mechanism and the lender accepts. Any other ex-ante efficient feasible mechanism gives an equilibrium as well. The neutral bargaining solution is, however, not ex-ante efficient whenever there exist "medium-risk" types who are not "bribed our of business" and for whom the project yields a loss ($v - pc < 0$). Thus the results from bargaining ex-ante seem to conflict with the neutral bargaining solution.

However, this argument does not take account of the possibility of renegotiation. Suppose ex-ante bargaining has led to the mechanism where the surplus is shared equally between all types. Now the firm gets her private information and learns that her project has a very low accident probability. She could then address the lender and propose the neutral bargaining solution μ_0. Since middle- and high-risk types can only lose if this proposal gets accepted, the lender should infer that he is facing a low-risk type firm and accept the offer. (After having excluded the possibility that high-risk types would make such an offer, he would actually expect to win.)

Thus the simple bargaining model where a party makes an offer ex-ante before learning her type can give interesting insights (see Laffont and Martimort (2002) for examples), but it is problematic if renegotiation could lead to a mechanism that is not ex-ante efficient. Once renegotiation is taken into account, the simplicity of the ex-ante approach is lost and it may be easier to analyze the interim bargaining problem, after the type has been revealed.

3.5.2 Perfect Bayesian Equilibria

Maskin and Tirole (1990) and Maskin and Tirole (1992) characterize perfect Bayesian equilibria in the three-stage game where an informed principal can make a take-it-or leave-it offer for the (direct) mechanism to be selected. Their analysis is based on the Rothschild-Stiglitz-Wilson allocation (short: RSW allocation). As remarked above, their approach also selects the strong solution as the unique perfect Bayesian equilibrium outcome for $c < v$. However, when the joint liability c exceeds the net value v of the project, results for the RSW allocation relative to the "no deal" allocation, where the lender gets zero for every type, are somewhat disappointing.[10] In particular, when there are types with accident probability less than 1 for whom the project does not yield a gain ($v - pc < 0$), the RSW allocation gives zero to each type of the firm. This is so because in any feasible mechanism where one type of the firm makes a positive gain, the types for whom the project yields a loss must also gain a positive amount because they can imitate. In any

[10] Further insights, though, might be obtained from studying the renegotiation game discussed in Maskin and Tirole (1992) and RSW allocations relative to other intial allocations.

such mechanism the lender must hence make a loss on the highest-risk type. Since the lender must get at least zero in the RSW allocation conditional on each type, each type of the firm must gain zero. The results of Maskin and Tirole (1992) imply that *every feasible* mechanism of the model yields a perfect Bayesian equilibrium in the three-stage game. The RSW allocation is interim efficient for the firm and it is the unique perfect Bayesian equilibrium of the three-stage if and only if the prior puts positive probability only on types of the firm for whom the project generates a loss. The equilibrium is then, of course, the one where trade never occurs ($Q(p) = 0$ and $w^0(p) = 0$ for all p).

When there are only types for whom the project yields a gain, the results have a little more structure. In the RSW allocation the lender cannot make a loss on the highest type T, so the highest risk type cannot gain more than $v - p_T c$. Lemma 1 c) implies that the RSW allocation yields $\frac{1-p_t}{1-p_T}(v - p_T c)$ for type t. The following simple calculation shows that the RSW allocation is interim efficient (and so the three-stage game has a unique perfect Bayesian equilibrium) if and only if there is only a single type and hence no problem of incomplete information.

$$\sum_{t=0}^{T} q_t \frac{1-p_t}{1-p_T}(v - p_T c) \leq \sum_{t=0}^{T} q_t (v - p_t c) \Leftrightarrow \sum_{t=0}^{T} \frac{q_t}{1-p_t}(p_T - p_t)(c - v) \leq 0$$

whereby equality occurs if and only if $p_t = p_T$ for all t and hence $T = 0$.

If there is more than one type, every feasible mechanism which Pareto-dominates for all types of the firms the RSW allocation yields an equilibrium outcome for the three-stage game. Again, the selection is not sufficiently strong to allow for an insightful comparative statics analysis.

3.5.3 Refinements for Signalling Games

The three-stage game just discussed is a signalling game. The findings in this subsection suggest that there is indeed a connection between Myerson's weighted bargaining solution and the refinement concepts for signalling games discussed in the literature. We need, however, a fairly strong refinement criterion, namely the FGP criterion based on Farrell (1985) and Grossman and Perry (1986) as discussed in Maskin and Tirole (1992), Section 5B. The relation to other refinement criteria needs further research.

The consistency of the neutral bargaining solution μ_0 with the FGB criterion follows in the case $v \leq c$ from Proposition 7 in Maskin and Tirole (1992). The explicit description of the neutral bargaining solution for $c > v$ implies:

Lemma 9 *Let μ be any interim efficient mechanism for the firm different from μ_0, the neutral bargaining solution if the lender has no bargaining power. Then there exists a type $0 < t \leq T$ with $v - p_t c > 0$ such that $U_f(\mu|p_\tau) \leq U_f(\mu_0|p_\tau)$ for all $\tau < t$ and $U_f(\mu|p_\tau) > U_f(\mu_0|p_\tau)$ for all $\tau \geq t$.*

As a straightforward consequence of the definitions and the lemma we obtain.

Proposition 10 *Consider the three stage game, where first the firm can propose a mechanism which is interim efficient for the firm, the lender can then accept or reject and thereafter the proposed mechanism, if accepted, is implemented. Then the neutral bargaining solution μ_0 when the firm has all the bargaining power is the only pure strategy perfect Bayesian equilibrium outcome of this game consistent with the FGP criterion.*

Proof. We show first that μ_0 defines such an equilibrium. This equilibrium is as follows. All types of the firm propose μ_0. The lender accepts this mechanism. If the firm would deviate and propose a different interim efficient mechanism μ, the lender would update his believes by assuming that all types who would strictly gain by having μ rather that μ_0 accepted have an equal likelihood to have deviated. Let t be the type determined for μ and μ_0 in the previous lemma. The lender's posterior is then given by $\{q_\tau^t\}$ from the t-bargaining problem. This belief is consistent with the FGP criterion. Since μ is interim efficient for the firm, the lender expects to break even when μ is played given his initial prior. He expects to make a loss from μ given his posterior. (The types $\tau < t$ are the most productive and they gain even less in μ than in μ_0. Since his posterior rules out these types, he must expect to make a loss.) Hence it is optimal for him to reject. Thus the described behavior describes a perfect Bayesian equilibrium consistent with the FGP criterion.

Consider now a perfect Bayesian equilibrium consistent with the FGP criterion where an interim efficient mechanism μ is offered. Suppose the firm deviates and offers μ_0 instead. The FGP criterion then requires the lender to update his belief by assuming that the deviation must have come with probability zero from the types who gain less in μ_0 than in μ, with equal probability from the types who gain more in μ_0 than in μ and with an equal or possibly lower probability from types who are indifferent. Since, by the lemma, he is eliminating only high risk types from his belief by updating, he expects to strictly gain in expectation by accepting μ_0. The equilibrium must, by the FGP criterion, hence be such that he accepts μ_0. However, then we cannot have an equilibrium because the lowest-risk type would always deviate and propose μ_0. This is a contradiction. ■

One can similarly "justify" the neutral bargaining solution for the bargaining power α by assuming that the firm must offer an interim efficient mechanism that gives the lender at least the fraction α of the maximally available surplus.

4 The Optimal Liability

We can now present our main result concerning the optimal joint liability in the pure adverse selection case studied here.

Theorem 11 *Either a full or a punitive joint liability maximizes social welfare. In particular, the deep pockets of the lender have to be employed to achieve the social optimum.*

Proof. The neutral bargaining solution is described in Theorems 7 and 8. Assume that initially the full liability $c = h > v$ is imposed and then gets reduced to a liability $\tilde{c} \geq v$. (The case $\tilde{c} < v$ is obvious.) Initially, the project is run with certainty only if it is socially worthwhile, i.e., when $v - ph \geq 0$ Let us write $t_1(c)$, $t_2(c)$, and $w^+(c)$ to indicate the dependence of these numbers on c. It is immediate that $t_1(c)$ and $t_2(c)$ are weakly increasing in c. We must have $w^+(\tilde{c}) > w^+(h)$ because, by Theorem 6, $w^+(\tilde{c})$ is the solution to a less constrained optimization problem than $w^+(h)$ (i.e., the same objective function is maximized over a smaller set). For all types t with $v - p_t c \geq 0$ the project is run with certainty both at the liabilities h and $\tilde{c}$, for types $t_2(\tilde{c}) \leq t$ it is never run. For all other types the probability of running the project increases from 0 to $(1-\alpha)\frac{w^+(\tilde{c})}{v+X}$ or 1 or from $(1-\alpha)\frac{w^+(h)}{v+X}$ to $(1-\alpha)\frac{w^+(\tilde{c})}{v+X}$ or 1.Thus social welfare cannot increase and hence either a full or a joint liability must be optimal. ■

Whether the optimal joint liability is strictly punitive or just equal to the damage costs is harder to say in general. If $t_1(h) = t_2(h)$ then full liability is optimal because it yields first best since $\sum_{t=0}^{T} q_t (v - p_t h)^+$ is the maximal social welfare. This is, for instance, always the case when there are only two types. Due to the discreetness of the type space we can, however, not deduce that the optimal liability is unique. The range of optimal liabilities may contain values above and below the damage costs, at least one value cannot be below.

To avoid these integer problems, let us look at the uniform distribution of types as a limit, as described in Example 1.[11] For this example we are going to show that a) the optimal liability is strictly punitive unless the lender has all the bargaining power and b) the optimal liability is decreasing and social welfare increasing in the bargaining power of the lender, at least as long as the optimal liability does not jump discontinuously.

We write $P_1 = v/c$, for the limit of p_{t_1} and $P_2 = \frac{2v+X}{v+c+X}$ for the limit of p_{t_2}. $P_1(c)$ and $P_2(c)$ are strictly increasing since $\frac{\partial P_1}{\partial c} = -\frac{v}{c^2} < 0$ and $\frac{\partial P_2}{\partial c} = -\frac{2v+X}{(v+c+X)} < 0$. We have $w^+ = \frac{v^2(v+c+X)}{v^2+c^2+c(c-v)+cX}$ and hence $\frac{\partial w^+}{\partial c} = -v^2\frac{2c^2+4cv-2v^2+4cX+X^2}{(v^2+2c^2-cv+cX)^2} < 0$.

When, for $c > v$, the weighted neutral bargaining solution for the bargaining power α of the lender is played, utilitarian social welfare is

$$SW = \int_0^{P_1} (v - ph)\, dp + (1-\alpha)\frac{w^+}{v+X}\int_{P_1}^{P_2} (v - ph)\, dp$$

[11] An older script, available from my WEB side, gives a detailed analysis for the case of three types.

Leibniz' rule yields

$$\begin{aligned}\frac{\partial SW}{\partial c} &= \frac{\partial P_1}{\partial c}(v - P_1 h) + (1-\alpha)\left[\frac{\partial w^+}{\partial c}\frac{1}{v+X}\int_{P_1}^{P_2}(v - ph)\,dp\right. \\ &\quad \left. + \frac{w^+}{v+X}\left(\frac{\partial P_2}{\partial c}(v - P_2 h) - \frac{\partial P_1}{\partial c}(v - P_1 h)\right)\right]\end{aligned}$$

and substitution shows $\frac{\partial SW}{\partial c}_{|c=h} > 0$ as long as $\alpha < 1$. Since the proof of the previous proposition still applies here, it follows that *the optimal liability must be strictly punitive* unless the lender has all the bargaining power. Since $h > v$, it is always optimal to employ the deep pockets of the lender.

I have not yet been able to show that the social welfare function is quasi-concave. Therefore the following argument is correct only as long as there is no discontinuous jump in the optimal joint liability c (and $\frac{\partial^2 SW}{\partial c^2} < 0$ holds at the optimum).

Let $c^* > h$ denote the optimal liability given as a solution to the first-order condition $\frac{\partial SW}{\partial c} = 0$. $\frac{\partial SW}{\partial c}$ takes the form $A + (1-\alpha)\,B$ where A and B do not depend on α. At the optimum A is positive since $c^* > h$ and so $B < 0$ for $\alpha < 1$ from the first-order condition. Since $\frac{\partial^2 SW}{\partial\alpha\partial c} = -B > 0$ the envelope theorem implies that *social welfare is* (at least locally) *increasing in α*. Since $\frac{\partial^2 SW}{\partial c^2} < 0$ at the optimum we obtain then

$$\frac{\partial c^*}{\partial \alpha} = -\frac{\partial^2 SW}{\partial\alpha\partial c}\Big/\frac{\partial^2 SW}{\partial c^2} < 0,$$

i.e., *the optimal joint liability is decreasing in the bargaining power α of the lender.*

5 Conclusion

We have been able to determine the weighted neutral bargaining solution for an adverse selection model where lenders lend to wealth-constrained firms who could cause a severe environmental accident. We have seen how adverse selection can create a distortion away from first-best because in the contract selected by the lender and the firm the potentially hazardous project is undertaken at a loss for medium-risk types of the firm in order to avoid the large compensation payments needed to deter these types from running the project. Overall, the comparative statics of the model considered here is simpler, but in sharp contrast to the comparative statics in Balkenborg (2001) for the moral hazard problem.

The analysis has a number of limitations. First of all, it is still a conjecture that the weighted neutral bargaining solution is unique for this model. Secondly, the conclusions would not be as extreme if the firm had some small amount of own capital to lose from running the project. This more realistic assumption was avoided here to keep the analysis simple. Thirdly, our analysis is made for a bilateral monopoly and it is not obvious how it extends to the case of several lenders and firms in competition.

A Proofs

For $c \geq v$ we apply Theorem 4 of Myerson (1984) to verify that we have indeed found the neutral bargaining solutions. We will not address the question of uniqueness of the neutral bargaining solution.

Because all types of each player are risk neutral, every mechanism $\mu = (Q(p), w^+(p), w^-(p), w^0(p))_{0 \leq p \leq 1}$ is payoff equivalent to a lottery $(\mu(d_i|p_t))_{\substack{1 \leq i \leq 5 \\ 1 \leq t \leq T}}$ over the following six mechanisms.

d_0: The project is not run and everyone gets zero. $u_f(d_0|p_t) = u_l(d_0|p_t) = 0$;
d_1: The project is not run and the firm receives all available cash. $u_f(d_1|p_t) = X$, $u_l(d_1|p_t) = -X$;
d_2: The project is run and the lender gets all available wealth. $u_f(d_2|p_t) = 0$, $u_l(d_2|p_t) = v - p_t c$;
d_3: The project is run and the firm gets all available wealth if no accident occurs but nothing otherwise. $u_f(d_3|p_t) = (1-p_t)(v+X)$, $u_l(d_2|p_t) = p_t(v-c) - (1-p_t)X$;
d_4: The project is run and the firm gets all available wealth if an accident occurs but nothing otherwise. $u_f(d_3|p_t) = p_t(v+X-c)$, $u_l(d_2|p_t) = -p_t X + (1-p_t)v$;
d_5: The project is run and the firm gets all available wealth. $u_f(d_5|p_t) = v + X - p_t c$, $u_l(d_2|p_t) = -X$.

With some abuse of notation we write $\mu = (\mu(d_i|p_t))_{\substack{0 \leq i \leq 5 \\ 1 \leq t \leq T}}$.

For $k = 1, 2, \cdots$ let $\left(\lambda_t^k\right)_{0 \leq t \leq T}$ be a strictly positive vector of weights with $\lambda_t^k \to 0$ for $k \to \infty$. Moreover, we assume $\lambda_t^k < r_t$ with r_t as in defined subsection 3.4. A mechanism μ that maximizes for given k

$$\left(\lambda_0^k + \alpha_0^k\right) U_f(\mu|p_0) + \sum_{t=1}^{T} \lambda_t^k U_f(\mu|p_t) + U_l(\mu) \tag{5}$$

with given $\alpha_0^k \geq 0$ among all individually rational and incentive compatible mechanisms is interim efficient. The Lagrangian for this linear programming problem can be written as

$$\begin{aligned} & \left(\lambda_0^k + \alpha_0^k\right) U_f(\mu|p_0) + \sum_{t=1}^{T} \lambda_t^k U_f(\mu|p_t) + U_l(\mu) \\ & + \sum_{t=1}^{T} \alpha_t^k \left(U_f(\mu|p_t) - U_f^*(\mu, p_{t-1}|p_t)\right) + \sum_{t=1}^{T} \beta_t^k \left(U_f(\mu|p_{t-1}) - U_f^*(\mu, p_t|p_{t-1})\right) \\ = & \sum_{t=0}^{T} sv(\mu|p_t) = \sum_{t=0}^{T} \sum_{i=0}^{5} \mu(d_i|p_t)\, sv(d_i|p_t) \end{aligned}$$

whereby

$$\begin{aligned}
sv(\mu|p_t) &= \left(\lambda_t^k+\alpha_t^k+\beta_{t+1}^k\right)U_f(\mu|p_t)+q_tU_l(\mu|p_t)\\
&\quad-\alpha_{t+1}^kU_f^*(\mu,p_t|p_{t+1})-\beta_t^kU_f^*(\mu,p_t|p_{t-1})\\
&= \left(\lambda_t^k+\alpha_t^k+\beta_{t+1}^k-\alpha_{t+1}^k-\beta_t^k-q_t\right)U_f(\mu|p_t)+q_tQ(p_t)(v-p_tc)\\
&\quad+\left(\alpha_{t+1}^k(p_{t+1}-p_t)-\beta_t^k(p_t-p_{t-1})\right)Q(p_t)\left(w^+(p_t)-w^-(p_t)\right)
\end{aligned}$$

provided the only binding constraints are the incentive constraints requiring that type $t+1$ cannot gain from imitating type t and vice versa. Hereby we use the convention $\alpha_{T+1}^k=\beta_1=0$ and obtain from the definitions[12]

$$\begin{aligned}
sv(d_0|p_t) &= 0\\
sv(d_1|p_t) &= \left(\lambda_t^k+\alpha_t^k+\beta_{t+1}^k-\alpha_t^{k+1}-\beta_t^k-q_t\right)X\\
sv(d_2|p_t) &= q_t(v-p_tc)\\
sv(d_3|p_t) &= \left(\lambda_t^k+\alpha_t^k+\beta_{t+1}^k-\alpha_{t+1}^k-\beta_t^k-q_t\right)(1-p_t)(v+X)+q_t(v-p_tc)\\
&\quad+\left(\alpha_{t+1}^k(p_{t+1}-p_t)-\beta_t^k(p_t-p_{t-1})\right)(v+X)\\
&= \Big(\left(\lambda_t^k-q_t\right)(1-p_t)+\alpha_t^k(1-p_t)+\beta_{t+1}^k(1-p_t)\\
&\quad-\alpha_{t+1}^k(1-p_{t+1})-\beta_t^k\left(1-p_{t-1}^k\right)(v+X)\Big)+q_t(v-p_tc)\\
sv(d_4|p_t) &= \left(\lambda_t^k+\alpha_t^k+\beta_{t+1}^k-\alpha_{t+1}^k-\beta_t^k-q_t\right)p_t(v+X-c)+q_t(v-p_tc)\\
&\quad+\left(\alpha_{t+1}^k(p_{t+1}-p_t)-\beta_t^k(p_t-p_{t-1})\right)(v+X-c)\\
sv(d_5|p_t) &= \left(\lambda_t^k+\alpha_t^k+\beta_{t+1}^k-\alpha_{t+1}^k-\beta_t^k-q_t\right)(v+X-p_tc)+q_t(v-p_tc)\\
&\quad+\left(\alpha_{t+1}^k(p_{t+1}-p_t)-\beta_t^k(p_t-p_{t-1})\right)p_tc
\end{aligned}$$

The dual problem is the problem of minimizing for given $\left(\lambda_t^k\right)$ and α_0^k the sum

$$\sum_{t=0}^{T}\max_{0\le i\le 5} sv(d_i|p_t)$$

with respect to $\alpha_1^k,\cdots,\alpha_T^k$, $\beta_1^k,\cdots,\beta_T^k\ge 0$.

We choose inductively for $t=T,T-1,\dots$

$$\begin{aligned}
\alpha_t^k &= \sum_{\tau=t}^{T}\frac{1-p_\tau^*}{1-p_t^*}\left(r_\tau-\lambda_\tau^k\right)+\beta_t^k\frac{1-p_{t-1}^*}{1-p_t^*}\\
\beta_t^k &= \begin{cases}\frac{1-p_t^*}{p_t^*-p_{t-1}^*}\left[r_t-q_t-\frac{p_{t+1}^*-p_t^*}{1-p_t^*}\alpha_{t+1}^k\right]^+ & \text{for}\quad t_1\le t<t_2\\ 0 & \text{else}\end{cases}
\end{aligned}$$

One verifies immediately (using $p_{t_2}=0$) the recursive formula

$$\alpha_t^k+\beta_{t+1}^k=r_t-\lambda_t+\alpha_{t+1}^k\frac{1-p_{t+1}^*}{1-p_t^*}+\beta_t^k\frac{1-p_{t-1}^*}{1-p_t^*}$$

[12]In the notation we suppress the dependency of $sv(d_i|p_t)$ on α_t^k and λ_t^k

Lemma 12 *With these choices we have*

$$q_t\left(v-p_tc\right)^+=\max_{0\le i\le 5} sv\left(d_i|p_t\right)=\begin{cases} sv\left(d_2|p_t\right) &=& sv\left(d_3|p_t\right) & \textit{for} \quad 0\le t<t_1 \\ sv\left(d_0|p_t\right) &=& sv\left(d_3|p_t\right) & \textit{for} \quad t_1\le t<t_2 \\ sv\left(d_0|p_t\right) &=& sv\left(d_1|p_t\right) & \textit{for} \quad t_2\le t\le T \end{cases}$$

Proof. We leave it to the reader to check that $\max_{4\le i\le 5} sv\left(d_i|p_t\right)\le \max_{0\le i\le 3} sv\left(d_i|p_t\right)$. We calculate $sv\left(d_1|p_t\right)$ and $sv\left(d_3|p_t\right)$ using the recursive formula for α_t^k.

Suppose $0\le t<t_1$. Then $\beta_t^k=0$. We obtain $sv\left(d_1|p_t\right)<0$ and $sv\left(d_3|p_t\right)=v-p_tc$ from the recursion formula, $r_t=q_t$ and the second expression for $sv\left(d_3|p_t\right)$ above.

Suppose $t_1\le t<t_2$. Then $\alpha_t^k+\beta_{t+1}^k=q_t\left(1-\frac{q_t(v-p_tc)}{(1-p_t)(v+X)}\right)-\lambda_t^k+\frac{1-p_{t+1}}{1-p_t}\alpha_{t+1}^k+\beta_t^k\frac{1-p_{t-1}^*}{1-p_t^*}$ and therefore $sv\left(d_3|p_t\right)=0$ whereas

$$sv\left(d_1|p_t\right)=\left(r_t-q_t+\frac{1-p_{t+1}}{1-p_t}\alpha_{t+1}^k-\left[r_t-q_t-\frac{p_{t+1}^*-p_t^*}{1-p_t^*}\alpha_{t+1}^k\right]^+\right)X\le 0$$

Suppose $t_2\le t\le T$. Then $\alpha_t^k=q_t-\lambda_t^k+\alpha_{t+1}^k$ and so $sv\left(d_1|p_t\right)=0$ whereas

$$sv\left(d_3|p_t\right)=q_t\left(v-p_tc\right)+\left(p_{t+1}-p_t\right)\alpha_{t+1}^k\left(v+X\right)<0.$$

■

We can choose now $\omega_f^k\left(p_t\right)$ and ω_l^k as the solution to the simultaneous system of equations

$$\begin{aligned}\left(\lambda_t^k+\alpha_t^k+\beta_{t-1}^k\right)\omega_f^k\left(p_t\right)-\alpha_{t+1}^k\omega_f^k\left(p_{t+1}\right)-\beta_t^k\omega_f^k\left(p_{t-1}\right) &= (1-\alpha)\,q_t\left(v-p_tc\right)^+ \\ \omega_l^k &= \omega_l=\alpha\sum_{t=0}^{T}q_t\left(v-p_tc\right)^+\end{aligned}$$

As $k\to\infty$ we obtain the limits $\alpha_t^k\to\alpha_t=\sum_{\tau=t}^{T}\frac{1-p_\tau^*}{1-p_t^*}r_\tau+\beta_t\frac{1-p_{t-1}}{1-p_t}$,

$$\beta_t^k\to\beta_t=\begin{cases}\frac{1-p_t^*}{p_t^*-p_{t-1}^*}\left[r_t-q_t-\frac{p_{t+1}^*-p_t^*}{1-p_t^*}\alpha_{t+1}\right]^+ & \text{for} \quad t_1\le t<t_2 \\ 0 & \text{else}\end{cases}$$

and $\omega_f^k\left(p_t\right)\to\omega_f\left(p_t\right)$ whereby

$$\left(\alpha_t+\beta_{t+1}\right)\omega_f\left(p_t\right)=(1-\alpha)\,q_t\left(v-p_tc\right)^++\alpha_{t+1}\omega_f\left(p_{t+1}\right)-\beta_t\omega_f\left(p_{t-1}\right)$$

from which we obtain as a unique solution (since $(v - p_t c)^+ = 0$ whenever $\beta_t \neq 0$)

$$\alpha_t \omega_f (p_t) = (1 - \alpha) \sum_{\tau=t}^{T} q_\tau (v - p_\tau c)^+$$

In particular we have

$$\begin{aligned} \omega_l &= \alpha \sum_{t=0}^{T} q_t (v - p_t c)^+ = \alpha \sum_{t=0}^{T} \max_{0<i\leq 5} sv (d_i|p_t) = U_l (\mu_\alpha) \\ \omega_f (p_0) &= (1 - p_0) \frac{(1 - \alpha) \sum_{t=0}^{T} q_t (v - p_t c)^+}{\sum_{t=0}^{T} r_t (1 - p_t^*)} = U_f (\mu_\alpha|p_t) \end{aligned}$$

Since μ_α is interim efficient (see the argument below), it follows from Theorem 4 in Myerson (1984) that μ_α is a neutral bargaining solution if $\omega_f (p_t) \leq U_f (\mu_\alpha|p_t)$ holds for all t.

To verify this, notice that our notation corresponds to the one used in Myerson's theorem as follows. We use p_t instead of t_i to indicate types. For the lowest risk-type p_0 we write $\lambda_0^k + \alpha_0^k$ instead of $\lambda^k (p_0)$. For all other types p_t we write λ_t^k instead $\lambda^k (p_t)$. Our analysis assumes that $\alpha^k (p_t|p_{t'})$ is zero except when $t' = t + 1$ or $t' = t - 1$ and so we have write α_t^k instead of $\alpha^k (p_t|p_{t-1})$ and β_t instead of $\alpha^k (p_{t-1}|p_t)$. We write $sv (d|p_t)$ instead of $\sum_{j=f,l} V_j \left(d, p_t, \lambda^k, \alpha^k\right)$.

For all $t \geq t_1$ we have $\omega_f (p_t) = 0 < U_f (\mu^\alpha|p_t)$. Now suppose $t < t_1$. Then $U_f (\mu_\alpha|p_t) = \frac{1-p_t}{1-p_0} U_f (\mu^\alpha|p_0) = \frac{1-p_t}{1-p_0} \omega_f (p_0)$. Thus we must show

$$\omega_f (p_t) \leq \frac{(1 - p_t)}{(1 - p_0)} \omega_f (p_0)$$

which is verified as follows

$$
\begin{aligned}
& \omega_f(p_t) \leq \frac{(1-p_t)}{(1-p_0)} \omega_f(p_0) \\
\Leftrightarrow \quad & \frac{(1-\alpha) \sum_{\tau=t}^T q_\tau (v - p_\tau c)^+}{\alpha_t} \leq \frac{(1-p_t)}{(1-p_0)} \frac{(1-\alpha) \sum_{\tau=0}^T q_\tau (v - p_\tau c)^+}{\alpha_0} \\
\Leftrightarrow \quad & \frac{\sum_{\tau=t}^T q_\tau (v - p_\tau c)^+}{\sum_{\tau=t}^T r_\tau (1-p_\tau^*)} \leq \frac{\sum_{\tau=0}^T q_\tau (v - p_\tau c)^+}{\sum_{\tau=0}^T r_\tau (1-p_\tau^*)} \\
\Leftrightarrow \quad & \left(\sum_{\tau=0}^T r_\tau (1-p_\tau^*) \right) \left(\sum_{\tau=t}^T q_\tau (v - p_\tau c)^+ \right) \\
\leq \quad & \left(\sum_{\tau=t}^T r_\tau (1-p_\tau^*) \right) \left(\sum_{\tau=0}^T q_\tau (v - p_\tau c)^+ \right) \\
\Leftrightarrow \quad & \left(\sum_{\tau=0}^{t-1} r_\tau (1-p_\tau^*) \right) \left(\sum_{\tau=t}^T q_\tau (v - p_\tau c)^+ \right) \\
\leq \quad & \left(\sum_{\tau=t}^T r_\tau (1-p_\tau^*) \right) \left(\sum_{\tau=0}^{t-1} q_\tau (v - p_\tau c)^+ \right) \\
\Leftrightarrow \quad & 0 \leq \sum_{\tau=t}^T \sum_{\sigma=0}^{t-1} \left[r_\tau (1-p_\tau^*) q_\sigma (v - p_\sigma c)^+ - r_\sigma (1-p_\sigma^*) q_\tau (v - p_\tau c)^+ \right] \\
\Leftrightarrow \quad & 0 \leq \sum_{\tau=t_1+1}^T \sum_{\sigma=0}^{t-1} r_\sigma r_\tau \left[(1-p_\tau^*) (v - p_\sigma c)^+ \right] \\
& + \sum_{\tau=t}^{t_1-1} \sum_{\sigma=0}^{t-1} q_\sigma q_\tau \left[(1-p_\tau)(v - p_\sigma c) - (1-p_\sigma)(v - p_\tau c) \right] \\
\Leftarrow \quad & \sum_{\tau=t}^{t_1-1} \sum_{\sigma=0}^{t-1} q_\sigma q_\tau (p_\tau - p_\sigma)(c - v) \qquad \checkmark
\end{aligned}
$$

Except for the interim efficiency of μ_α we have now shown that μ_α is the weighted neutral bargaining solution.

For the remaining arguments of the paper one has to consider variants of the Lagrangian

$$
\begin{aligned}
& \alpha_0 U_f(\mu|p_0) + U_l(\mu) + \sum_{t=1}^T \alpha_t \left(U_f(\mu|p_t) - U_f^*(\mu, p_{t-1}|p_t) \right) \\
& + \sum_{t=1}^T \beta_t \left(U_f(\mu|p_{t-1}) - U_f^*(\mu, p_t|p_{t-1}) \right)
\end{aligned}
$$

and observe that the $\mu_\alpha(d_i|p_t)$ (for the relevant α) together with the Lagrangian multipliers α_t and β_t, as calculated above, form admissible solutions for a variety of maximization problems and their duals. By the duality theory for linear programming they are hence solutions to the respective optimization problems. In this way we can show, for instance, that μ_α maximizes $U_f(\mu|p_0)$ subject to $U_l(\mu) \geq \alpha S(c)$ and the incentive constraints. This completes together with the above lengthy calculation Proposition 6 and we obtain in turn Theorem 5. By fixing the values of $U_f(\mu|p_\tau)$ and $U_l(\mu|p_\tau)$, the dual variables α_τ etc. for all $\tau < t$ with a given t one can similarly show that μ_α maximizes the lenders expected payoff in the t-bargaining problem subject to the incentive constraints and the constraint that type t gets at least $U_f(\mu_\alpha|p_t)$. From this result it follows immediately that μ_α is interim efficient. This line of arguments can also be used to derive Lemma 9.

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